Boson propagators on the light front

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Contents

Apresentação

Este livro foi idealizado tendo em vista nosso propósito de ilustrar a descrição das partículas elementares e dos campos sob a ótica da forma frente de luz para a dinâmica das partículas, primeiramente introduzida por P.A.M.Dirac em 1949. Para tanto, valemo-nos da descrição particular dos bósons livres em interação e dos mesmos em estados ligados. Essa especificidade resulta da grande abrangência que o tema permite e impomo-nos à essa restrição por questões de ordem prática.

Nosso objetivo ao preparar este material foi fornecer aos alunos de pós-graduação em ciências exatas uma ferramenta da Teoria Quântica de Campos que tem muitos pontos fortes, mas ao mesmo tempo, que demanda cuidados especiais pelas sutilezas que apresenta, de modo a estimulá-los e a desafiá-los na vanguarda da pesquisa básica nesse setor. Paralelamente a isto, cremos que a abordagem que adotamos permite aos demais alunos uma introdução ao assunto suficientemente detalhada para prepará-lo a ingressar nessa área de pesquisa razão pela qual o texto originalmente foi escrito em inglês.

Os capítulos que compõem esse volume estão divididos em cinco partes distintas: No capítulo 1 introduzimos a nomenclatura, a identificação, a notação, as convenções e as leis de transformação do plano nulo e das coordenadas da frente de luz em relação ao espaço-tempo quadridimensional usual. Enfatizamos as peculiaridades desse plano nulo, suas vantagens e desvantagens como plataforma para a descrição das interações fundamentais da matéria. No capítulo 2 introduzimos a descrição dos campos de spin inteiro - os bósons - livres e interagentes via propagadores covariantes projetados na frente de luz. Sendo os propagadores os entes matemáticos básicos na construção da dinâmica de uma partícula no espaço-tempo, sua descrição na frente de luz será fundamental para a construção de grandezas físicas de interesse como correntes e equações dos estados ligados. No capítulo 3 tratamos então especificamente o caso de dois bósons livres e em interação até termos de quarta ordem perturbativa. No capítulo 4 estudamos o estado ligado utilizando todo o ferramental matemático desenvolvido nos capítulos anteriores e construímos a equação de Bethe-Salpeter na frente de luz. Finalmente, no capítulo 5 consideramos como essa técnica pode ser importante na investigação dos bósons de gauge que obedecem a uma simetria de gauge, seja abeliana ou não abeliana.

Este projeto é parte integrante de um projeto de pesquisa mais amplo que conta com o apoio financeiro das agencias de fomento à pesquisa do Estado de

São Paulo - FAPESP - e do Estado de Minas Gerais - FAPEMIG - além dos apoios parciais das Empresas de Tecnologia GT e FMC sediadas em Santa Rita-MG e da Fundação de Ensino e Pesquisa de Itajubá, mantenedora do Centro Universitário de Itajubá Universitas.

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Foreword

This book was born from our idealization having in view our purpose to illustrate the description of elementary particles and fields through the front form perspective for particle dynamics first introduced by P.A.M.Dirac in 1949. To this end we particularize our focus of attention into interacting free bosons and these in bound states. This specification is somewhat demanded by the sheer extension of topics allowed within the light front theme and we have chosen to restrict ourselves in this manner for practical purposes.

Our objective in preparing this material was to provide graduate students in sciences with a tool from Quantum Field Theory that has many strong points, but which at the same time, demands special care for the sutleties that it has, so as to stimulate and challenge them to pursue research in this line. Paralel to this, we believe that the approach we have adopted makes it possible to other students to have an introduction to the subject with sufficient details to prepare them to enter into this research area, the reason why we have originally written the manuscript in English.

