Trends in Physics

Festschrift in homage to Prof. José Maria Filardo Bassalo

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Editora

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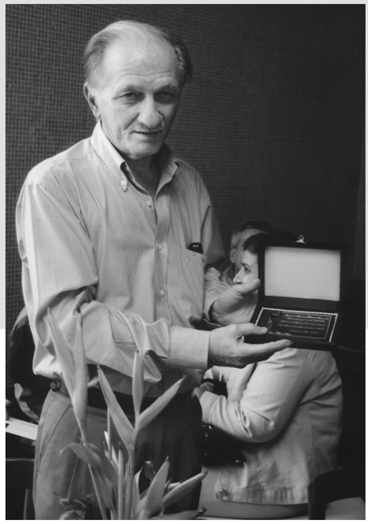
Summary

Professor José Maria
Filardo Bassalo in
the ASP opening
ceremony.

ASP opening ceremony.
From left to right: L. C. B. Crispino,
M. S. D. Cattani, J. M. F. Bassalo,
M. O. C. Gomes, J. F. de Souza, R.
Dall'Agnol and V. S. da S. Alves

Foreword



Trends in Physics is the Proceedings of the *Amazonian Symposium on Physics*, held in Belém-Pará-Brazil, in September 9^{th}-12^{th} 2008, as a *festschrift* for José Maria Filardo Bassalo, on occasion of his 73^{rd} birthday.

Born in Belém in September 10^{th} 1935, Bassalo has played a fundamental role in the development of Physics in the northern region of Brazil. In fact, initially as a high school teacher and afterwards as a professor at the university, he has motivated and inspired a whole generation of young students. Being mathematically inclined it was natural for him to pursue the career of civil engineer at the old school of engineering of the state of Pará where he graduated in 1958. After that, he continued studying physics on his own until 1965 when he decided to obtain the physicist degree at the University of Brasilia. Brasília, the *University for the Development*, a dream of renowned educators and scientists was at that time in a political turmoil, the military invaded the university, professors went on strike and resigned. Bassalo had to postpone his goals for some years. In 1968 Bassalo went to the University of São Paulo where he obtained the titles of Master and Doctor in Physics, in 1973 and 1975, respectively.

The professional career of Bassalo begun in 1961 as an instructor of Physics, in an incipient group of Physics at the University of Pará called *Núcleo de Física e Matemática.* He became associate professor in 1978 and, through an open competition, full professor of Physics in 1989. In September 10th 2005 he was compulsorily retired.

At the Department of Physics of the University of Pará, Bassalo worked on all relevant aspects of the academic life, teaching, doing research and divulging physics. He lectured at both undergraduate and graduate levels and was the advisor of many master degree thesis and dissertations. Altogether he is the author of about 40 scientific works and 196 articles on the History of Physics published in Brazil and abroad, as well as 21 books (more details on his academic life can be found at http://www.bassalo.com.br).

One must highlight his enthusiasm, writing articles and participating in workshops, to create an institution for the development of science and technology in the Amazon. This institution, Instituto de Ciência e Tecnologia da Amazônia (ICTA), would investigate and use scientific means to find solutions for the peculiar technological problems of the region. With comparable zeal he endeavored to generate an agency for support of research (Fundação de Amparo à Pesquisa do Estado do Pará – FAPESPA) with scope similar to other existing organizations (FAPESP, FAPERJ, FAPEMIG, FAPEPE, etc.).

Compulsorily retired from the University of Pará, Bassalo, together with his wife and friends have created an agency, Fundação Minerva, which intends to disseminate the humanist and techno-scientific cultures, to encourage the younger generations to fight for the Amazonia development. He continues to do research in Physics, publishing articles on the History and Philosophy of Physics.

For all that has been said, Bassalo is in fact a role-model as a Professor, researcher and civil citizen. Securely, he will always be remembered by future generations.

***The Editors*.**

Foward Physics: Phenomenological and Experimental Comments

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Departamento de Física Nuclear e Altas Energias (DFNAE)
Instituto de Física Armando Dias Tavares (IFADT)
Universidade do Estado do Rio de Janeiro (UERJ)

This paper was written for the José Maria Filardo Bassalo's Festschrift. We intend to describe the current status of Forward Physics focusing in the experimental side and in view of the coming era of LHC experiments.

1. Introduction

Forward Physics is the common name for the all near beam physics in a Collider. Several processes produced in the proximity of the beam play an important role on the Physics of strong interactions. Forward Physics includes topics like: elastic scattering, total cross section measurements, photon-photon physics, low-x QCD physics, diffraction, glueballs, centauros and some observed phenomena in Cosmic Ray Physics. The oldest one is diffraction.

Diffraction was first observed in Optics by Leonardo da Vinci (1452-1519).[1] In Particle Physics it was introduced at the time of Regge model. There are many books [1, 2] and papers [3] with a good number of references about the subject, including the nice paper of Good and Walker [4].

The elastic scattering and total cross section at high-energies are expected to be measured by TOTEM [5] and CMS Collaboration [6] at CERN Large Hadron Collider (LHC).

The photon-photon Physics is a very interesting topic. Photons, different

1 An interesting reference: "Leonardo da Vinci -An Artabras Book" -Reynal and Company in Association with William Marrow and Company, N.Y..(1965) Chapter: Leonardo's Optics, Domenico Argentieri (pag. 405) says the following: *"I did a great discovery founding the transcription of Gian Battista Venturini which was in the Reggio Emilia Library"*. The text shown clearly that Leonardo observed the diffraction phenomena but made a "wrong interpretation".

of gluons, does not interact directly with each other. Therefore, in principle, we can think that this feature would eliminate many of the complications presented in gluon-gluon interactions. But, at high energies, photons have a "hadronic behavior" which turn on their physics very exciting. There are many related topics like the Higgs decay, $H \rightarrow \gamma\gamma$, and the dileptons production, $\gamma\gamma \rightarrow \ell\bar{\ell}$ [7]. Besides, as quarks can couple with photons, many hadronic states can decay too in $\gamma\gamma$ [8], like in the "classic" pion decay, $\pi^o \rightarrow \gamma\gamma$.

Low-x Physics include many subjects like diffraction. This is a very important fact because define very well the specific phase space region to work and to observe forward physics phenomena.

Although, as we said, diffraction is one of the oldest topics in Physics, there are many recent results on diffraction in Particle Physics from Fermilab TEVATRON [9] and DESY HERA [10]. The most important is the hard diffraction, discovery at CERN ISR [11]. We expect in the near future that LHC will produce new data that will allow the confirmation and the comparison with the recent results of nowadays.

The main characteristics of diffractive processes are:

(i) weak energy dependence;

(ii) strong t dependence;[2]

(iii) vacuum quantum numbers exchange.[3]

Besides these characteristics, diffractive processes can be exclusive or inclusive. The principal processes which are interesting to look for are: elastic scattering ($a + b \rightarrow a + b$); beam dissociation before target interaction ($a + b \rightarrow a^* + b$); target dissociation ($a + b \rightarrow a + b^*$); double diffractive dissociation ($a + b \rightarrow a^* + b^*$) and inclusive diffraction ($a + b \rightarrow a + X$ or $a + b + c... + X$).

Still as diffractive production, we can find the glueballs. They are interesting hadrons predicted by Quantum Chromodynamics (QCD) [12], in the sense that interactions of gluons would produce new color objects made only of gluons. We would say that it is a direct consequence of the non-abelian characteristic of QCD.

Finally, there are the centauros events, the most exotic objects. They were discovered in cosmic ray experiments [13]. The main characteristics of centauros events are the low electromagnetic activity and a large number (~ 100) of hadrons production. The LHC experiments will have the conditions to observe this type of process.

2 *t* is the square four-momentum transferred from the initial to the final proton.

3 The vacuum numbers are: *P* (parity) = +1, *G* (G-parity) = +1, *I* (isospin) = 0, ξ (Regge signature) = 1.

2 Phenomenological and Experimental Questions

We do not intend to write a review for each subtopic of forward Physics. We would like rather to give a good idea about each subject.

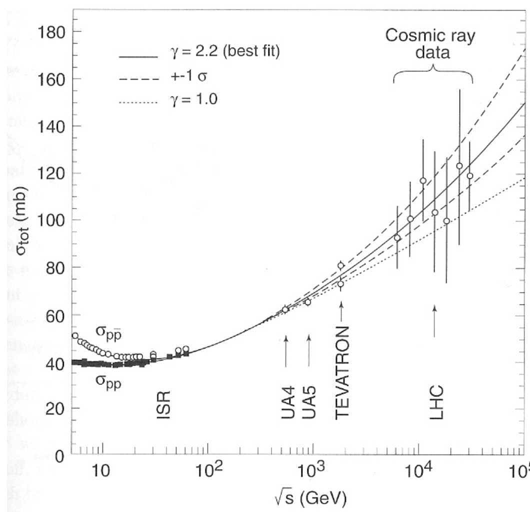
2.1 Elastic Scattering and Total Cross Section

There are many attractive points to study elastic scattering in hadron Colliders. By definition, in an elastic scattering the final particles are identical to the initial particles, and we have many theoretical models to explain this kind of process [14]. From Regge model, an elastic scattering at high-energy is explained by a Pomeron exchange,[4] and according to QCD, Pomeron is a hadron constituted dominantly by gluons, such in an elastic scattering the valence quarks from the incident hadrons coupling directly with gluons of Pomeron producing the same initial quarks in the final states.[5]

Total cross section measurement discrepancies were found in two experiments at TEVATRON [15], CDF and E811, and can be a serious problem related to theoretical bounds, as unitarity [16] and Froissart bound [17]. Therefore is important to do good measurements with elastic scattering in LHC experiments with TOTEM.

The calculation of total cross section is still an important issue. The elastic scattering is directly connected to the total cross section via the optical theorem. Total cross section (σ_{Tot}) measurements made by some experiments in hadron scattering are shown in Figure 1.

Figure 1:
Measurements of σ_{Tot} in hadron scattering [2].

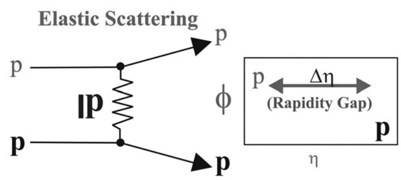


4 Pomeron is a Regge trajectory with vacuum numbers.

5 The interested reader can find a few papers in [2, 14].

The cross section in proton-proton scattering is large and can be done in a short time with a expected low luminosity (~ 10^{28} cm^{-2} s^{-1}). The detectors of TOTEM are essentially Roman Pots with silicon detectors of ~ (4 cm x 4 cm) and have to operate near to the beam, approximatively 0.5 mm. Figure 2 shows the diagram of an elastic proton-proton scattering (pp) at high-energy, with Pomeron exchange, and the lego plot with a large rapidity interval ($\Delta\eta$) without particle production, which is the signature of an elastic hadron-hadron scattering at high-energy.

Figure 2:
Diagram of a proton-proton elastic scattering, and the representation in $\eta \times \varphi$ space, expected for LHC experiments.



2.2 Photon-photon Physics

The photon-photon physics has been studied in several scattering processes [18] like $e^+ e^-$, e^- p, $\gamma\gamma$, γp and pp. With the advent of LHC experiments the Physics of these processes will become again very exciting. It will allow several studies with different approaches in electroweak and strong interactions. The most expected is the possibility of the Higgs production, $\gamma\gamma \rightarrow H \rightarrow b\bar{b}$ ° β, to be observed in LHC experiments, independent of its low rate in comparison with other process in pp Collider. Dilepton production ($\gamma\gamma \rightarrow \ell\bar{\ell}$) is also another topic to be studied [19]. The photon present an interesting "property". It can have point-like interactions with the proton and in this case we call a direct process, but it can also have an interaction as an object constituted by a bunch of partons, in this case we call a resolved process. Based in this fact, we sometime read in the literature "the hadronic behavior of the photon", and we can consider the interaction among "partons coming from photons" with those coming from protons.

Before the ending of this short summary of photon-photon Physics, we would like to call attention for heavy ions scattering [20] at LHC, which is predict to have a high luminosity for the photon-photon process.

Finally, we can point out a number of topics involving photon-photon physics: (i) vector meson production, (ii) several processes using vector dominance model, (iii) photon structure function, (iv) heavy quarks photon production and so on [21].

2.3 Low-x QCD Physics

For didactic reasons let us introduce the usual phenomenological potential (V) representing the interaction between two partons as $V \simeq -\frac{4}{3}\frac{\alpha_s}{r} + kr.$[6] The first term represent the perturbative aspect of the interaction or the short-distance region and the second term represent the nonperturbative aspect or the long-distance region. The low-x physics[7] is related to the phenomena that occur at the interface of these two high-energy physics region. To the interested reader there are two good references to note about low-x studies [22].

It will be very important for CMS studies in Forward Physics to have sub-detectors like Castor and FP420 in order to get the possibility to do precise measurements at low-x Physics region.

The studies of structure functions of proton on HERA shown the gluon dominance in comparison with u quarks, in the low-x region [23].

2.4 Soft and Hard Diffraction

The study of diffractive processes in Particle Physics began with the work of the group of Landau [24] and the paper of Good and Walker [4]. A model that describe very well the spectrum of the soft diffraction, based on Good and Walker ideas, at the energies of the ISR, is given in [25]. A good review with experimental and theoretical results can be found in [26].

We have done many progress in strong interactions, with help of QCD, but still not enough to understand very important aspects and, therefore, to really consider QCD as the general theory of the strong interactions. The nuclear and hadronic sectors are still plenty of open problems like diffraction as a hadronic phenomenon. In fact the LHC will inaugurate a new era of observations and we hope to clarify some of the questions of the Standard Model and beyond. More recently, enormous progress was made [27], but still not so useful for the experiments to test the effective gluon reggeization and other theoretical progresses. It would be very useful to have a theory to be tested with clear experimental parameters to be measured. Hence, from the experimental point of view, it is absolutely necessary to build appropriated detectors with the capacity to observe

6 α_s is the strong coupling, k is a numerical factor, and r is the distance between the partons.

7 The variable x is the ratio $\frac{P_{parton}}{P_{proston}}$.

and to test the existence of glueballs [12], centauros [13] and all near beam phenomena. In our point of view, what is more important is to get experimental results to really reach progress in these subjects.

For instance, it is important to know the rate of the diffractive and non diffractive cross sections, in reference to the total cross sections, to understand the importance of each component production of the global heavy flavor production. The distribution of the events of high and low $_{PT}$ will guide us to determine these rates.

Strong interactions has crossing a long way and many models and theories have tried to give us a real description of these interactions. With the evolution of experimental results, came from the big machines, we had the possibility to have at least two good theories. First, was the Regge model, that dominated all phenomenology during the decade of 70 and 80. But this theory was conceived on the framework of *S* matrix program and at the same time all results pointed to the low p_T Physics. It was the discovery of J/ψ [28] that gave the starting to use QCD as the basic theory to describe the interactions between quarks and gluons. Recently, theoreticians are working to unify these two descriptions, one exclusive for low p_T physics, and another intending to be the general description of the strong interactions. This means reggeization of the gluon [29]. Again, this was motivated mainly by the discovery of the hard diffraction [30, 31].

One of the more exciting problems of the actuality is the discovery of Higgs boson. With LHC the possibility to observe this object increases a lot. Also, from the side of Forward Physics, special detectors are being proposed to observe Higgs diffractively produced and good calculations prove its availability [32].

While soft diffraction is characterized by small t production and the absence of jets associated, the hard diffraction is identified by the jet associated production. In both cases, the Pomeron exchange is required to explain the diffraction processes.

It is usual to identify a large rapidity gap[8] making a lego plot with the pseudo-rapidity φ and the azimuthal angle ϕ of the scattering process (Figure 2). To do that, experimentally, one way it to follow the steps:

1. adjust the experimental points of a distribution of the multiplicity in one of the calorimeters, like the electromagnetic calorimeter, by a negative

8 Absence of particles in a limited range of rapidity.

binomial function in order to get a good fit;
2. make all corrections due to the existing backgrounds;
3. calculate the rate production corresponding to the diffractive process using the following expression

$$f = \sum_{n=0}^{n_0 - 1} \frac{\text{data } (n) - \text{ fit } (n)}{N_{\text{total}}(\text{data})} \tag{1}$$

where, n is the average multiplicity;
4. if possible, repeat the steps above for each appropriated detector.

The dominant idea about diffractive processes was given by the authors of the reference [30], where the hard diffraction is described by two jets j_1 and j_2 of high p_T , and one jet spectator opposed to the other two jets.

The hard diffraction, after its discover by UA8, knew a quick development [31]. During a decade the experiments of HERA and TEVATRON [33] showed the importance of studying the subject. Nevertheless, by the lack of the appropriated experimental environment we could not go too deeply in the open questions linked to the diffractive production.

To get a more complete idea about the hard diffraction status we have to go to the talks given at LISHEP2000, LISHEP2002, and references therein [34].

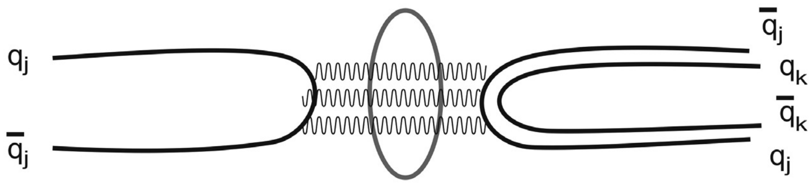
We expect now at CMs to answer some of the questions about the double Pomeron exchange, heavy flavor diffractively produced, distributions on high and low t, W/Z production and so on.

Theoretically, we know that the missing of data gives too much freedom to the models. Many progress was done in fact to understand the reggeization of the hard Pomeron on QCD, some proposals for unification and studies of transition from soft to hard interaction [34].

2.5 Glueballs

As a consequence of the interaction between gluons we can consider the possibility to form bound states or gluonic resonances. These objects are called glueballs. We would like to call attention for two aspects of these objects. Its relation with Pomeron or diffraction and the old hadronic phenomenology [12]. The OZI rule establishes that "disconnected quarks diagrams imply very suppressed processes". From QCD, we know that gluons interact with quarks, therefore we can bridge with gluons the connections between quarks like shown in the Figure 3.

Figure 3:
A possible diagram to connect quarks via gluons.



And as gluons interact with gluons too it is possible that they form a bound state or a resonance, a glueball which decay into final hadrons. This can be observed in the decay of the $\varphi \rightarrow \rho\pi$ and $J/\psi \rightarrow$ hadrons. Both decays are suppressed and there are other examples.

In diffraction we have a hypothesis well supported by the phenomenology: Pomeron and glue-balls may be are the same objects. Based on that we think that the best way to produce glueballs is the dynamic of the double Pomeron exchange. Since this diffraction topology implies in a fusion of two Pomerons the probability to form glueballs is bigger than other mechanisms, independent of its low cross section.

The phenomenology of glueballs continue to be an interesting Particle Physics topic. A large number of theoretical and experimental papers [35] show the interest of the physicists for the subject. Also a large number of experiments have trying to look for glueballs presenting several candidates but not yet an state well defined. The principal difficulty is to separate the glueballs from the competitive θθømesons produced in same phase space region. One way to give a preference to look for glueballs are the states called "oddballs". They have quantum numbers like $J^{PC} = O^{+-}, O^{--}, 1^{-+}, 2^{+-}, 3^{-+}$ ....

There are some new calculations [36] of glueball masses using gauge/gravity theory to the states 0^{++} , 1^{--}, 2^{++}, but do not show how they can be separates from the $q\bar{q}$ states existing for these J^{PC} quantum numbers. The same problem is found in another paper [37] that asked for pseudo scalar glueballs, but do not point out the way to isolated these states from θθøexcited states. We hope to see some signal on the LHC experiments.

2.6 Centauros

Among the non observed objects the most exotic are the centauros [13] which was observed by two different groups of Cosmic Ray Physics [38]. Centauros was first reported by Lattes [13], in 1973, and confirmed by Pamir collaboration in 1984. The main characteristics of centauro events are: (i) a moderated large number of hadrons and (ii) a week electromagnetic activity. These can be summarized in the following itens [39]:

1. $\sum E_h >> \sum E_\gamma$
2. $N_h >> N_\gamma$
3. $Q_h = \frac{\sum E_h}{\sum E_{both}} > 0.5 \qquad \left(\sum E_{both} = \sum E_h + \sum E_\gamma\right)$

The LHC is the only accelerator with the capability of energy to produce centauros. The set of detectors of CMS Collaboration in near beam position is also another very positive factor. The CASTOR detector [41] was projected also for centauro observations.

As centauros are observed diffractively in Cosmic Rays experiments, we think that maybe the best diffractive mechanism to produce centauros would be the double Pomeron exchange. And it is not difficult to get a first trigger for centauros in the complex of detectors of CMS. It has to be required that we have at same time a strong activity in hadron calorimeter and a very poor activity in the electromagnetic calorimeter. With CASTOR detector this trigger would be much better.

There are many theoretical tentative of explanation for centauro events. In the references given here the reader can easily find several descriptions. We think that the most important now is to find in Accelerator Physics the centauro production to be possible to progress in this matter.

Final Comments

To have a complete view of the fundamental interactions we have to go to the whole phase space available in the colliders and in special with LHC new era of experiments at the highest energy in Particle Accelerator Physics. To sweep the whole phase space we have to take in account the forward physics. A lot of progress was made during the last decades, from theoretical point of view. Some experimental results help us to motivate to build new detectors to be use on the experiments of LHC. These new detectors around LHC detectors, and in particular the CMS are: Roman Pots [40], CASTOR [41], FP420 [42], ZDC [43] and Micro Stations [44].

The Roman Pots are devices for detection of the scattered protons, and when triggered with the calorimeters or the central detectors can be use as a very good signature for diffractive events. There is not only one type of Roman Pots. We send the interested reader to the references given in [40].

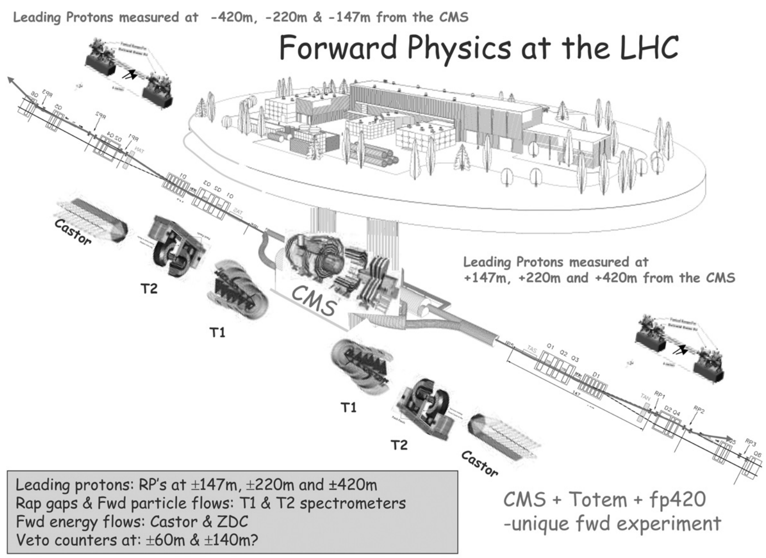
The CASTOR detector is a nice calorimeter built recently for the CMS experiment. The best description can find on references given in [41].

The FP420 is being designed and will be built for LHC for observation of Higgs and photon-photon Physics. See references given in [42].

The ZDC (Zero Detector Collider) is a very special calorimeter put in very near to the beam [43].

Finally, the Micro Stations will be put on the beam line, and are very similar to the Roman Pots. The Figure 4 shows all these detectors in the approximate local where they will be in the beam line.

Figure 4:
Picture taken from a talk of Risto Orava, in a CMS Week.



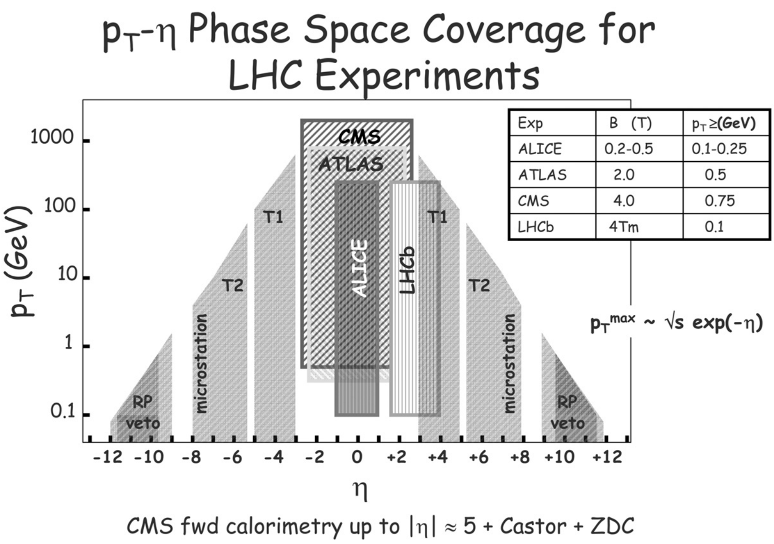
One important and particularly crucial problem on the LHC experiments will be the pile-up, because hard diffractive events constitute only a very small fraction of the total events turning the analysis very painful due the possible suppression of the rapidity gaps. Studies in Forward Physics is a great job to be done in the near future due to the number of possible new results.

For the new experiments of LHC (CMS, ATLAS, ALICE and LHCb), the level of the energy of the proton proton interactions (14 TeV) will permit not only to upgrade all results of diffractive Physics but create a new and large spectrum of results. For example, the diffractive production of Higgs [32] and higher diffractive masses will allow us to investigate much better the centauros existence [45].

It is interesting to see the availability of the possible phase space to be used for Forward Physics. The range of η and p_T shown in the Figure 5 is very promising and motivate us to expect good results. We never had a such number of specific detectors to do Forward Physics.

We would like to conclude saying that the forward physics experiments and the understanding of strong interactions will be very promising in the near future.

Figure 5:
All experiments range for p_T and η at LHC beam line.



Exp	B (T)	$p_T \geq$(GeV)
ALICE	0.2-0.5	0.1-0.25
ATLAS	2.0	0.5
CMS	4.0	0.75
LHCb	4Tm	0.1

Acknowledgments

We would like to thank Dr. Luis Carlos B. Crispino for giving us the opportunity to write this paper dedicated to our friend, José Maria Filardo Bassalo. For one of us (A. S.), it was a great honour to have had Bassalo as a colleague in the University of Brasília (UnB), before 1964, where we had a so short moment but very enthusiastic for science and human company. We knew at that time that we were very happy with all what represented the UnB in our lives. It was a time of really human construction. Our so good days in Brasília was a very nice time and stimulant for science development in Brazil. We would like to acknowledge CNPq and FAPERJ for the partial financial support of this work. Up today we continue still to learn a lot from Bassalo's papers, academic books, and from those unique writings called Crônicas da Física [46].

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The field concepts of Faraday and Maxwell

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We discuss how Faraday (1791-1867) and Maxwell (1831-1879) defined the field concept. According to them magnetic field was a region of the space close to magnetized bodies and electric field was a region of the space close to electrified bodies.

1. Introduction

In modern textbooks there is a great polisemy as regards the meaning of *electric field* and *magnetic field*, [1], [2], [3] and [4]. Field appears defined as a region of space, as a vectorial function, as something which propagates in space, as something which stores or contains energy and momentum, as a substance that mediates interactions between gross bodies etc.

Here we analyze how the field concept was presented by Faraday and Maxwell, as these two authors are normally considered the modern initiators of this concept. Although we restrict our analysis to these famous scientists, we agree with Heilbron when he mentioned that "the electricians of 1780 lacked the word but not the concept, which they called 'sphere of influence', *sphaera activitatis*, or *Wirkungkreis*", [5].

2. Faraday's Field Concept

Michael Faraday (1791-1867) utilized the word 'field' for the first time in November 7th, 1845, in his Diary, [6, Note 17]. But much before this time he had utilized expressions like 'magnetic curves' or 'lines of magnetic forces'. For instance, in the paper of 1831 in which he described his discovery of electromagnetic induction he presented this law in terms of a wire cutting the magnetic curves, which he defined as follows, [7, p. 281, §114]: "By magnetic curves, I mean the lines of magnetic forces, however modified by the juxtaposition of poles, which would be depicted by iron filings; or those to which a very small magnetic needle would form a tangent." In 1845 he gave this definition with the following words, [7, p. 595, §2149]: "But before I proceed to them, I will define the meaning I connect with certain terms which I shall have occasion to use: thus, by *line of magnetic force*, or *magnetic line of force*, or *magnetic curve*,I mean that exercise of magnetic force which is exerted in the lines usually called magnetic curves, and which equally exist as passing from or to magnetic poles, or forming concentric circles round an electric current. By *line of electric force*, I mean the force exerted in the lines joining two bodies, acting on each other according to the principles of static electric induction (1161, &c.), which may also be either in curved or straight lines." The most clear definition of these lines was presented by Faraday in a paper published in 1852, [7, p. 758, §3071]:

> A line of magnetic force may be defined as that line which is described by a very small magnetic needle, when it is so moved in either direction correspondent to its length, that the needle is constantly a tangent to the line of motion; or it is that line along which, if a transverse wire be moved in either direction, there is no tendency to the formation of any current in the wire, whilst if moved in any other direction there is such a tendency; or it is that line which coincides with the direction of the magnecrystallic axis of a crystal of bismuth, which is carried in either direction along it. The direction of these lines about and amongst magnets and electric currents, is easily represented and understood, in a general manner, by the ordinary use of iron filings.

Faraday began to mention the magnetic *field* in his publications presented to the Royal Society in 1845 and published in 1846, [7,p. 608,§2247, our emphasis]: "Another magnet which I have had made has the horseshoe form. [...] the poles are, of course, 6 inches apart, the ends are planed true, and against these move two short bars of soft iron [...] The ends of these bars form the opposite poles of contrary name; the *magnetic field* between them can be made of greater or smaller extent and the intensity of the lines of magnetic force be proportionately varied." The magnetic field is here related to the region between two magnetized bars. On §2252 he defined the axial and equatorial directions along or across the lines of magnetic force, [7, p. 608, §2252]: "I shall have such frequent occasion to refer to two chief directions of position across the magnetic field, that to avoid periphrasis, I will here ask leave to use a term or two, conditionally. One of these directions is that from pole to pole, or along the line of magnetic force; I will call it the *axial direction*: the other is the direction perpendicular to this, and across the line of magnetic force; and for the time, and as respects the space between the poles, I will call it the *equatorial direction*."

In many other places he utilized the expression 'magnetic field', [7]: pp. 634-6, §2463 to §2475; p. 690, §2806 to §2810; p. 694, §2831; p. 777, §3171 etc.

A clear definition of what he understood by a *magnetic field* appeared only in a paper read in 1850 at the Royal Society and published in 1851, [7, p. 690, §2806, our emphasis]:

> I will now endeavour to consider what the influence is which paramagnetic and diamagnetic bodies, viewed as conductors (2797), exert upon the lines of force in a magnetic field. *Any portion of space traversed by lines of magnetic power, may be taken as such a field*, and there is probably no space without them. The condition of the field may vary in intensity of power, from place to place, either along the lines or across them; but it will be better to assume for the present consideration a field of equal force throughout, and I have formerly described how this may, for a certain limited space, be produced (2465). In such a field the power does not vary either along or across the lines, but the distinction of direction is as great and important as ever, and has been already marked and expressed by the term axial and equatorial, according as it is either parallel or transverse to the magnetic axis.

That is, for Faraday a magnetic field may be taken as any portion of space traversed by lines of magnetic power. And these lines of magnetic power can be visualized by iron filings.