The chapters that compose this volume are divided into five distinct sections. In chapter 1 we introduce the nomenclature, the identification, the notation, the conventions and the laws of transformation for null plane and light front coordinates in relation to the usual four-dimensional space-time. We emphasize some peculiarities of this null plane, its advantages and disadvantages as a tool to describe matter 's fundamental interactions. In chapter 2 we deal with spin 1 fields - the bosons - both free and interacting ones via covariant propagators projected onto the light front. Since propagators are the basic mathematical entities to build the dynamics of a particle in the space-time, its description in the light front is pivotal in the building of physical quantities of interest such as currents and bound state equaions. In chapter 3 we specifically treat the case of two free and interacting bosons up to the fourth perturbative order. In chapter 4 we study the bound state using all the mathematical tools developed in the previous chapters and construct the Bethe-Salpeter equation in the light front. Finally, in chapter 5 we consider how this technique can be of importance in the investigation of gauge bosons which obey gauge symmetry, be it Abelian or non Abelian.

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Summary

The scope and aim of this work is to describe the two-body interaction mediated by a particle (either the scalar or the gauge boson) within the light-front formulation. To do this, first of all we point out the importance of propagators and Green functions in Quantum Mechanics. Then we project the covariant quantum propagator onto the light front time to get the propagator for scalar particles in these coordinates. This operator propagates the wave function from $x^+ = 0$ to $x^+ > 0$. It corresponds to the definition of the time ordering operation in the light front time x^+. We calculate the light-front Green's function for 2 interacting bosons propagating forward in x^+. We also show how to write down the light front Green's function from the Feynman propagator and finally make a generalization to N bosons. The same technique and procedure we will use to obtain the light front gauge boson propagator for gauge fields.

We are going to employ the technique developed for N scalar and gauge bosons to study the Green's function of two bodies in the ladder approximation for the dynamics defined on the light-front. In this context, we shall leave out perturbative corrections that can be decomposed into one body treatment.

Finally, from the two body interaction mediated via scalar bosons we want to define the correction to the Green's function originated from such an interaction.

Introduction

In Classical Mechanics (CM), we characterize the state of a physical system by a point in the four dimensional space-time, (x_i, t), where x_i with $i = 1, 2, 3$ and t are respectively the space and the time coordinates. If we parametrize the time coordinate, we can describe the same system in the phase space (q, p), with $q = (q_1, q_2, ..., q_n)$ indicating collectively the degrees of freedom for such a system and $p = (p_1, p_2, ..., p_n)$ their respective conjugate momenta. Time evolution of the system is then described by a line either in the space-time or in the phase space $(q(t), p(t))$ which represents the set of values for all degrees of freedom and respective conjugate momenta at a given instant of time t. This time evolution is generated by Hamilton's equation. In this strictly deterministic theory, the state of the systemn at a given time is fixed by the initial conditions, as for example, at $t = 0$.

In Quantum Mechanics (QM), or wave mechanics, on the other hand, the state of a system is characterized by the wave function $\Psi(x, t)$, and its evolution is governed by Schrödinger's equation. The wave function must be known at each point of the space-time (x, t), so that it requires a continuous infinite values to be described, with the wave function taking the role of a field.

In a scattering process we have a wave function $\Psi(x, 0)$, where we fix the initial conditions at $t = 0$, coming into a point which has a force field or a particle, so that the typical question that arises is: Which is the wave function representation $\Psi(x, t)$ after the interaction with the scattering center?

In order to answer this question, we use the generalized Huygens principle [1]. If a wave function $\Psi(x, t)$ is known at a given time t, it is acceptable to assume that the wave function $\Psi(x', t')$ that emerges from the scattering centre located at (x, t) and propagates from position x to x' in a time $t' - t$, be proportional to the amplitude of the wave function $\Psi(x, t)$. The proportionality contant is defined as $iG(x', t'; x, t)$, so that we have in a mathematical notation:

$$\Psi(x', t') = i \int d^3x \; G(x', t'; x, t) \, \Psi(x, t),$$

where $t' > t$.