It is interesting to note that Faraday, mainly from 1851 onwards, considered the lines of magnetic power as representatives of local processes, perhaps states of the ether, as we can see of the passage that follows, [7, p. 759, §3075]:

> I desire to restrict the meaning of the term *line of force*, so that it shall imply no more than the condition of the force in any given place, as to strength and direction; and not to include (at present) any idea of the nature of the physical cause of the phenomena; or to be tied up with, or in any way dependent on, such an idea. Still, there is no impropriety in endeavouring to conceive the method in which the physical forces are either excited, or exist, or are transmitted; nor, when these by experiment and comparison are ascertained in any given degree, in representing them by any

method which we adopt to represent themere forces, provided no erroris thereby introduced. On the contrary, when the natural truth and the conventional representation of it most closely agree, then are we most advanced in our knowledge. The emission and the ether theories present such cases in relation to light. The idea of a fluid or of two fluids is the same for electricity; and there the further idea of a current has been raised, which indeed has such hold on the mind as occasionally to embarrass the science as respects the true character of the physical agencies, and may be doing so, even now, to a degree which we at present little suspect. The same is the case with the idea of a magnetic fluid or fluids, or with the assumption of magnetic centres of action of which the resultants are at the poles. How the magnetic force is transferred through bodies or through space we know not: — whether the result is merely action at a distance, as in the case of gravity; or by some intermediate agency, as in the cases of light, heat, the electric current, and (as I believe) static electric action. The idea of magnetic fluids, as applied by some, or of magnetic centres of action, does not include that of the latter kind of transmission, but the idea of lines of force does. Nevertheless, because a particular method of representing the forces does not include such a mode of transmission, the latter is not therefore disproved; and that method of representation which harmonizes with it may be the most true to nature. The general conclusion of philosophers seems to be that such cases are by far the most numerous, and for my own part, considering the relation of a vacuum to the magnetic force and the general character of magnetic phenomena external to the magnet, I am more inclined to the notion that in the transmission of the force there is such an action, external to the magnet, than that the effects are merely attraction and repulsion at a distance. Such an action may be function of the ether; for it is not all unlikely that, if there be an ether, it should have other uses than simply the conveyance of radiations (2591, 2787). [...]

Although Faraday does not seem to have utilized the expression electric field in his works, he would probably understand it as any portion of space traversed by lines of electric force (as we saw before, he defined these lines in §2149).

3. Maxwell's Field Concept

James Clerk Maxwell (1831-1879) followed closely Faraday's ideas and tried to express them mathematically. In a paper published in 1864 called "A dynamical theory of the electromagnetic field", he wrote, [8, p. 527]:

> (3) The theory I propose may therefore be called a theory of the *Electromagnetic Field*, because it has to do with the space in the neighbourhood of the electric and magnetic bodies, and it may be called a *Dynamical* Theory, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.
>
> (4) The electromagnetic field is that part of space which contains and surrounds bodies in electric or magnetic conditions.

In his *A Treatise on Electricity and Magnetism* originally published in 1873, we find the same definition of *field* as a region of space surrounding electrified or magnetized bodies. On §44 he wrote, [9, Vol. 1, p. 47]: "The electric field is the portion of space in the neighbourhood of electrified bodies, considered with reference to electric phenomena." A similar definition is given in §476 as regards the magnetic field generated by a wire carrying a steady current (Oersted's experiment), [9, Vol. 2, p. 139, our emphasis]: "It appears therefore that in the space surrounding a wire transmitting an electric current a magnet is acted on by forces dependent on the position of the wire and on the strength of the current. *The space in which these forces act may therefore be considered as a magnetic field*, and we may study it in the same way as we have already studied the field in the neighbourhood of ordinary magnets, by tracing the course of the lines of magnetic force, and measuring the intensity of the force at every point."

It is important to mention that in Maxwell's theory concept of space, as we can observe, for example, in the passage that follows [9, Vol. 2, p. 158, §502]:

> The ideas which I have attempted to follow out are those of action through a medium from one portion to the contiguous portion. These ideas were much employed by Faraday, and the development of them in a mathematical form, and the comparison of the results with known facts, have been my aim in several published papers.

4. Conclusion

From these quotations we can see that Faraday and Maxwell defined the electric and magnetic fields as a region of space in the neighbourhood of electrified and magnetized bodies, respectively. Moreover, it appears as a reasonable hypothesis in the works of these two authors that this space was completely filled with something like an ether.

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Time dependence of Rn atom inside a Carbon nanotube

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This system consists of a nanomotor of a relaxing Rn atom inside of a rigid and static nanotube. We calculated temporal thermodynamic properties of these devices as molar specific heat and entropy variation at 300K. The simulation was made by classic molecular dynamics with standard parameterization.

1. Introduction

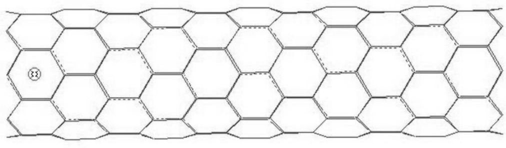
The molecular motor study is still in the beginning. Although its importance to nanomedicine and nanorobots, there are a few works about it. There are many molecular motors in nature, for example, the kinesin motor. [1,2] There are linear molecular motors[3] and rotatory motors.[4] Also, Legoas et al [5] did the first theoretical classical molecular dynamics simulation of the molecular motor. Guo et al [6] showed that energy dissipation exists in the molecular motor. Also, we calculated thermodynamics properties like entropy and efficiency. [7,8]

In this work, we propose to simulate one molecular motor using a Radonium atom (Rn) and zigzag carbon nanostube (CNT) to study molar specific heat, molar entropy variation, efficency changing with the time at 300 K.

2. Methodology

We propose a computational system with the Rn atom relaxing inside of the CNT (Figure 1). We also verified that the initial position very close to the extremity results a strong van der Waals gradient of energy potential, i.e. occurs an acting force in the probe.

Figure 1
The initial position of the Rn atom probe (Rn) at the left side inside of the carbon nanotube (CNT), which oscillates like a molecular motor.



The CNT has 192 atoms, 23.96 Angstroms of length and 6.04 Angstroms of diameter. The Rn atom has 1.34 Angstroms of atomic radius.

We did the simulation at vacuum and a run time of 8ns, with no cool time and step size equal to 1 fs and temperature of 300 K with the same methodology as Guo et al [6] and A.M. Chaves Neto. [7,8]

3. Results and Discussions

Figure 2 shows the energies and temperature *in situ* (TEMP) behavior of molecular motor at 300K. We can see that when the kinetic energy (EKIN) decreases with the time, the energy potential (EPOT) goes down. The EKIN varies directly proportional to the temperature for all time, as expected for the high temperature systems. There are many oscillations and the CNT almost stops several times. The EPOT is almost constant for all time. The EKIN maximum value is 250 kcal/mol, EPOT maximum is 150 kcal/mol, ETOT is 750 kcal/mol and TEMP maximum is 1280 K. The energy lost (ETOT-(EKIN+EPOT) is 350 kcal/mol. We observed that the energies decrease very slowly with the time because the energy and TEMP values at 0 ps is smaller than at 6ps.

Figure 2
Kinetic energy (EKIN), potential energy (EPOT), total energy (ETOT) (kcal/mol) and temperature *in situ* (TEMP) (Kelvin) versus time at initial temperature 300 K.

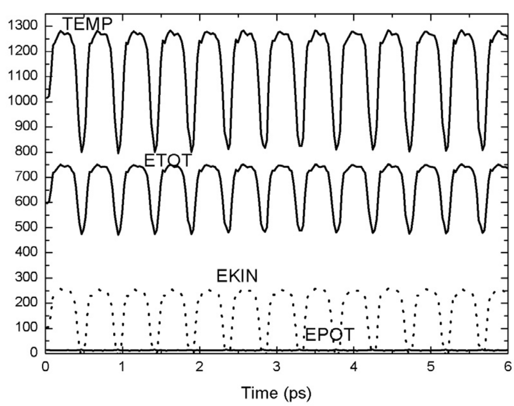


Figure 3 shows the molar specific heat of this molecular motor versus time at 300K. There is a peak at 0 ps and it decreases because there is a loss of energy by the atoms of this system. After time 120 ps, the molar specific heat turns on asymptotically constant.

Figure 3
Molar specific heat versus time at initial temperature 300 K.

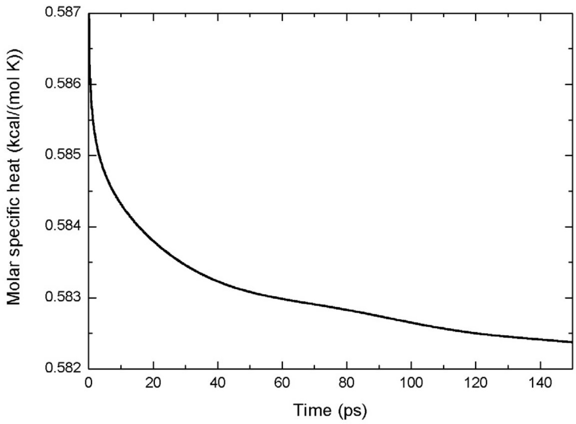


Figure 4 presents the molar entropy variation versus time at initial temperature of 300 K. There are many oscillations because of the repulsion between atoms of the CNT+Rn system. There is a period of increase in the molar entropy variation. It is caused by the thermal energy gained by external temperature received by this motor.

Figure 4
Molar entropy variation (kcal/ (mol K)) versus time at initial temperature 300 K.

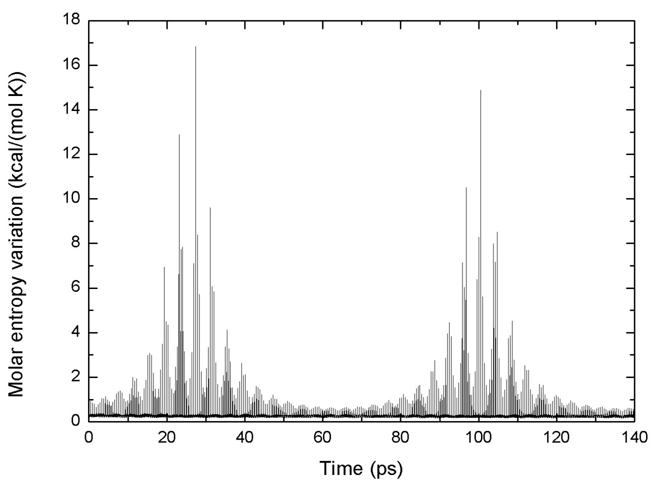
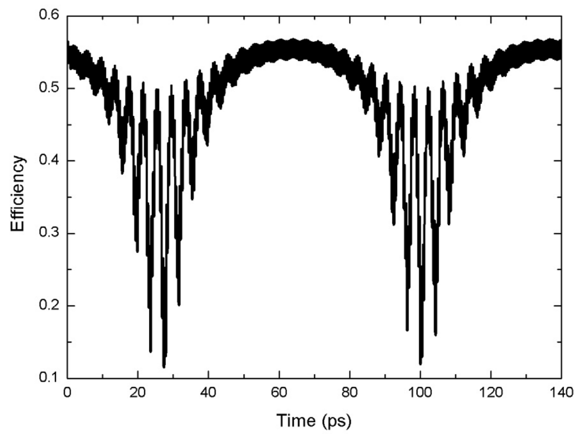


Figure 5 displays the efficiency (EKIN/ETOT) of this motor at initial temperature 300K. The efficiency oscillates a lot because of the collisions between CNT+Rn atoms. The efficiency decreases in the collisions of CNT with the branch at 37 ps and 100 ps. At 0 ps; 65ps and 140 ps there is a maximum efficiency gained by the external heat.

Figure 5
Efficiency versus time at initial temperature 300 K.



Comparing the Figures 2, 4 and 5 we can see when the Rn atom stops at the CNT extremity, the molar entropy variation increases. But, the efficiency and the rate EKIN/TEMP decrease. The molar entropy variation is inversely proportional to the efficiency; it means that there is a delay of the efficiency and the exchange of information of the atoms.

4. Conclusions

We concluded that the external temperature gives EKIN to the probe in this motor. There is high loss of energy before the time of 80 ps, after this time it tries to have a constant value. The oscillating efficiency found here is very low and far from the Carnot motor. The EPOT decreases and the EKIN increases like a harmonic oscillator. Also, we saw that there is a delay of the efficiency and the exchange of information of the atoms. Here, the rate EKIN/TEMP changes with the initial temperature. This system like this molecular motor here could be used like a specific clock to work at picoseconds to nanoseconds, because the period does not vary very much and there is a little loss of energy with the time.

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How can classical chaos be reconciled with quantum order?

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This work shows how the extremely irregular character of classical chaos can be reconciled with the smooth and wavelike nature of phenomena on the atomic scale. It is demonstrated that a wave packet under continuous quantum measurement displays both chaotic and non-chaotic features. From a path-integral formalism to describe a continuously measured quantum oscillator's position by a classical particle, a wave packet approach is formulated to establish a direct correlation between a classical variable X with a quantum variable x.
The Lyapunov characteristic exponents for the trajectories of classical particle and the quantum wave packet center of mass are calculated and their chaoticities are demonstrated to be about the same. Nonetheless, the width of the wave packet exhibits a non-chaotic behavior and allows for the possibility to beat the standard quantum limit by means of transient, contractive states.

Unlike classical chaos, there is no universally accepted definition of quantum chaos. [1] The main problem is that, due to Heisenberg's uncertainty principle, there is no unambiguous way in quantum mechanics to evaluate the degree of chaoticity via Lyapunov exponents and Poincar´e sections from phase space as in classical mechanics. Therefore, researchers have tried to characterize quantum chaos in different ways, and a variety of problems has been considered between classical and quantum systems under the conditions where classical chaos is present.[2]-[18] Nonetheless, how can the extremely irregular character of classical chaos be reconciled with the smooth and wavelike nature of phenomena on the atomic scale? This paper shows that a wave packet under continuous quantum measurement can display both chaotic and non-chaotic features so long as the classical variable is linearly coupled to the quantum variable.

Among the many approaches formulated in the quest for chaotic signals in quantum mechanics, Ballentine et al.[19] have argued that quantum-classical correspondence in chaotic systems can be preserved when stated in terms of the expectation values of dynamical variables. Also, Tomsovic and Heller [20] have demonstrated that quasiclassical methods are successful for times longer than was previously believed possible, and that even after this divergence time, the differences remain modest, typically 5%-10%. Nevertheless, attempts to clearly characterize notions of chaos in quantum dynamics as a formal semiclassical limit still remain elusive. To shed some insight on this problem, a wave packet approach is developed here having in mind the fact that in order to extract classical trajectories, one expects observed quantum systems to obey classical dynamics at the macroscopic level. This wave packet approach is also motivated by the advent of short-pulsed lasers which have made possible the production and dectection of coherent superpositions of quantum-mechanical electron states for a variety of physical systems. Once produced, such superpositions can result in the formation of localized electron wave packets.

The evolution and dynamics of wave packets are the subject of much current investigation in many areas of both physics and chemistry.[21, 22] In molecular physics, a new realm of phenomena involving wave packets has opened up with the emergence of femtosecond pulse technology.[23] Wave packets have also been produced in semiconductor quantum well systems.[24] The use of wave packets to analyze the dynamics of quantum mechanical systems is crucial for the study of the classical-quantum interface. Further, the description of a real scattering event should correspond to the established experimental observation of localized particles approaching a scattering center and subsequently receding from it. This entails the construction of a wave packet state and the analysis of its evolution in time.

Starting from a path-integral formalism to describe a continuously measured quantum particle's position by a classical particle, a wave packet approach is formulated here to establish a direct correlation between a classical variable X with a quantum variable x. The Lyapunov characteristic exponents for the trajectories of classical particle and the quantum wave packet center of mass are calculated and their chaoticities are demonstrated to be about the same. Nonetheless, the width of the wave packet exhibits a non-chaotic behavior and allows for the possibility to beat the standard quantum limit by means of transient, contractive states. [25] In particular, the periodically driven Duffing oscillator, which has become a classic model for analysis of nonlinear phenomena,[2, 3, 26] is studied, and its classical chaos is shown to crossover into the quantum regime. The theory presented below focuses on some unresolved features posed by chaos and on the correspondence principle which is the main focus of many new experiments with excited atomic and molecular systems. These experiments can directly probe the realm of high quantum numbers or classically chaotic motion.

The renewed interest in coupling classical systems to quantum ones has been revived by a number of authors[27, 28, 29] who have examined continuous quantum measurements. The question of coupling classical variables to quantum variables is intimately connected to the question of how certain variables become classical in the first place.[4] In reality, there are no fundamentally classical systems, only quantum systems that are effectively classical under certain conditions. One must start from the underlying quantum theory of the whole composite system and then derive the effective form of the classical theory. The starting point is to think of the classical particle as continuously monitoring the quantum particle's position and responding to the measured value. To this end, consider a classical particle of mass M with position X in a nonlinear potential, the periodically driven Duffing oscillator, coupled to a quantum oscillator of frequency ω and mass m:

$$M\ddot{X}(t) + BX^3(t) - AX(t) + \lambda\bar{x}(t) = \Lambda\cos(\Omega t)\,, \tag{1}$$

where $\bar{x}(t)$ is associated with the measurement record of the quantum system. The evolution of the wave function of the quantum system ψ can be expressed at first in terms of the path-integral for the unnormalized wave function:

$$\psi(x',t') = \int \mathcal{D}[x(t)]\exp\left[\frac{i}{\hbar}\int_0^{t'} dt\left(\frac{1}{2}m\dot{x}^2(t) - \frac{1}{2}m\omega^2x^2(t) - \lambda x(t)X(t)\right)\right] \times \exp\left(-\int_0^{t'} dt\frac{[x(t)-\bar{x}(t)]^2}{4\sigma^2(t)}\right)\psi(x_0,0), \tag{2}$$

where the path integral is over paths $x(t)$ satisfying $x(0) = x_0$ and $x(t')= x'$. The quantity σ in the equation above represents the resolution of the effective measurement of the particle by the classical system, as indicated by previous works.[27, 28, 29] However, some differences here are worth mentioning. One is the time dependence of the quantity $\sigma(t)$: most importantly is the novelty that the general resolution of the measurement evolves according to a nonlinear differential equation. Another difference relates to the dimension of the quantity $\sigma(t)$: it should be considered only proportional to the actual position uncertainty in the measurement of the quantum particle. So, an explicit connection to a wave packet approach can be established by writing $\sigma^2(t)= \tau\delta^2(t)$, where δ and τ have dimensions of space and time, respectively. This point can be further elucidated by approximating the last term of Equation (2) around an average time $\bar{t}$,i. e., ~ exp – [($x(\bar{t}) - \bar{x}(\bar{t}$

$))^2/4\delta(\bar{t}\,)^2]\exp(-\bar{t}/\tau)$, where $\delta(t)$ clearly stands for the position uncertainty (width of the wave packet) and τ characterizes the time constant (relaxation time) of the measurement.

Now, the square of the absolute value of Equation (2) yields the probability density for different measurement outputs at different times and from this equation the associated Schr¨odinger equation describing the system undergoing continuous measurement can be written as:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + \left(\frac{1}{2}m\omega^2x^2 + \lambda x X(t)\right)\psi(x,t) - \frac{i\hbar}{4\tau}\left(\frac{[x-\bar{x}(t)]^2}{\delta^2(t)} - 1\right)\psi(x,t). \tag{3}$$

Next, a solution to this equation can be found by considering previous findings [26, 27] which have shown that continuous position measurement produces and maintains localization in phase space as a necessary result of the information it provides. In addition to localizing the state, a continuous position measurement can also introduce noise in its evolution: the measured value $\bar{x}\,(t)$ can be associated with a mean value $<x(t)>$ plus a noise-dependent component $\xi(t)$. So, in order to obtain a semiclassical protocol one must be in a regime in which the localization is relatively strong and the noise sufficiently weak. However, a protocol based on a complete hierarchy of stochastic equations associated with the average value of the position $<x(t)>$ makes it difficult to obtain an analytic solution to the problem.[27, 30] The details of the variances and resulting noise strength permit only partial solutions based on varying ¯h and steady state regimes. Therefore, a formalism that keeps the measurement record quantity $\bar{x}$ (t) without dealing with the details of the variances can circumvent this difficult task and give a direct description of the evolution of the quantum system. This rationale entails a wave packet solution around the measurement record $\bar{x}\,(t)$ as follows:

$$|\psi(x,t)| = \left[2\pi\delta^2(t)\right]^{-1/4}\exp\left(-\frac{[x-\bar{x}(t)]^2}{4\delta^2(t)}\right). \tag{4}$$

This minimum-uncertainty wave packet solution is further supported by recent, alternative stochastic approaches[26] which have demonstrated that individual quantum trajectories remain minimum-uncertainty localized wave packets for all times: the localization being stronger the smaller $\hbar$ becomes. Similar localization properties hold also for a variety of quantum trajectory methods[31, 32, 33] where the mean uncertainty product $M[\Delta x \Delta p]/\hbar$ remains close to 1 almost independent of $\hbar$, thus corroborating the minimum-uncertainty ansatz (4). These quantum trajectory methods have been used extensively in recent years due their intimate connection to continuous measurement.

Employing a method of variation of coefficients,[34] the auxiliary functions of time $\delta(t)$and $\bar{x}(t)$ of the wave packet (4) can be shown to conform to the following equations:

$$\ddot{\bar{x}}(t) + \omega^2 \bar{x}(t) + \left(\frac{\lambda}{m}\right) X(t) = 0. \tag{5}$$

and

$$\ddot{\delta}(t) + \frac{1}{\tau}\dot{\delta}(t) + \left(\omega^2 + \frac{1}{4\tau^2}\right)\delta(t) = \frac{\hbar^2}{4m^2\delta^3(t)} \tag{6}$$

Equations (5) and (6) show that a continuous measurement of a quantum oscillator gives specific features to its evolution: the appearance of distinct classical and quantum elements. This measurement consists of monitoring the position of the quantum system and the result is the measured path $\bar{x}(t)$for t within an uncertainty $\delta(t)$. The solutions to these equations are presented as follows.

First, Equation (5) demonstrates the claim that continuous measurement can effectively obtain classical mechanics from quantum mechanics. This issue can be substantiated by the numerical solutions of the coupled equations (1) and (5).[36] The error tolerances used to limit the local truncation error are here set to be 10 digits less than the working precision, which is fine for our purposes. By this means, solutions to the coupled system of equations (1) and (5) are implemented, and the Lyapunov exponents for both systems are calculated. Using this high-precision solution one can plot the phase-plane distance for the two solutions as t increases. So, the usual signature of chaos is found and illustrated in phase-space (See Fig. 1). Although these trajectories are quite different, the Lyapunov characteristic exponent for the quantum regime is found to be about the same as that of the classical regime. The divergence of trajectories with a 10^{-6} perturbation in the initial conditions is plotted in Fig. 2. The Lyapunov exponents that separate different time scales of motion are established for both classical and quantum solutions as follows:

$$\lambda_{(cl,qu)} = \lim_{\substack{t\to\infty \\ \Delta(0)\to 0}} \left\{ \ln\left[\Delta_{(cl,qu)}(t)/\Delta_{(cl,qu)}(0)\right]/t \right\}, \tag{7}$$

where

$$\Delta_{(cl,qu)}(t) = \left\{ [(X^+, \bar{x}^+) - (X^-, \bar{x}^-)]^2 + [(\dot{X}^+, \dot{\bar{x}}^+) - (\dot{X}^-, \dot{\bar{x}}^-)]^2 \right\}^{1/2} \tag{8}$$

represents the renormalized classical, quantum Euclidean distances of the trajectories in phase space, respectively. Equation (7) describes explicitly the asymptotic rate of exponential divergence of the classical and quantum trajectories evolving from two initially close initial conditions, respectively. It appears that, at least initially, the logarithmic divergence of trajectories with a very small perturbation in the initial conditions is roughly linear on this plot, indicating an exponential relationship. To find the exponent we need to find a line that fits the logarithm of the data. Thus, it is appropriate to use only the data up to the point where the difference is of order one. Although a perturbation causes exponential divergence locally, solutions near this initial condition are attracted to a strange attractor, which is a bounded set with zero area. Since this set is bounded, the divergence cannot continue indefinitely. A regression on the data gets us a reasonable exponential function to model the divergence: for the classical case, $8 \times 10^{-7} e^{0.17(1)t}$ and for the quantum case $5 \times 10^{-7} e^{0.16(8)t}$. Thus, the behavior of a quantum wave packet center of mass and the monitoring classical coordinate are equally chaotic and the Lyapunov exponents for both cases is found to be: $\lambda_{qu} \simeq \lambda_{cl} = 0.17$.

On the other hand, Equation (6) shows that the width of the wave packet exhibits a non-chaotic behavior. Fig. 3 depicts in phase space the complete time evolution of the wave packet width $\sigma(t)$: it shows a transient, non-chaotic squeezed-state solution. In this context, a solution to Equation (6) for a free particle ($\omega = 0$) supports qualitatively Yuen's conclusions [25] so far as showing the possibility to beat the standard quantum limit by means of transient, contractive states. Extensive deliberations on how to defend or beat the standard quantum limit for both discrete and continuous measurements of the position of a quantum particle can be found in the literature.[37]–[42] Accurate measurements of the position of a particle is of much interest in the context of gravitational-wave detection where questions have arisen as to whether there are fundamental quantum mechanical limits on detection sensitivity. The point here is that discrete or continuous measurements may introduce squeezing that affects subsequent measurements. The features presented in Fig. 3 are not encountered in previous works.[27, 28, 29] Besides, the resolution squared [$\sigma^2(t) = \tau\delta^2(t)$] of the measurement can reach a stationary regime, namely:

$$\sigma_o^2 = \frac{\hbar\tau/m}{(1+4\omega^2\tau^2)^{1/2}}, \tag{9}$$

which indicates that localization can occur on a time scale which might be extremely short compared to the oscillator's frequency ω. For the low-frequency limit $\omega\tau << 1$ (the free particle limit $\omega = 0$), this result reduces to $\sigma_0^2 = \hbar\tau^2/m$. On the other hand, for the high-frequency limit $\omega\tau >> 1$, $\sigma_0^2 = \hbar\tau/2m\omega$. These results show that the resolution σ of the effective measurement increases as the characteristic time constant τ (relaxation time) increases.

To conclude, this work has developed a wave packet approach from a path-integral formalism to describe a continuously measured quantum particle's position by a classical particle and to establish a direct correlation between a classical variable X with a quantum variable $\bar{x}$. It shows how the extremely irregular character of classical chaos can be reconciled with the smooth and wavelike nature of phenomena by demonstrating that a wave packet under continuous quantum measurement displays both chaotic and non-chaotic features. The Lyapunov characteristic exponents for the trajectories of classical particle and the quantum wave packet center of mass are calculated and their chaoticities are demonstrated to be about the same. On the other hand, the width of the wave packet exhibits a non-chaotic behavior and allows for the possibility to beat the standard quantum limit by means of transient, contractive states.

Figure 1:
Comparison between the classical and quantum signatures of chaos in phase-space for M =Ω= A = B = λ = ω =1and Λ = 1⁄3. The top graph plots in phase space P versus X which represent the momentum and position of the classical particle. The bottom graph plots in phase space $\overline{p}$ versus $\overline{x}$ which represent the momentum and position of the wave packet center of mass.

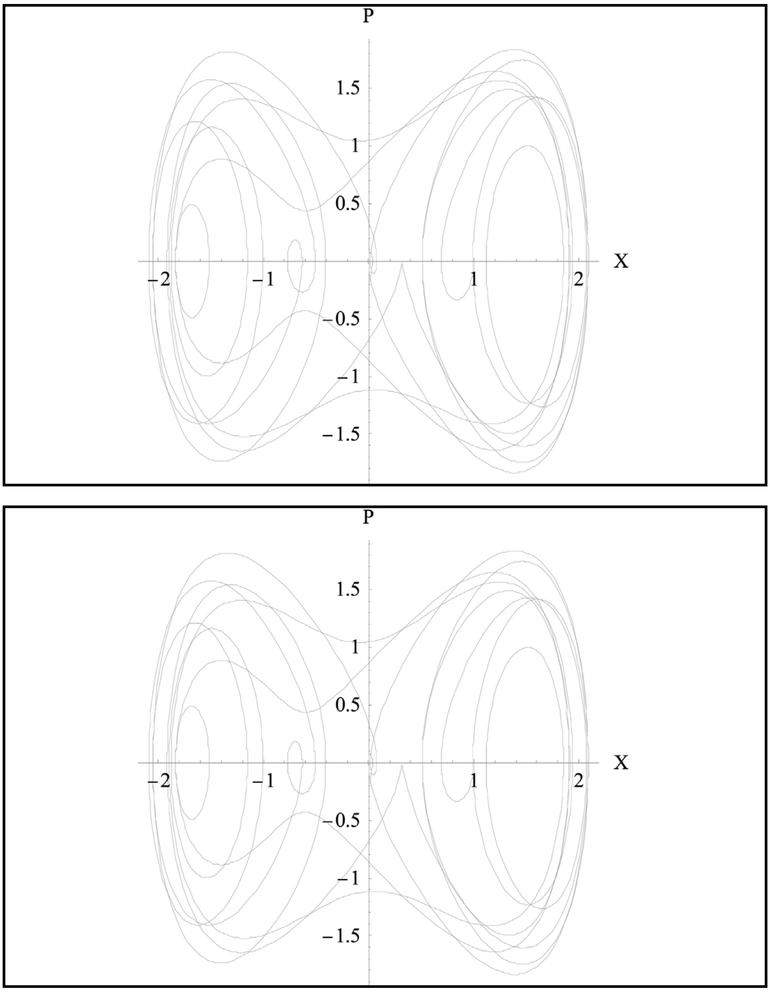


Figure 2:
The divergences of classical (solid line) and quantum (dashed line) positions X and $\bar{x}$ for Fig. 1 with a 10^{-6} perturbation in the initial conditions. These are solutions to Equations (1) and (5). The parameters used are the same as in Fig. 1.

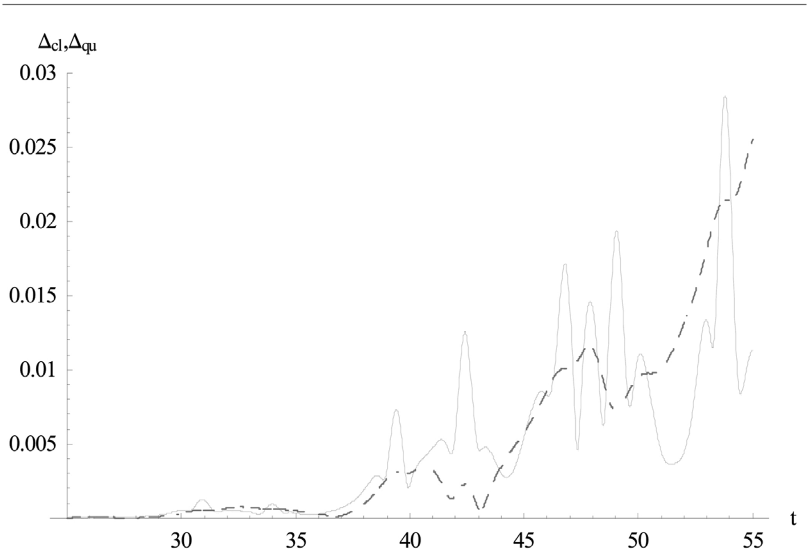
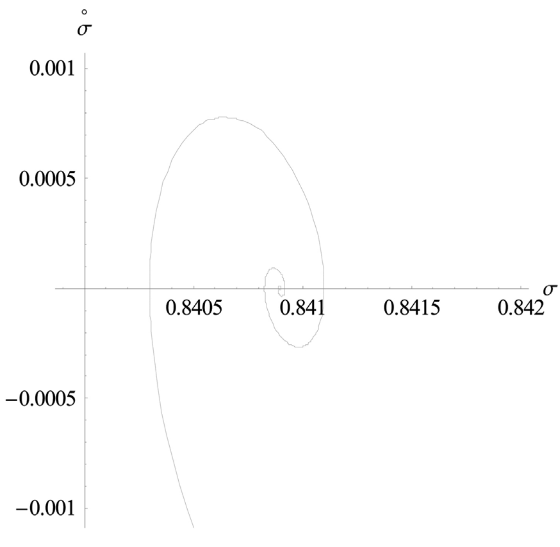


Figure 3:
The complete dynamics in phase space $\dot{\sigma}(t)$ versus $\sigma(t)$ for $\hbar\tau^2/m = 1$ and $\omega\tau = 1/3$. This graph depicts the general solution to Equation (6) which represents the width $\delta(t)$ of the wave packet: $\sigma^2(t) = \tau\delta^2(t)$.