Here $\Psi(x', t')$ defined at position x' and time t' is the wave function that emerges from the scattering centre. The quantity $G(x', t'; x, t)$ is called the Green's function or propagator [1]. Knowing this G means to solve completely the scattering problem. In other words, knowing the Green's function is equivalent to solving of the Schrödinger's equation.

Now, is it possible to describe a physical system in any space-time hypersurface with initial conditions defined in a hypersurface different from $t = 0$? The answer to this question is positive and it was given long ago in 1949 by Dirac [2]. In his work, he proposes three distinct forms that could describe the dynamics of relativistic systems, two of which do not use time to describe the dynamical properties of simple systems. These different forms received different names: instant, point and front forms. The instant form is the usual relativistic dynamics described in terms of boundary conditions set at $t = 0$ while the front form uses the hyperplane of Minkowski space that contains the trajectory of light (light-cone).

In this work we explore the concept of covariant quantum propagator written down in terms of light front coordinates and obtain the propagator and Green's function in the light-front for a time interval x^+ where $x^+ = t + z$, is the light-front "time". In principle, this is equivalent to the canonical quantization in the light front [3, 4, 5, 6, 7, 8, 9]. Kogut and Soper [5] also makes use of this way of constructing quantities in the light front: starting with 4-dimensional amplitudes or equations, they integrate over $k^- = k^0 - k^3$, which plays the role of "energy" and corresponds to processes described by amplitudes or equations in "time" x^+. With this, the relative time between particles disappear and only the global propagation of intermediate state is allowed. The global propagation of the intermediate state is the "time" translation of the physical system between two instants x^+ and $x^{'+}$.

We focus our attention in the study of quantum propagators and bound states of two quantum bodies in the light front. In four-dimensional theories the Bethe-Salpeter [10] amplitude, which represents the bound state of two bodies, satisfies a covariant homogeneous integral equation with the kernel defined for all irreducible processes to two bodies, and with self-energy corrections to the external propagators of each particle [10, 11]. In principle it is possible to project the Bethe-Salpeter amplitude and equation for times x^+, eliminating the relative time between particles in intermediate states and external propagators as well. However, the description of the Bethe-Salpeter equation for times x^+, or as we will use in the light front, even for simple kernels as in the "ladder" approximation is very complex as we shall shortly see along the next chapters.

Weinberg in 1966 [12] discussed in a model of two bosons exchanging a third boson, the representation of the four-dimensional Bethe-Salpeter equation in the ladder approximation in the infinite momentum frame. He concluded that he could write a three-dimensional homogeneous integral equation, where two bosons exchanged only one boson in the kernel of such an equation. From the viewpoint of infinite momentum frame, the system is described in terms of transverse coordinates to the speed of this frame and the coordinate along the light trajectory. This description could simply be translated in terms of coordinates defined in the plane $x^+ = 0$ which is invariant by Lorentz transformations along the z direction [13]. However, the three-dimensional representation of the Bethe-Salpeter equation discovered by Weinberg showed itself to be only an approximation [5].

Here we are going to obtain a systematic expansion of Bethe-Salpeter's equation, completing the work of Weinberg for the "ladder" approximation [14, 15]. The idea that it will be applied along this book is the construction of the Green's function for the two body interacting system in the light front, starting from the covariant propagator and projecting it for equal times x^+, for all particles, thus eliminating the relative time. The Green's function in the light front satisfies a set of coupled equations which we argue are equivalent to the Bethe-Salpeter one. From the point of view of projection onto times x^+, the intermediate processes that occur in the Bethe-Salpeter equation, correspond to the propagation of intermediate states of many particles in time x^+. These intermediate states are defined in the Fock space of the quantized theory in $x^+ = 0$ [9].

In this book we also discuss the canonical quantization in $x^+ = 0$, using largely Dirac's idea, in that it is possible to define the dynamics of a quantum system for a given x^+. Therefore, starting from covariant propagators, or Green's functions with n legs [11], we can define the Green's function or propagator for the quantum system for times x^+.