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Stable Magnetic Levitation

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The present work is proposed to expose an explain experiments involving stable magnetic levitation. Despite the Earnshaw´ s theorem, that avoids the levitation of magnetic and electrical charges in the presence of static electromagnetic and gravitational felds, one show that there is at least two situations where the existence of diamagnetic material provides a stable levitation confguration. We show also magnetic energy and forces if change when in the presence of materials diamagnetism. In this sense, some experimental devides - composed by magnets, supports and diamagnetic boundaries - were set up and succeed in yielding stable levitation of magnetic bodies.

Introduction

The observation of levitating bodies has awoken great interest over the latest years, both by the fascination inherent to this phenomenon and the innumerous possibilities of technological applications in systems where it is extremely desirable to eliminate any type of mechanic contact, friction or dissipation. The history of this discovery goes back to 1939 when W.Braunbeck [1] succeeded in levitating small pieces of graphite in a region permeated by the feld of a permanent magnet. Some years after (1947) V. Arkadiev [2] used a lead disk immersed into liquid helium (a superconductor) to levitate diminutive magnets. Since the publication of this pioneer result the experiments with superconductor levitation received great attention, increased more yet in the 80's years with the discovery of the high-Tc superconductivity [3] .These new materials have become the magnetic levitation process especially stable due to the pinning mechanism usual to the type-II superconductors [4]. In 1996 the observation of a new kind of magnetic levitation caught the attention of scientifc community the levitron that consists basically in a magnetic dipole in shape of a top, spinning and Floating above a fixed base with permanent magnetization. The interesting fact is that the mathematical formulation used to explain the Levitron^{TM} [5] [6]can be also employed with success for approaching the levitation of diamagnetic samples.

The levitation of purely magnetic origin is a motion of surprise between the physicists due to the well know Earnshaw's theorem [7] (1842) that establishes the impossibility of fnding a stable equilibrium point inside a confguration of static felds (of electric, magnetic or gravitational nature), a consequence of Maxwell 's equations. By another way, one can say that the earnshaw's theorem does not allow the existence of a minimum for the potential magnetic energy, as it is necessary for observation of a stable equilibrium point. The presence of diamagnetic materials inside felds, however, modifes the shape of the potential energy, creating the possibility of a minimum point without violation of the Earnshaw's theorem [8], [9] ,[8].

Diamagnetic materials have negative magnetic susceptibility, repulse and are repulsed by a strong magnetic feld. When an external magnetic feld acts on the electrons of a diamagnetic sample, they tend to adjust their orbital motion in such a way to create small permanent currents that oppose to the external feld. The graphite and the bismuth are the two materials that present the major (absolute) magnetic susceptibility, being therefore the most suitable for accomplishment of levitation experiments (for requiring magnetic felds of minor intensity) [10].

The first evidence that diamagnetic materials could act as a particular case beyond the generality of the Earnshw's theorem was given by Lord Kelvin in 1847 [11], eight years after the publication of this theorem. At the time he demonstrated qualitatively that these materials could be put in stable equilibrium when immersed into magnetic felds. The theoretical and experimental demonstration of this possibility was given in 1939 by W.Braunbeck [1], in whose experiments were verifed the achievable character of the process. After this publication, there were several attempts of obtaining levitation with usage of diamagnetic materials. In 1956 Boerdijk used a horizontal graphite sheet (inside a magnetic feld) to stabilize the levitation of a magnet above it [12]; in 1981 Ponizovskii presented some devices constructed to yield the levitation of diamagnets inside magnetic felds, and he also utilized pyrolytic graphite at stabilizing confgurations for accomplishing levitation of small magnets [13]. It is important observe that both Boerdijk and Ponizovskii exposed the effect of the diamagnetic stabilization, but without developing a satisfactory theory to explain it. At the beginning of 90s, this issue gained again notoriety after that E. Beaugnon and R.Tournier achieved the levitation of drops of water and ethanol, pieces of wood, plastics and other organic materials [14]. This work showed that the major parts of organic materials are levitate, since all have almost the same diamagnetic specifc susceptibility. In 1996 Weilert at al. Levitated drops of liquid helium in order to study noncoalescence effects [15].

Renewed the interests by the theme, some other works have arisen: A. Geim at al. [16] and M.Berry and A. Geim obtained the levitation of various diamagnetic samples ñincluding an alive frog ñinside a solenoid subjected to intense magnetic feld of 16T . In this last article it is accomplished the evaluation of stability zones (regions where can occur the stable levitation inside the solenoid) based on the formalism developed to the Levitron. Finally, more recently, Geim at al. And Simon & Geim presented experiment and theory related to the levitation of small magnets inside experimental setups with diamagnetic stabilizing boundaries (disks and cylindrical sheets) [16]. The use of these boundaries allows the constitution of a region of horizontal and vertical stability.

1. Earnshaw's Theorem

In 1842 Samuel Earnhaw published a theorem of large validity where particles wich interact by any type or combination of $1/r^2$ forces can have no stable equilibrium position. Earnshaw´s theorem depends depends on a mathematical property of $1/r$ type energy potencials. The Laplacian of any sum of $1/r$ type potentials is zero, or $\Delta^2 \sum k_i/r_i = 0$. This means that at any point where there is force balance ($-\sum k_i/r_i = 0$), the equilibrium is unstable because there can no local minimum in the potencial energy [16]. Instead of a minimum in three dimensions, the energy potencial surface is a saddle (fig.1). If the equilibrium is stable in one plane, it is unstable in the orthogonal direction. An example of the application of Theorema Earnshaw is an interaction between two dipolos with charge equal to $\pm m$ separated by a distance d, with $M = ms$. The feld $\vec{B}(r)$ created by such dipolos [17], [18]is

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \frac{m}{r^3} \hat{u}_r \tag{1}$$

Adding the terms of each dipole at y e z, we have

$$\vec{B} \simeq \frac{M\mu_0}{4\pi r^5} \left\{ (2z^2 - y^2)\hat{u}_z + 3yz\hat{u}_y \right\} \tag{2}$$

The energy potential of magnetic dipole directed along the *y* axis is given by

$$U = -M.B = -\frac{M^2\mu_0}{4\pi r^5}(2z^2 - y^2) \qquad (3)$$

The plot of the equation 3 is show in de picture 1.
Varying the energy potencial for the *y* and *z* the force in the dipole is

$$F = -\nabla U = \frac{M^2\mu_0}{4\pi r^7}\left\{(9y^2 - 6z^2)z\hat{u}_z + (3y^2 - 12z^2)y\hat{u}_y\right\} \qquad (4)$$

The plot the force components vertical e horizontal are shows in the fgure 2 e 3. note that when there is a minimum stable in a components vertical force there is a minimum unstable in the horizontal components that prove the Earnshaw´s theorem in ferromagnetic materials.

Earnshaw´s theorem applies to a test particle, charged and/or a magnet, located at some position in free space with only divergence- and curl-free felds. No combination of electrostatic, magnetostaic, or gravitational forces can create the three-dimensional potencial well necessary for stable levitation in free space. The theorem also applies to any rigid array of magnet or charges. In the magnetostatic context instabilility may be thus demonstrated [19]. The equation of a magnetostatic feld, with uniform magnetisation M, are

Figure 1:
Energy potential of the interaction between two dipoles with charge equal $\pm m$. It surface is a saddle.

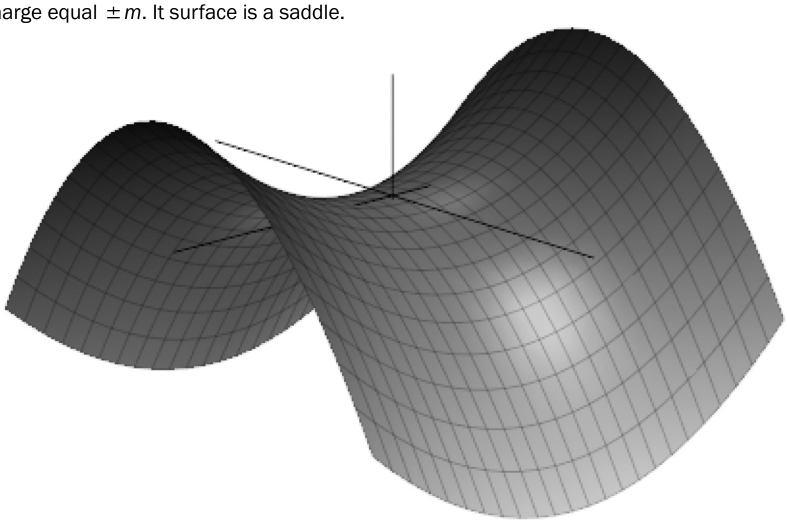


Figure 2:
Vrtical component force of the interaction between two dipoles with charge equal $\pm m$.

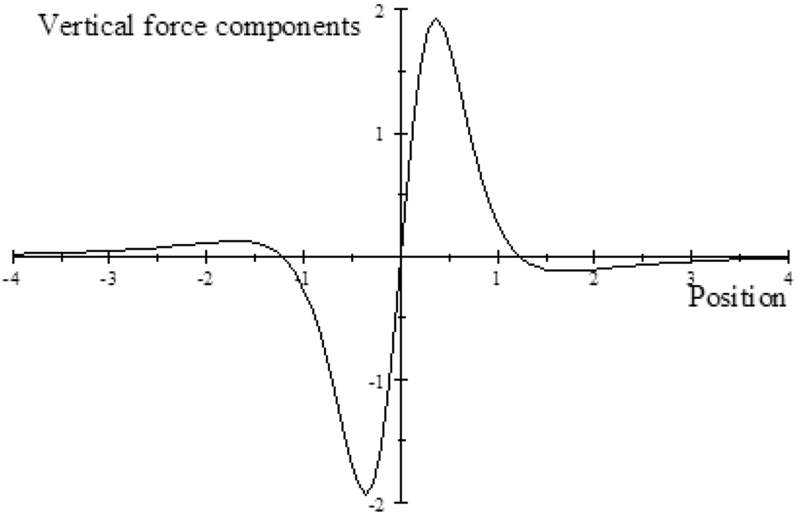
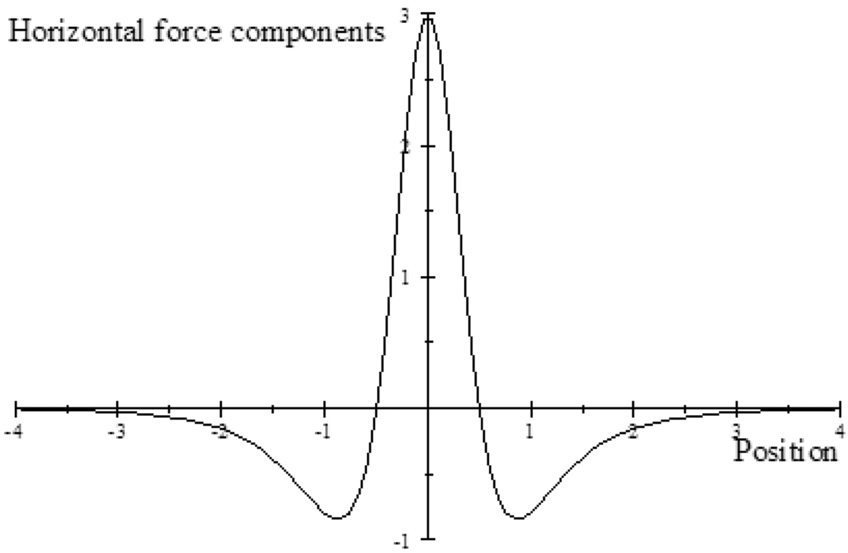


Figure 3:
Horizontal component force of the interaction between two dipoles with charge equal $\pm m$.



$$\oint_s \vec{B}.d\vec{s} = 0, \text{ and} \tag{5}$$

$$\oint_s \vec{B}.d\vec{s} = \mu_o \vec{j}_s \tag{6}$$

with $\vec{B}$ magnetic induction of the feld, μ_o magnetic permeabilitidy in vacuum, and $\vec{j}_s$ the surface current. For equation (5) the divergence of $\vec{B}$ is null so the components of force $\vec{F}$ are

$$\nabla F = (\partial F_x/\partial x) + (\partial F_y/\partial y) + (\partial F_z/\partial z) = 0. \tag{7}$$

But in order for the force F_x, F_y, F_z to be stable it is necessary that

$$\partial F_x/\partial x < 0, \quad \partial F_y/\partial y < 0, \quad \partial F_z/\partial z < 0. \tag{8}$$

Thus, if the equations (8) is valid, the equilibrium is unstable at least in one direction. The same conclusion can be reached by considering that for the potential V of $\vec{B}$ it is $\nabla^2 V = 0$, thus it not have a minimum, that implies that:

$$\vec{\nabla}.\vec{F}(\vec{r}) < 0 \text{ then } \nabla^2 U(r) > 0. \tag{9}$$

This equations (9) gives us the condition of stability. The energy of a magnetic material in the gravitational feld with volume V and magnetic susceptibility X subject to a magnetic feld $\vec{B}$ is[16]

$$U = -\frac{\chi B^2 V}{2\mu_0} + mgz \tag{10}$$

To balance the force of gravity, using that $\vec{F} = -\vec{\nabla} U(\mathrm{r})$, we require that

$$\frac{\chi V}{2\mu_0}\nabla B^2 = mg\hat{u}_z \implies B\nabla B = \mu_0 g\frac{1}{\chi}\frac{m}{V}\hat{u}_z \implies B\nabla B = \mu_0 g\frac{\rho}{\chi}\hat{u}_z \tag{11}$$

where ρ is the mass density of the material to be levited and $\hat{u}_z$ is the unity vector in the vertical direction. Stability requires, using $\nabla^2 U(\mathrm{r}) > 0$, that at the levitation point,

$$\nabla^2 U(r) = -\frac{\chi V}{2\mu_0}\nabla^2 B^2, \tag{12}$$

and we can see that only diamagnets, which susceptibility $\chi < 0$, can satisfy the stability condition. This stability condition, while necessary, is not suffcient. For stability, we must have positive curvature in the energy surface en every direction. We can write this condition for diamagnets as

$$\frac{\partial^2 B}{\partial^2 z} > 0 \text{ vertical stability} \tag{13}$$

$$\frac{\partial^2 B}{\partial^2 y} > 0, \ \frac{\partial^2 B}{\partial^2 x} > 0 \text{ horizontal stability} \tag{14}$$

For a cilindrically geometry we can expand the feld around the poit de levitation in terms of B_z componenent and its derivates,

$$B_z = B_0 + \left[\frac{\partial B}{\partial z}\right]_{z=z_0} z + \frac{1}{2}\left[\frac{\partial^2 B_z}{\partial^2 z}\right]_{z=z_0} z^2 - \frac{1}{4}\left[\frac{\partial^2 B_z}{\partial^2 z}\right]_{z=z_0} r^2 + ..., \text{ and} \tag{15}$$

$$B_r = -\frac{1}{2}\left[\frac{\partial B}{\partial z}\right]_{z=z_0} r - \frac{1}{2}\left[\frac{\partial^2 B_z}{\partial^2 z}\right]_{z=z_0} rz + ... \tag{16}$$

with square magnitude

$$\begin{aligned} B^2 &= B_0 + 2B_0\left[\frac{\partial B}{\partial z}\right]_{z=z_0} z + \left\{B_0\left[\frac{\partial^2 B_z}{\partial^2 z}\right]_{z=z_0} + \left[\frac{\partial B}{\partial z}\right]^2_{z=z_0}\right\} z^2 \\ &+ \frac{1}{4}\left\{\left[\frac{\partial B}{\partial z}\right]^2_{z=z_0} - 2B_0\frac{1}{2}\left[\frac{\partial^2 B_z}{\partial^2 z}\right]_{z=z_0}\right\} r^2 + ... \end{aligned} \tag{17}$$