The observation of the structure of bound states can be done through the points of electroweak proofs. The compound states of electromagnetic current contain complex processes even in the approximation where a photon is absorbed by one of the particles that compose the bound state, when described in terms of global time x^+. In essence this is a consequence of the possibility that in the instant x^+ in which the photon is absorbed, the quantum system is propagating in a Fock space with many particles. In terms of the wave function component of two bodies of the bound state, this contribution to the electromagnetic current can be interpreted as a two body operator, as we shall see.

In the description of the electromagnetic current in the light front, we have also the concept of "good" or "+" component of the current [16, 17, 18, 19, 20], for which, in the Drell-Yan frame of reference [9] ($q^+ = q^0 + q^3 = 0$, transfered momentum), the suppression of the pair terms is maximum [19, 21, 22, 23]. However, for the other components in the Drell-Yan frame of reference there is not a suppression of the pair term and in particular for the "$-$" component there are contributions where the photon produces a pair of particle and antiparticle for $q^+ \to 0$ [24]. These processes are represented by diagrams of the "Z" type, as a consequence of the time ordering of the system's description in the time x^+. We can also interpret the contribution of the pair creation process $q^+ \to 0$ as a contribution from the "zero mode" [25, 26]. In general, even the "good" component of the electromagnetic current, in systems of compound fermions, can have a pair contribution in the Drell-Yan frame of reference [25]. The instantaneous term of the fermionic propagator in the time x^+ originates a contribution to the pair term [25].

In the following we briefly present the organization and the contents of chapters: In the first chapter we define the light front coordinates and present the hyperplane known as the null plane ($x^+ = 0$), and with these we define the canonically conjugate momenta and the properties of the scalar product. We then write down Einstein's relation for mass and energy in terms of the null plane components discussing also some of the advantages and disadvantages proper to this approach.

In chapter two the covariant quantum propagator projected onto the light front, that is, for time intervals x^+ is obtained for scalar free particles.

In the third chapter we study the Green function for a system of two interacting bosons in the light front, that is, given the perturbative correction to the covariant propagator for two bosons interacting with the exchange of one, two or n intermediate bosons, we evaluate the corresponding representation in the light front. Once the light front Green function for systems with N particles plus $N-2$ intermediate bosons is evaluated, we construct a system of infinite hierarchical coupled equations for these Green functions, which when solved perturbatively rebuild the corresponding covariant propagators.

In chapter four we construct the integral equation for the vertex of the bound state of two bosons in the null plane, including in the kernel the interaction up to the fourth order in the coupling constant. The truncation of the hierarchical equations for the Green function gives base to the systematic construction of Bethe-Salpeter equation in the light front.

In chapter five we consider gauge bosons.

Chapter 1

The null plane

In his pioneering work, Dirac [2] showed that it is possible to construct dynamical forms starting from a description of the initial state of a relativistic system in any space-time surface, whose distances between points are not connected causally. The dynamical evolution corresponds to the system following a trajectory through the hypersurfaces. For example, the hypersurface $t = 0$ is our three-dimensional space. It is invariant under translations and rotations. However, in any transformation of inertial frame of reference that involves "boosts" the time coordinate is modified and therefore the hypersurface $t = 0$. Other hypersurfaces can be invariant by some type of "boost"; it is the case of the hyperplane called null plane, defined by $x^+ = t + z = 0$, which is the "time" coordinate for the light front. For example, a "boost" in the z-direction does not modify the null plane.

In the following, we define the position and momentum coordinates in relation to the null plane, rewrite in these new coordinates the momenta canonically conjugate and the property of scalar products. We write Einstein's relation for mass and energy in the components of canonically conjugate momenta of the coordinates in the light front and conclude this chapter with a discussion about advantages and difficulties associated to this formalism.

1.1 Light front coordinates

A point in the four-dimensional space-time is defined by the set of numbers (x^0, x^1, x^2, x^3), where x^0 is the time coordinate and $\mathbf{x} = (x^1, x^2, x^3)$ is the three-dimensional vector with space coordinates $x^1 = x$, $x^2 = y$ e $x^3 = z$. Observe that we adopt here the usual convention to take the speed of light as $c = 1$.