In terms of this last equation, one can rewrite the stability conditions (13) e (14) in cylindrical coordinates

$$\frac{\partial^2 B}{\partial^2 z} > 0 \implies (B_0 B_z^{''} + B_z^{'2} > 0 \text{ vertical stability} \tag{18}$$

$$\frac{\partial^2 B^2}{\partial r^2} > 0 \implies (B_z^{'2} - 2B_0 B_z^{''}) > 0 \text{ horizontal stability} \tag{19}$$

where $B_z^{'} = \left[\frac{\partial B}{\partial z}\right]_{z=z_0}$ and $B_z^{''} = \left[\frac{\partial^2 B_z}{\partial^2 z}\right]_{z=z_0}$. Therefore, for diamagnetic samples it is always possible to find out small regions (around the point $B_z'' = 0$) where the stability equations (18) e (19) are satisfed and the levitation becomes possible (see fgure 4). At the case of magnetic feld its intetection energy with a magnetic feld depends linearly on the magnitude of the feld $B(r\,;\,z)$, so that its total energy is given by

$$U(r) = -M.B + mgz \tag{20}$$

From (17) one can take the magnitude of $B(r\,;\,z)$:

$$B = B_o + B_z^{'} z + \frac{1}{2} B_z^{''} z^2 + \frac{1}{4}\left(\frac{B_z^{'2}}{2B_0} - B_z^{''}\right) r^2 \tag{21}$$

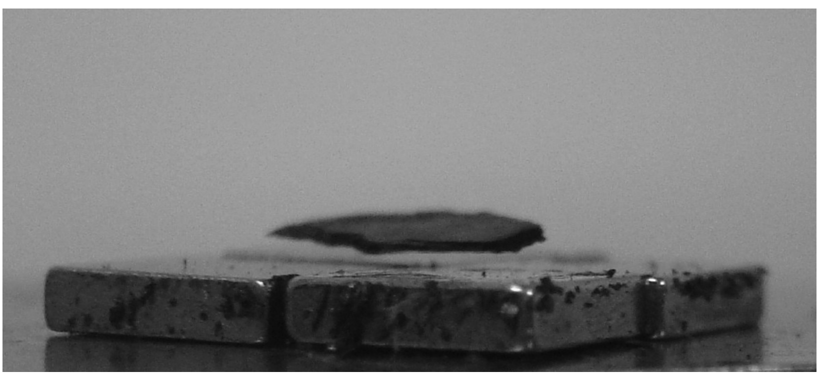
which substituted into (20) leads to the following expression for the total potencial energy of the dipole:

$$U(r) = -M \left\{ B_0 + \left[B_z^{'} - \frac{mg}{M} \right].z + \frac{1}{2} B_z^{''} z^2 + \frac{1}{4} \left(\frac{B^{'2}}{2B_0} - B^{''} \right) r^2 \right\} \tag{22}$$

In the last equation the term linear in z must be null to assure the vertical equilibrium between the weight and the lifting magnetic force. The new stability condition now reads

$$-\frac{M}{2} B_z > 0 \tag{23}$$

Figure 4:
Piece of graphite on a strong magnetic feld
- Department of physics - UFMA.



$$\frac{M}{4}(B'' - \frac{B'_z}{2B_0}) > 0 \tag{24}$$

At the proximity of the ináection point ($B''_z = 0$), around which it happens the levitation of last case, the second derivate can be negative or positive; if it is negative, the negative stability is assured, but not horizontal; if it is positive, there i no vertical satbility. In this case, the equilibrium will be achieved only after the introduction of stabilizing diamagnetic boundaries[20].

2 Magnetic levitation with borders diamagnetic

The influence of the diamagnetic sheets, with is show in the figure (5) and (6) upon the test magnetic can be through a technice similar to the images [16], [21] since a dipole near a diamagnetic sheet induces currents, whose effects may reproduced by the presence of an image charge with magnetization reduced in:

$$\frac{\mu - 1}{\mu + 1}\vec{M}. \tag{25}$$

For χ<< 1 the magnetization of the charge image is reduced to half of the original and in the on a perfect diamagnetic sheet (superconductor sheet, $\chi = -1$, diamagnetic perfect), one has total reflection, resulting in an image charge with same original magnetization.

Supposing a dipole feld aproximation, one can determine the magnetic feld $\vec{B}_i$ of the image charge, whose contribution to the energy of the test dipole $\vec{M}$ is:

$$U = -\frac{1}{2}\vec{M}.\vec{B} \tag{26}$$

Figure 5: Confguring a system with borders diamagnetic composed of two sheets.

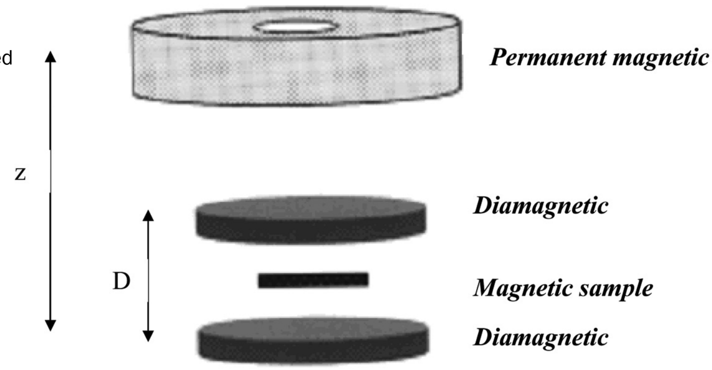


Figure 6:
Confguring a system with borders diamagnetic composed of one sheet.

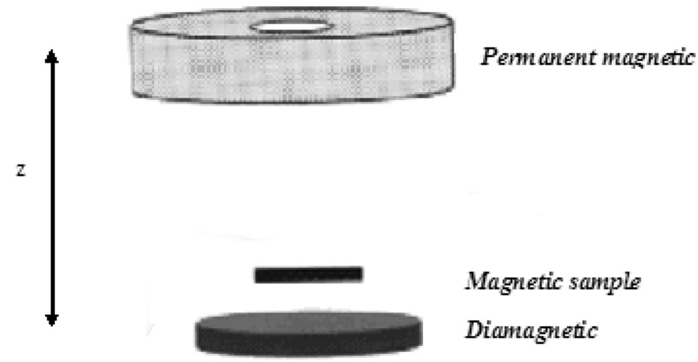
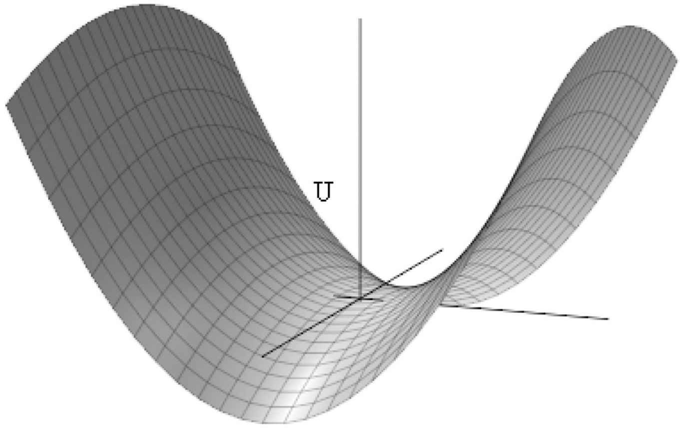


Figure 7:
Energy potential of the interaction between two dipolos with charge equal $\pm m$ when puts the diamagnetic sheets.



Let D be the separation between the two parallel diamagnetic sheets and carrying out an expression of the dipole felds around the levitation point, one obtain the following contribution to the potential energy of the test dipole:

$$U(z) = \frac{6\mu_0 M^2 \left|\chi\right|}{\pi D^5} z^2 = kz^2. \tag{27}$$

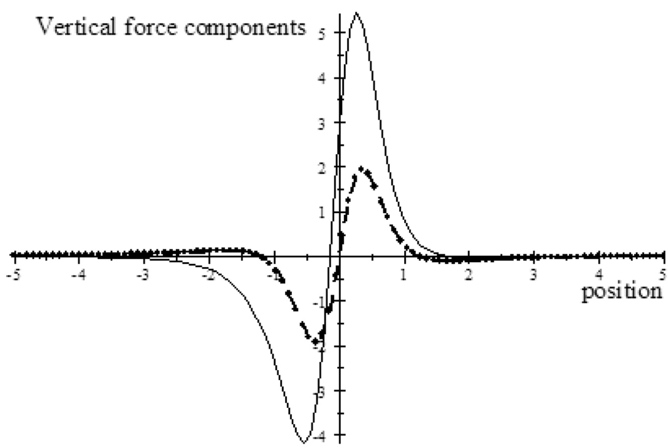
Observe that the magnetic energy potential leads to the appearance of a restorative force $F = -kz$ because the diamagnetic sheet. Also note that this energy will also transform the point of saddle of Figure 1 in a little bit of potential resulting in a stable equilibrium of the magnetic sample.The figure 7 shows the graphic potential of magnetic energy when puts the diamagnetic plates and added a term proportional to the square of the distance int the equation 3. When we add a linear term in the equation of horizontal component of force (equation 4) observed that the graphic approaches the graphic component of the vertical force due to material diamagnetic, as can be seen in the picture 8.

The inclusion of the equation (27) into the equations (22), (23) e (27) leads to an interval for the second derivative so that there can occur levitation:

$$B_z'^2/2B_0 < B_z'' < 2k/M. \tag{28}$$

The equation (28) encloses an interval for values of B_z'' for wich the stable levitation is achievable.

Figure 8:
Verticals components force of the interaction between two dipoles with charge $\pm m$. Line - when puts the diamagnetic sheets.



3. How to build a magnetic levitation with borders diamagnetic

To build a magnetic levitate with borders diamagnetic we use Bismuth and graphite. In the case of bismuth was made a disc of 0.5 cm in height and diameter of 4.0 cm. For the disks of graphite was used graphite powder pressed to 15 ton and placed in an oven at 900°C degrees. To the field we use of ferrite and rare earth magnets. To vary of the feld we use a support with a screw to close the distance and to vary the magnetic feld. At both experiment carried out succeeded in obtaining stable levitation as shown in Figures 9, 10 and 13. The fgure 11, 12 and 14 shows close view.

4 Conclusion

In this work we have done a review of stable magnetic levitation using materials diamagnetism. As an example of Earnshaw's theorem was given the behavior of potential energy and force magnetic of the interaction of a system composed of two dipolos with load equal to $\pm m$ separated by a distance d, with $M = ms$. Then noticed that the surface of magnetic energy potential having a minimum unstable, ie a point of cell (Figure 1) which represents that when you have a stable equilibrium in a one direction in the others the equilibrium is unstable. From the energy equation calculate the strength toward yez through the equation $\vec{F} = -\vec{\nabla} U$ (Figures 2 and 3). Also note (see Figure 2 and 3), when it has a stable equilibrium in the vertical compenents force the horizontal components force the equilibrium is unstable. When we add the equation plates diamagnetism magnetic energy potential is increased by a term proportional to z^2 so that the total energy will be changing to an area of less stable (Figure 7). From the new equation of energy potential can again calculate the component of force in the horizontal direction and noticed that you are approaching the component of force in vertical direction without the plates diamagnetism (Figure 8), occurring in this way the equilibrium stable in a small range along the axis z. We also present how to build this device and also noticed that there is no need for two plates (fgure 13) to have a stable equilibrium.

Figure 9:
Experimental setup composed by grafphite sheets.

Figure 10:
Experimental setup composed by Bismuth sheets.

Figure 11:
Close view into fgure 9.

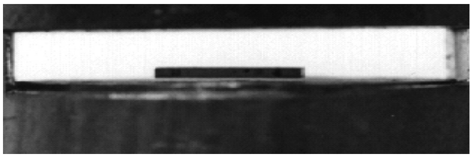


Figure 12:
Close view into fgure 10.

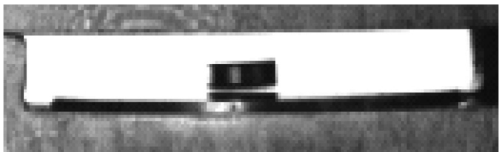


Figure 13:
Experimental setup composed only a graphite sheets.

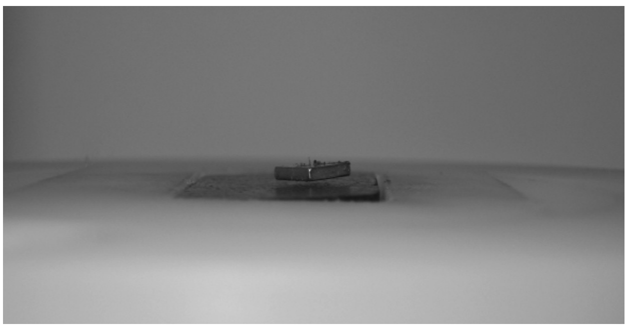
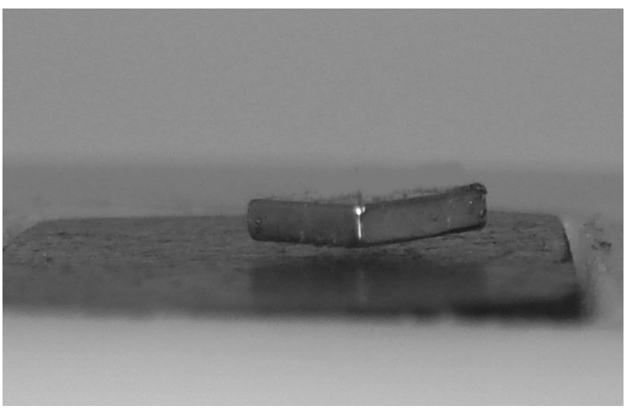


Figure 14:
Close view into figure 13.



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The Dynamical Casimir Effect

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In the present work we comment some results obtained by our research group in the UFPA (Federal University of Pará), related to theoretical aspects of the Dynamical Casimir effect. In this context, we discuss the problem of a real massless scalar field in a two-dimensional spacetime, satisfying Dirichlet or Neumann boundary condition at the instantaneous position of a moving boundary. For a relativistic law of motion, our investigations revealed that Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is invariant under time translations. Exact formulas for the energy density of the field and the radiation force on the boundary for vacuum, thermal and a coherent state, are analyzed. We also comment the investigations on the field inside a non-static cavity.

Besides the present authors, our research group has as members: João Paulo da Silva Alves (MSc student),Marcus Danilo F. B. da Costa (undergraduate student), Itamara Campos (undergraduate student), Alessandra N. Braga (undergraduate student) and Jeferson Danilo Lima Silva (undergraduate student).
Our group dedicate this work to Prof. José Maria Fillardo Bassalo, who gave a fundamental boost to its creation.

1. Introduction

In the 1970s the first papers related to the quantum problem of the radiation force acting on mirrors moving in vacuum were published (see Refs. [1-7]). Fullling and Davies [3] studied the moving mirror radiation problem in the context of a real scalar field in a two dimensional spacetime, with the Dirichlet boundary condition imposed to the field at the moving boundary position $x = z(t)$, in the context of the particular set of mirror trajectories for which $z(t < 0) = 0$, as shown in Fig. 1. They obtained the following exact formula for the finite physical part of the expected value of the energy-momentum tensor in the right side of the mirror (regions I and II in Fig. 1), assuming the initial state as the vacuum (we assume throughout this paper $h = c = k_B = 1$):

$$T_{vac} = -1/24\pi\,[p'''(u)/p'(u) - (3/2)p''(u)^2/p'(u)^2], \qquad (1)$$

where u and $p(u)$ are defined next, respectively in Eqs. (9) and (11) Their results revealed that the radiation is originated at the mirror and propagates away from it. This effect, and also others ones related to the reconstruction of the quantum state of the field due to the time dependence of the external conditions, are known as the Dynamical Casimir effect.

Ford and Vilenkin [8] developed a perturbative method which can be applied to mirrors moving in small displacements and with nonrelativistic velocities. In this approximation, they obtained, for a real scalar field in a two dimensional spacetime, that the radiation force is proportional to the third time derivative of the mirror law of motion, which is the nonrelativistic limit of the result obtained in Ref. [3]:

$$F_{vac}(t) \approx \dddot{z}/6\pi. \tag{2}$$

Ford and Vilenkin also applied their method to a scalar field in four-dimensional spacetime, obtaining a force proportional to the fifth time derivative of the displacement of the mirror.

For a non-static cavity with a moving boundary, Moore [1], in the context of a real massless scalar field in a two dimensional spacetime, investigated the radiation generated in this case. Imposing Dirichlet boundary condition to the field and also a prescribed law for the movement of a boundary, Moore obtained the following exact formula for the expected value of the energy-momentum tensor, assuming the initial field state as the vacuum:

$$\mathcal{T}_{\text{vac}} = -f(v) - f(u), \tag{3}$$

where

$$f = \frac{|\gamma|^2}{24\pi}\left\{\frac{R'''}{R'} - \frac{3}{2}\left(\frac{R''}{R'}\right)^2 + \pi^2\frac{1}{2}R'^2\right\}. \tag{4}$$

In the above equation the derivatives are taken with respect to the argument of the R function, solution of the following functional equation:

$$R[t + L(t)] - R[t - L(t)] = 2, \tag{5}$$

usually called Moore's equation, for which thereis no general technique of analytical solution. Law [9] obtainedan exact analytic solution for the Moore equation for a particular resonant movement of the boundary (see also Refs. [10]). Cole and Schieve [11] proposed a numerical method to solve exactly the Moore equation for a general law of motion of the boundary. Approximate analytical solutions of the Moore equation were also obtained, for instance, by Dodonov-Klimov-Nikonov [12] and Dalvit-Mazzitelli [13]. With approaches different of those adopted by Moore [1] and Fulling-Davies [3], perturbative methods where developed to solve the problem of a quantum field in oscillating cavities[14].

The first works investigating the problem of the creation of particles by moving mirrors with initial states different from vacuum were also published thirty years ago [5], showing that the presence of real particles in the initial state amplifies the phenomenon of particle creation. Jaekel and Reynaud [16] obtained for the scalar field in 1+1 dimensions the following thermal contribution to the dissipative force, proportional to the velocity of the mirror, valid in the nonrelativistic limit:

$$F^{(T)} \approx F^{(T)}_{(0)} = -2\pi T^2 \dot{z}/3. \tag{6}$$

Thermal effects have also been considered in Refs. [17–23]. The coherent state is another initial field state which has been considered [23–25], as well as thesuperposition of coherent states used to study decoherence via the dynamical Casimir effect [26].

Furthermore, attention has been given to the role that boundary conditions play on the dynamical Casimir effect. In the static Casimir effect, different boundary conditions can change the sign of the Casimir force [27]. The role of Dirichlet and Neumann conditions on the static Casimir force has been investigated in Ref. [28] in the context of the scalar field. Recently, several works have investigated the influence of different boundary conditions on the dynamical Casimir effect [24, 29, 30]. In this context, Farina, Maia Neto and one of the present authors, showed that Dirichlet and Neumann boundary conditions yield the same force on a moving mirror when the initial field state is symmetrical under time translations [24]. However, the validity of this conclusion, obtained in the context of the Ford-Vilenkin approach [8], was restricted to the nonrelativistic and small displacement approximations. In the work [23], instead of following the approximate approach considered in Ref. [24], three of the present authors used the exact approach proposed by Fulling and Davies [3], and showed that, also for relativistic laws of motion, Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is symmetrical under time translations. In the present work we review the main results obtained in Ref. [23], and also comment other results obtained by the research group in the UFPA.

2. One mirror problem

As done in Ref. [23] we start by reviewing the exact field solution for Dirichlet and Neumann dynamical boundary conditions [2–4]. Let us consider the field satisfying the Klein-Gordon equation: $\left(\partial_t^2 - \partial_x^2\right)\phi\left(t,x\right) = 0$, and obeying Dirichlet $(\phi'(t',x')|_{boundary} = 0)$ or Neumann $(\partial_{x}{}' \phi'(t',x')|_{boundary} = 0)$ condition, taken in the instantaneously co-moving Lorentz frame, at the moving boundary position $x = z(t)$. We examine the particular set of mirror trajectories for which $z(t < 0) = 0$, as shown in Fig. 1. Using the appropriate Lorentz transformation, Dirichlet and Neumann conditions can be written in terms of quantities in the laboratory inertial frame as follows: $\phi[t,z(t)] = 0$ and $\{[\dot{z}(t)\partial_t + \partial_x]\phi(t,x)\}|_{x=z(t)} = 0$, respectively. The mode field solution can be obtained by exploiting the conformal invariance of the Klein-Gordon equation, and can be written as:

$$\hat{\phi}\left(t,x\right) = \int_0^\infty d\omega\left[\hat{a}_\omega\phi_\omega + \hat{a}_\omega^\dagger\phi_\omega^*\right], \tag{7}$$

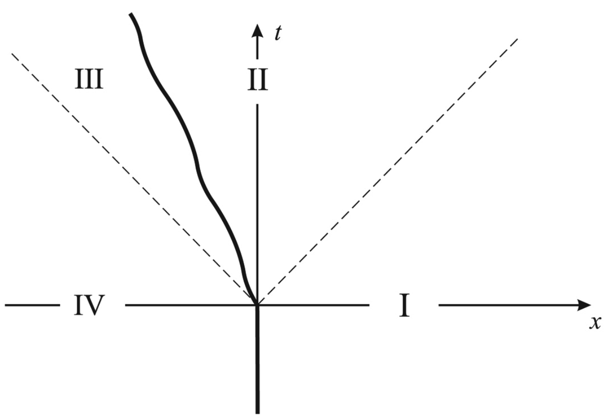
where

$$\phi_\omega\left(t,x\right) = \left(4\pi\omega\right)^{-\frac{1}{2}}\left[\gamma e^{-i\omega\pi r(v)} + \gamma^* e^{-i\omega\pi p(u)}\right] \tag{8}$$

form a complete set of positive-frequency solutions, and

$$u = t - x,\ v = t + x. \tag{9}$$

Figure 1:
Moving mirror trajectory. The dashed lines are null-lines separating the regions I from II, and III from IV.



In Eq. (8), we introduce a notation which enables us to put into a single formula the solutions for Dirichlet and Neumann boundary conditions, and also the solutions for the right hand side (regions I and II in Fig.1) and the left side (regions III and IV) of the mirror. In this sense, for $\gamma = 1$, Eq. (8) gives the Neumann solution, whereas for $\gamma = i$ we have the solution for Dirichlet boundary condition. For the regions I and II showed in Fig. 1:

$$r(v) = v \tag{10}$$

$$2\tau(u) - u = f^{1}(u) \equiv p(u), \tag{11}$$

where $\tau(u)$ can be obtained from:

$$\tau(u) - z[\tau(u)] = u. \tag{12}$$

For the regions III and IV:

$$p(u) = u \tag{13}$$

$$2\tau(v) - v = g^{-1}(u) \equiv r(v) \tag{14}$$

where

$$\tau(v) + z[\tau(v)] = v. \tag{15}$$

As causality requires, the field in the regions I and IV is not affected by the boundary motion[3], so that p and r are also chosen to be identity functions in these static regions. Hereafter we consider the averages $\langle\ldots\rangle$ taken over an arbitrary initial field state (regions I and IV) assumed here, for simplicity, as being the same one for both sides of the mirror. From the field solution (8), we can obtain the exact formulas for the expected value of the energy density operator $T = \langle\hat{T}_{00}(t,x)\rangle$, and the net force $F(t)$ acting on the moving boundary defined by (since $T_{00} = T_{11}$ in this model):

$$F(t) = T[t,z(t)]^{(-)} - T[t,z(t)]^{(+)}, \tag{16}$$

where the superscript "+" indicates the regions I and II, whereas "-" indicates the regions III and IV in Fig. 1. The net radiation force F acting on the moving mirror is given by [23]:

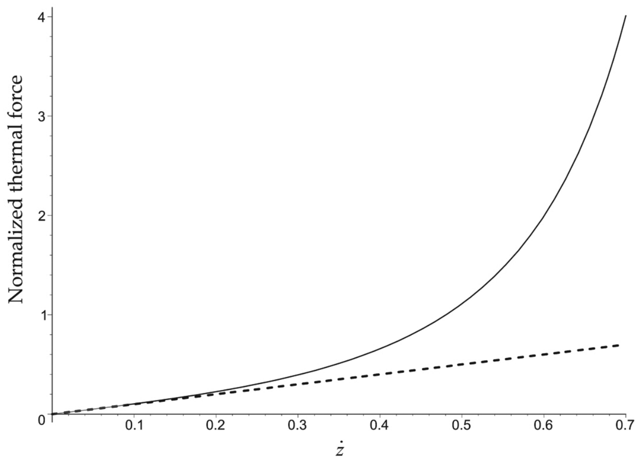
$$F = F_{vac} + F_{\langle \hat{a}^{\dagger}\hat{a}\rangle} + F_{\langle \hat{a}\hat{a}\rangle}, \tag{17}$$

where:

$$F_{vac} \;=\; |\gamma|^2 \left(1+\dot{z}^2\right) \left[(\ddot{z}^2\dot{z}/2\pi)/\left(1-\dot{z}^2\right)^4 + (\dddot{z}/6\pi)/\left(1-\dot{z}^2\right)^3\right], \tag{18}$$

$$F_{\langle \hat{a}^{\dagger}\hat{a}\rangle} = 2\int_0^{\infty}\int_0^{\infty} d\omega d\omega' \left\langle \hat{a}^{\dagger}_{\omega'}\hat{a}_{\omega}\right\rangle \mathcal{F}(\omega,\omega',|\gamma|,|\gamma|) + \text{c.c.}\ , \tag{19}$$

Figure 2:
Normalized exact thermal force $|F^{(T)}|/\sigma T$ (solid line) and approximate thermal force $|F^{(T)}_{(0)}|/\sigma T$ (dashed line), both valid for Dirichlet and Neumann boundary conditions, plotted as functions of the mirror velocity $\dot{z}$.



$$F_{\langle \hat{a}\hat{a}\rangle} = -\int_0^{\infty}\int_0^{\infty} d\omega d\omega' \left\langle \hat{a}_{\omega}\hat{a}_{\omega'}\right\rangle \mathcal{F}(\omega,\omega',\gamma^*,\gamma) + \text{c.c.}\ , \tag{20}$$

and

$$\mathcal{F}(\omega,\omega',\rho,\lambda) \;=\; \frac{\sqrt{\omega\omega'}}{4\pi}\left\{-\rho^2\frac{(1+\dot{z})^2}{(1-\dot{z})^2}\, e^{-i(\omega-\omega')p[t-z(t)]} + \right.$$

$$\rho^2 e^{-i(\omega-\omega')[t-z(t)]} - [(z,\dot{z},p,\rho)\to(-z,-\dot{z},r,\lambda)]\Bigg\} \qquad (21)$$

We see that F_{vac} and $F_{\langle\hat{a}^\dagger\hat{a}\rangle}$ depend on $|\gamma|^2$, which has the same value for Dirichlet and Neumann conditions. On the other hand $F_{\langle\hat{a}\ \hat{a}\rangle}$ depends on γ^{*2} (or γ^2), which differs by a sign in Dirichlet and Neumann cases. Noting that the term $\mathcal{C}_{\langle\hat{a}\hat{a}\rangle}$ $(\langle\hat{a}\hat{a}\rangle\neq 0)$is non-symmetric under time translations, we can generalize the result found in Ref. [24], concluding that for a general (relativistic) law of motion, Dirichlet and Neumann boundary conditions yield the same radiation force on a moving mirror when the initial field state is symmetric under time translations. In the non-relativistic limit, we recover $F_{vac}(t) \approx |\gamma|^2\dddot{z}/6\pi$, found in Ref. [8] for Dirichlet, and in Ref. [24] for Neumann condition.

Let us examine the radiation force on moving mirrors when there are real particles in the initial state of the field. We start with the thermal bath with temperature T, which is an example of invariant field state under time translations. For this state we need to take into account that $\left\langle\hat{a}^\dagger_{\omega'}\hat{a}_\omega\right\rangle = \bar{n}(\omega)\delta(\omega-\omega')$ where $\bar{n}(\omega)=1/(e^{\hbar\omega/T}-1)$, with the Boltzmann constant equal to 1. We get $\mathcal{T}_{\langle\hat{a}\hat{a}\rangle}=0$ and $\mathcal{T}_{\langle\hat{a}^\dagger\hat{a}\rangle}$, renamed as the energy density $\mathcal{T}^{(T)}_{\langle\hat{a}^\dagger\hat{a}\rangle}$, is given by: $\mathcal{T}^{(T)}_{\langle\hat{a}^\dagger\hat{a}\rangle} = |\gamma|^2\pi T^2/12\left[r'(v)^2+p'(u)^2\right]$. The force $F\hat{a}\hat{a}$= 0, whereas $F_{\langle\hat{a}^\dagger\hat{a}\rangle}$, renamed as the net thermal force $F^{(T)}$, is given by:

$$F^{(T)} = -\sigma_T\left[\dot{z}\frac{(1+\dot{z}^2)}{(1-\dot{z}^2)^2}\right] = -\sigma_T\sum_{n=0}^{\infty}(2n+1)\dot{z}^{2n+1}, \qquad (22)$$

where $\sigma_T=2|\gamma|^2\pi T^2/3$ is the viscosity coefficient. This exact formula is a generalization of the one obtained in Ref. [16]. The above series can be truncated in n = 0, leading to the approximate formula: $F^{(T)} \approx F^{(T)}_{(0)} = -2|\gamma|^2\pi T^2\dot{z}/3$, in agreement with Ref. [16] (for Dirichlet), and also with Ref. [24] (for Neumann boundary condition). In the nonrelativistic context, this approximate formula is in good agreement with the exact value. For relativistically moving mirrors, corrections to the approximate formula can become necessary, as it can be seen in Fig. 2.

Let us now consider the coherent state, as an example of a non invariant state under time translations. The coherent state of amplitude α is defined as an eigenstate of the annihilation operator: $\hat{a}_\omega|\alpha\rangle = \alpha\delta(\omega-\omega_0)|\alpha\rangle$, where α = $|\alpha|$ exp $(i\theta)$ and ω_0 is the frequency of the excited mode. The exact forces

$F_{\langle\hat{a}^\dagger\hat{a}\rangle}$ and $F_{\langle\hat{a}\ \hat{a}\rangle}$, relabeled as the coherent forces $F^{(\alpha)}_{\langle\hat{a}^\dagger\hat{a}\rangle}$ and $F^{(\alpha)}_{\langle\hat{a}\hat{a}\rangle}$ respectively, are given by:

$$F^{(\alpha)}_{\langle\hat{a}^\dagger\hat{a}\rangle} = -\frac{4|\gamma|^2}{\pi}\omega_0\,|\alpha|^2\,\dot{z}\big(1+\dot{z}^2\big)/\big(1-\dot{z}^2\big)^2,$$

Figure 3:
We plot the exact coherent force as function of time, for the Dirichlet case. We consider $\alpha = \exp(i\pi/2)$, $\omega_0 = 10$, and the following values of the velocity: -10^{-8} (dotted line), -10^{-2} (dashed line)and -8×10^{-1} (solid line). The dotted and solid lines exhibit the coherent force multiplied by the factors 5×10^5 and $1/125$ respectively.