In the light front, time and space coordinates are mixed up and we define the new coordinates as follows:

$$x^{+} = x^{0} + x^{3}, \tag{1.1}$$
$$x^{-} = x^{0} - x^{3}, \tag{1.2}$$
$$\vec{x}^{\perp} = x^{1}\overrightarrow{i} + x^{2}\overrightarrow{j}, \tag{1.3}$$

where $\overrightarrow{i}$ and $\overrightarrow{j}$ are unit vectors in the direction of x and y coordinates respectively.

The null plane is defined by $x^{+} = 0$ Fig.(1.1), that is, this condition defines a hyperplane that is tangent to the light-cone, the reason why many authors call the hypersurface simply by light-cone.

The initial boundary conditions for the dynamics in the light front are defined on this hyperplane. The axis x^{+} is perpendicular to the plane $x^{+} = 0$. Therefore a displacement of such hyperplane for $x^{+} > 0$ is analogous to the displacement of a plane in $t = 0$ to $t > 0$ of the four-dimensional space-time. With this analogy, we recognize x^{+} as the time in the null plane.

The canonically conjugate momenta are given respectively by:

$$k^{+} = k^{0} + k^{3}, \tag{1.4}$$
$$k^{-} = k^{0} - k^{3}, \tag{1.5}$$
$$\vec{k}_{\perp} = \left(k^{1}, k^{2}\right). \tag{1.6}$$

The scalar product in the light front coordinares is given by:

$$a^{\mu}b_{\mu} = \frac{1}{2}\left(a^{+}b^{-} + a^{-}b^{+}\right) - \vec{a}_{\perp}\cdot\vec{b}_{\perp}, \tag{1.7}$$

where $\vec{a}_{\perp}$ and $\vec{b}_{\perp}$ are the transverse components of the four vectors. All four vectors, including non vector quantities such as Dirac gamma matrices γ^{μ} can be expressed in terms of these components $(-, +, \perp)$.

From (1.7) we can obtain the scalar product $k^{\mu}x_{\mu}$ in the light-front representation, as:

$$k^{\mu}x_{\mu} = \frac{1}{2}\left(k^{+}x^{-} + k^{-}x^{+}\right) - \overrightarrow{k}_{\perp}\cdot\overrightarrow{x}_{\perp}. \tag{1.8}$$

In relation to the space-time described in terms of coordinates t and $\mathbf{x}$, k^{0} represents the energy, while the component k^{-} has an analogous meaning in the light-cone representation. This can be recognized by comparing the scalar product (1.8), in the null-plane with that of the usual coordinates. Therefore, we recognize k^{-} as being the "energy" of a particle moving along the "time" x^{+} of the null-plane.

Also, in the Minkowski space we have the relation between rest mass and energy for a free particle given by

$$k^{\mu}k_{\mu} = m^{2}. \tag{1.9}$$

Using the relation (1.7) in (1.9), we have

$$k^{\mu}k_{\mu} = \frac{1}{2}\left(k^{+}k^{-} + k^{-}k^{+}\right) - \overrightarrow{k}_{\perp}\overrightarrow{k}_{\perp},$$

so that

$$k^- = \frac{\overrightarrow{k}_\perp^2 + m^2}{k^+}. \tag{1.10}$$

Note that the energy of a free particle is given by $k^0 = \pm\sqrt{m^2 + \mathbf{k^2}}$, which shows us a quadratic dependence between k^0 and $\mathbf{k}$ in contrast to the linear dependence between $(k^+)^{-1}$ and k^- as seen in (1.10) [13].

This last result is very important. It means that particles progagating forward in the null-plane time have positive k^+. As we will see in the coming chapters, an energy integration in the null-plane arises from projecting the propagation of the physical system in time x^+, which implies that in the Bethe-Salpether equation and in the perturbative expansion of the propagator for two particles between two intants of time x^+, non-vanishing contributions will appear only for $k^+ > 0$.

1.2 Advantages and difficulties associated with the formalism

From the quantum standpoint, we can define the initial state of a system in any space-time hypersurface for which lengths between two points are spacelike, and therefore without a causal connection. The quantum evolution of such a system is given by the operator associated with the "time" translation of that hypersurface.

One advantage of using the light front description for such a quantum system is that we can perform "boosts" without involving its dynamics, that is, do Lorentz transformations that maintain the null plane invariant [13]. In this way we can describe the wave function in various frames of reference obtained with kinematical transformations from the system's rest frame.

In this line we can see frequent use of hadronic wave functions in the light front to study relativistic systems. The extent of its applications goes from systems with few nucleons [19, 20, 23, 27, 28] to studies about the structure of light hadrons [9, 22, 29, 30, 31, 32, 33]. This is due to the possibility of treating wave functions in the null plane with a few number of degrees of freedom.

However, within the limitation of a fixed number of particles, it is not taken into account a mixture of more complex components of the Fock space, in the belief that such contributions to the wave functions, resulting from those mixtures are small, so that it would be reasonable to think of them in terms of a fixed number of particles [29]. From the viewpoint of quantum theory of fields, however, truncation in the Fock space is problematic [34].

In general, change of reference frames through "boosts" leads to the creation of pairs so that this treatment becomes incovenient since the number of particles in the new, "boosted" frame is different. It is therefore desirable to find an approximate description in which the number of particles does not change with the change of frames of reference, preserving thus the physical description of the system.

Using the light front coordinates as a tool, the problem is solved as far as kinematical "boosts" are concerned [34]. The generators for the kinematical transformations do not produce pairs as they are applied to the quantum state since they do not contain any dynamics. As a consequence, the invariance in relation to those kinematical transformations for physical observables such as cross sections, electroweak form factors, etc. is also evidently preserved.

However, covariance through kinematical "boosts" in the light front is limited. For example, it is not possible to guarantee that the matrix elements of the electromagnetic current of a bound state keep the rotational and the gauge invariance without including, in a rigorous sense, more complex components of the Fock space [24].

Pursuing Dirac's idea which gives base to describe the evolution of a physical system between hypersurfaces of the space-time with constant time, i.e., $x^+ =$ constant, we are going to derive the free propagators, or covariant Green's functions of n-legs, the Green function that represents the global time evolution of the system between two instants x^+. This projection for the propagation of the physical system for global time x^+, which eliminates relative times among particles, is a restriction that does not make the quantum description incomplete, but allows us to access directly in the various physical processes, the contents of the intermediate states in terms of their components in the Fock space, allowing approximations that truncate such a Fock space without loosing covariance through kinematical transformations that maintain the null plane invariant [34].

In the next chapter we will start the study of the Green function in the light front obtained from the projection of the quantum propagator at global times x^+ for bosonic systems.

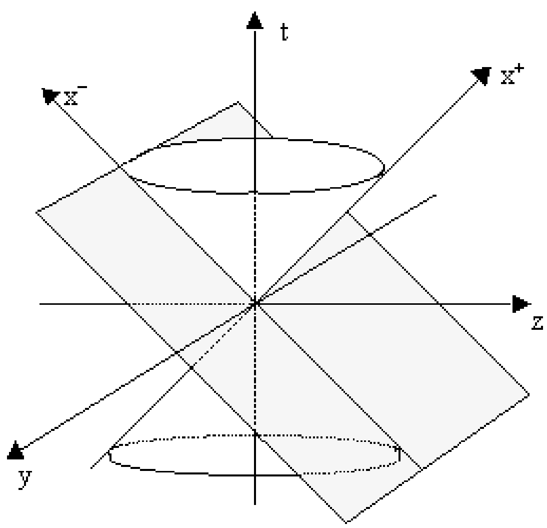

Figure 1.1: The null plane is defined by $x^+ = 0$.