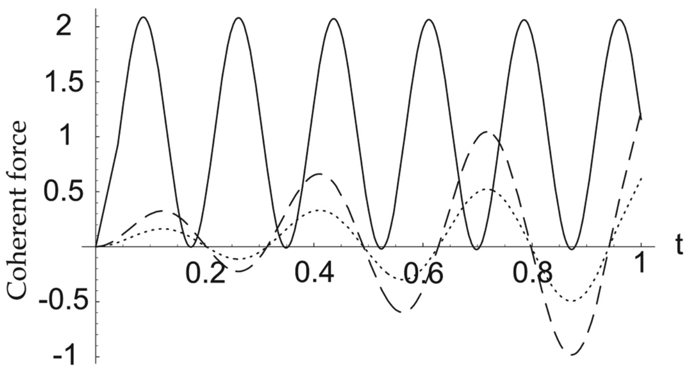
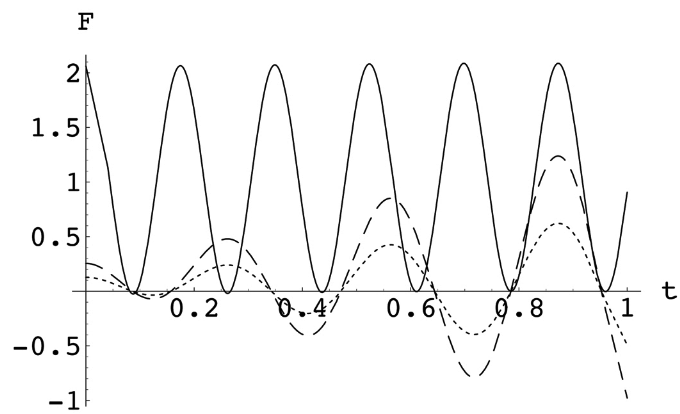


Figure 4:
We plot the exact coherent force as function of time, for the Neumann case. We consider $\alpha = \exp(i\pi/2)$, $\omega_0 = 10$, and the following values of the velocity: -10^{-8} (dotted line), -10^{-2} (dashed line)and -8×10^{-1} (solid line). The dotted and solid lines exhibit the coherent force multiplied by the factors 5×10^5 and $1/125$ respectively.



$$F^{(\alpha)}_{\langle\hat{a}\hat{a}\rangle} = -\frac{\omega_0}{4\pi}\,|\alpha|^2\,e^{-2i(\omega_0 t-\theta)}\left\{\gamma^2\left[e^{2i\omega_0 z(t)}\left(\frac{1-\dot{z}}{1+\dot{z}}\right)^2 - e^{-2i\omega_0 z(t)}\right]\right.$$
$$\left. -\,\gamma^{*2}\left[\left(\frac{1+\dot{z}}{1-\dot{z}}\right)^2 e^{-2i\omega_0 z(t)} - e^{2i\omega_0 z(t)}\right]\right\} + \text{c.c.}. \tag{23}$$

If we consider simultaneously nonrelativistic velocities and small displacements (in the sense considered in Ref. [24]), according to what is required by Ford-Vilenkin approach[8], the force $F^{(\alpha)} = F^{(\alpha)}_{\langle\hat{a}^\dagger\hat{a}\rangle} + F^{(\alpha)}_{\langle\hat{a}\hat{a}\rangle}$ can be approximated as:

$$F^{(\alpha)} \approx -\frac{2\omega_0}{\pi}\,|\alpha|^2\left\{2|\gamma|^2\dot{z}\,(t) - \left(\gamma^2+\gamma^{*2}\right)\right.$$
$$\left.\left[\cos\left(2\omega_0 t-2\theta\right)\dot{z}\,(t) - \sin\left(2\omega_0 t-2\theta\right)\omega_0 z\,(t)\right]\right\}. \tag{24}$$

In Figs. 3 and 4 (see Refs. [23, 33]) we show the exact coherent force as a function of time for the Dirichlet boundary condition and different values of the mirror velocity. Assuming the mirror moving with uniform velocity toward the negative direction of the x-axis, the Fig. 3 shows the force oscillating $F^{(\alpha)}$ and the graph shifting to the positive region of the vertical axis as the mirror velocity grows, becoming the force more intense and opposite to the motion. For the Neumann boundary condition, the force oscillates in a different manner, but exhibits analogous shift for relativistic velocities.

Thermal and coherent forces in the context of the relativistic particular trajectory proposed by Walker-Davies [35], have also been investigated [36, 37].

3. Two mirrors

Our research group in the UFPA has also investigated the time evolution of the energy density for a real massless scalar field in a two-dimensional space-time, inside an oscillating cavity [30, 31, 33, 34, 38 – 41]. We have considered the field satisfying the Klein-Gordon equation and obeying conditions imposed at the static boundary located at $x = 0$, and also at the moving boundary's position at $x = L(t)$, where $x = L(t)$ is a prescribed law for the moving boundary and $L(t< 0) = L_0$, with L_0 being the length of the cavity in the static situation. We have investigated four types of boundary conditions. The Dirichlet-Neumann (DN) boundary condition imposes Dirichlet condition at the static boundary, whereas the space derivative of the field taken in the instantaneously co-moving Lorentz frame vanishing (Neumann condition) at the moving boundary's position. We have also considered: Dirichlet-Dirichlet (DD), Neumann-Neumann (NN) and Neumann-Dirichlet (ND) boundary conditions.

In Refs. [30, 38] the model investigated consisted of one static boundary imposing the Neumann condition to the field, and a second one in a non-relativistic oscillatory motion, with small amplitude, imposing the Dirichlet condition to the field. The frequency of oscillation was considered twice the frequency of the first mode of the static cavity, resulting in a situation of parametric resonance. Following the procedure developed by Dodonov and Klimov [42], we studied the analytical solution for this problem, computing the number of created particles, the generation rate and the energy in the cavity. We have shown that, for this kind of oscillating cavity, particles are created in all allowed modes, in contrast to the corresponding Dirichlet-Dirichlet(DD) case, where just particles related to the odd modes are created [42]. A possible qualitative explanation for this fact was wondered: in the DD cavity, the oscillating frequency, which is by assumption twice the frequency of the first mode of the corresponding static cavity, is also twice the difference of two adjacent energy levels of the static cavity. Consequently, there would be a kind of indirect resonance with all modes whose frequencies differ from the fundamental one by an integer number times the frequency of oscillation. In this case, this means that only the odd modes will be excited. On the other hand, in the case of the Neumann-Dirichlet (ND) cavity, the oscillating frequency is not twice, but equal to the difference of two adjacent energy levels of the corresponding static cavity. Hence, for this case, all modes will be excited.

In Ref. [31], considering Neunmann and Dirichlet boundary conditions, we investigated the behavior of the energy density for a real massless scalar field in a two-dimensional space-time, inside an oscillating cavity with mixed boundary conditions. The time evolution of the energy density was investigated taking as basis two methods for solving the Moore equation: the numerical method proposed by cole and Schieve [11, 43], and the approximate analytical solution obtained by Dodonov [12] and Dalvit-Mazzitelli [13]. We calculated the exact solution for the field in the mixed cavity and obtained the renormalized expected value of the energy-momentum tensor, taken with respect to the vacuum "in" state. The same formula for the expected value of the energy-momentum tensor was also obtained for the ND case, so that, despite the distinction between the field modes for the Dirichlet-Neumann (DN) and ND cases, the dynamical Casimir effect is the same. From this conclusion we pointed out that the results found in [30] for ND case are also extendable to the DN case. We also pointed to a remarkable difference: for particular laws of motion for the oscillating boundary with the same length of the static cavity, we can have no particle created in DD case, in contrast to resonant particle creation in the mixed one.

Figure 5:
The energy density T in an oscillating cavity, for the initial field in the coherent state with $|\alpha|=1$ and $\theta = 7\pi/8$. The solid line corresponds T^{DD}, whereas the dotted line corresponds to T^{NN}. The law of motion considered is $L(t) = L_0[1+0.01\times\sin(2\pi t/L_0)]$, and the time is $t = 15L_0$.

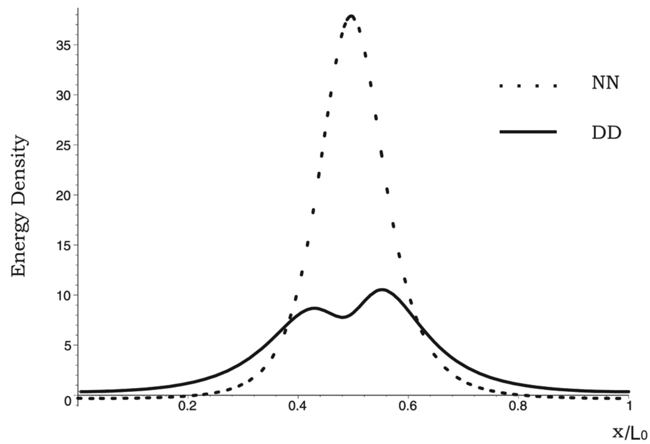
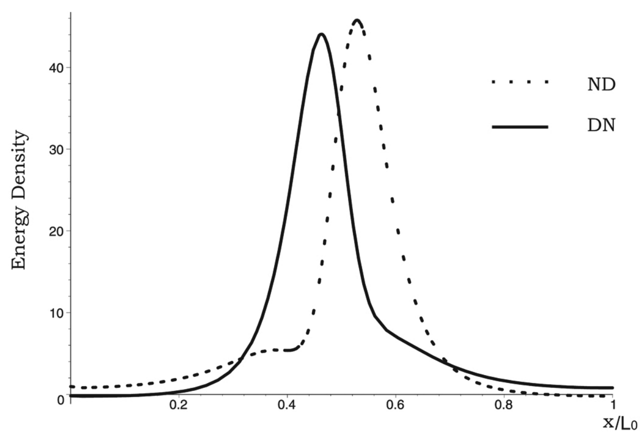


Figure 6:
The energy density T in an oscillating cavity, for the initial field in the coherent state with $|\alpha|=1$ and $\theta = 7\pi/8$. The solid line corresponds T^{DN}, whereas the dotted line corresponds to T^{ND}. The law of motion considered is $L(t) = L_0[1+0.01\times\sin(2\pi t/L_0)]$, and the time is $t = 15L_0$.



We have also obtained the exact formulas for the energy density inside the cavity for an arbitrary initial field state [33, 34, 41]. Next, we show some results for the coherent state. For the coherent state the energy density is showed in Fig. 5 for DD and NN cases, and also in Fig. 6 for DN and ND cases.

4. Final comments

At the moment, we are investigating the Robin boundary condition [44], possible extensions of our solutions to 3+1 dimensions, electromagnetic and massive fields, and also to more realistic models for the oscillating cavities.

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A note on space dimensionality constraints relied on Anthropic arguments: Methane structure and the origin of life

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Perhaps it is true that the hope that physical research can resolve the philosophical problems of space is just as vain as the hope that philosophical thought can resolve the physical problems of space.
Max Jammer

Anthropic arguments have been proposed, independently, by philosophers and scientists to explain why we perceive a three dimensional Universe [1]. Some of them will be briefly reviewed in this note and a possible relationship between methane structure, the origin of life and space dimensionality will be pointed out.

Kant's conjecture [2] that space three-dimensionality may, in some way, be related to Newton's inverse square law of Gravitation was the first step in this direction. Even though it has been shown [3] that Kant did not actually succeed in proving this conjecture – indeed, he just concluded that there should be a relationship between this law and *extension* –, his contribution has the very merit of suggesting that the problem of dimensionality can also be treated in the framework of Physics and does not belong exclusively to the domain of Mathematics, neither to that of pure philosophical speculation. A deeper comprehension of Kant's conjecture had to wait the rise of field theory.

As a second step, one can quote the work of William Paley [4], which can be considered the first attempt to shed light on the space dimensionality problem clearly from Anthropic arguments. In his work, Paley analyzes the consequences of changes in the form of Newton's gravitational law and of the stability of the solar system on human existence. Starting from a teleological thesis, his speculations take into account a number of mathematical arguments for an anthropocentric design of the World, which rest all upon the stability of the planetary orbits in our solar system and on a Newtonian mechanical *Weltanschauung*, as should be expected at that time.

In the twentieth century, the idea of how space dimensionality follows from the stability of planetary orbits in the solar system was revisited in Ehrenfest's seminal papers [5-6]. Ehrenfest discusses several physical phenomena, where qualitative differences between three-dimensional ($\Re^3$) and other *n*-dimensional ($\Re^n$) spaces are found, such as the existence of stable planetary orbits and the stability of atoms and molecules. These aspects, which distinguish the $\Re^3$ Physics from the $\Re^n$ one, are called by him "singular aspects" and his works were aimed at stressing them. A crucial assumption is built in the main ideas contained in [5], namely that it is possible to make the formal extension $\Re^3 \rightarrow \Re^n$ for a certain law of Physics and, then, find one or more principles that, in conjunction with this law, can be used to single out the proper dimensionality of space. For this approach to be carried out, in general, one has to decide how such formal extension will be done. Frequently, the form of a differential equation – which usually describes a particular physical phenomenon in a three-dimensional space – is preserved and its validity for an arbitrary number of dimensions is *postulated*. Then, "singular aspects" may be brought into evidence by analyzing the higher dimensional mathematical solutions. For example, the Newtonian gravitational potential for a $\Re^n$-space, $V(r) \propto r^{2-n}$, is the solution of the generalized Laplace-Poisson equation,

$$\sum_{i=1}^{n} \frac{\partial^2 V}{\partial x_i^2} = k\rho,$$

in an *n*-dimensional space. Based on the general solution of the above equation, assumed to correctly describe planetary motion in a space with *n* dimensions, Ehrenfest has postulated the stability of orbital motion under central forces in order to constraint the number of dimensions. This general procedure was also followed by Whitrow [7]. Tangherlini [8] noted that this approach could be broaden by proposing that for the Newton–Kepler problem, generalized to $\Re^n$ space, the principle to determine the spatial dimensionality could be summarized in the postulate that *there should be stable bound states orbits* – or "states" – for the equation of motion governing the interaction of bodies, treated as material points. This will be generically called, from now on, the *stability postulate*. In his first paper [8.a], Tangherlini showed that the essential results of the Ehrenfest–Whitrow investigation are unchanged when Newton's gravitational theory is replaced by General Relativity. Application of this same idea to the stability of hydrogen atom, described by a generalized Schrödinger equation, leads to the same kind of constraint in a very huge and different spatial scale.

In its essence, Ehrenfest's approach for planetary motion relies on two postulates: a) Poisson equation for any space dimensionality correctly explains the same phenomenon it describes in three dimensions; and b) the stability of the mechanical orbits should hold in the higher dimensional space. For him the former is the *causa formalis* and the later, the *causa efficiens* of space dimensionality. Actually, both are typical ingredients of any Anthropic constraint imposed on dimensionality. In spite of the fact that this kind of approach strongly reflects the recognition of our ignorance being complete and assumes a 'Principle of Similarity' – using the expression adopted in [1], namely that alternative physical laws should mirror their actual form in three dimensions as closely as possible – it seems a very hard task to avoid it as long as dimensionality is to be understood in the realm of Physics.

In any case, the previous results can be summarized by saying that only in universes in which gravity abides by an inverse square law could the solar system remain in a stable state over long time-scales. We will turn back to this point but, at this stage, it is important to stress that some epistemological and methodological aspects of this general approach based on the *stability postulates* were criticized in [9].

This briefly reviews how the *stability postulate* is used to cast some light on the problem of spatial dimensions.

It is important to stress that there is a third and decisive ingredient explicitly required or implicitly assumed every time a method which effectively connects the number of dimensions to some physical property is suggested. This is actually the most delicate part of any method one can propose for discussing the problem of spatial dimensions, and it will be shown that it is invariably connected to some version of Anthropic principle.

From the beginning, we would like to say that we are convinced that it is impossible to disentangle questions concerning this subject from some (any) kind of formalism representing a physical law, just because, as Jammer put it clearly [4], "*... it is clear that the structure of the space of Physics is not, (...), anything given in nature or independent of human thought. It is a function of our conceptual scheme.*"

This means that we should accept that the physical concepts and the concept of reality itself acquire sense only within a theoretical construction where they can be discussed and realized. So far as the problem of space dimensions is considered, we must carefully examine the consequences of this fundamental point for the obvious fact that looking back to the History of Physics we soon realize that all theories and systems were built up, as Bertrand Russell said, assuming that "*the limitation of the dimensions to three is* (...) *empirical.*" [10]. Although this point has, in fact, motivated several works on the problem of spatial dimensions, it constitutes itself, at the same time, one of the main difficulties for discussing it, because the three-dimensionality of space is never questioned a *prior*i when the physical law which is considered as the starting point for the search of any "singular aspect" is established.

Even taking this criticism into account, a review of the literature on this subject leads us to regard to the Anthropic Principle as *an almost unavoidable* approach to the problem of space dimensionality when we want to explain why dimensionality is three and not another number. In any case, this is essentially related to Jammer's idea just recalled above. Thus, to the best of our knowledge, this epistemological limitation seems to be inherent to this problem (so far as we understand it) and, in a certain sense, is well illustrated and justified by the following Grassmann's words:

> *"The concept of space can in no way be produced by thought, but always stands over against it as a given thing. He who tries to maintain the opposite must undertake the task of deducing the necessity of the three dimensions of space from the pure laws of thought, a task whose solution presents itself as impossible." [11].*

These words just reinforce our conviction that the structure of physical space – in particular its dimensionality – is a function of our conceptual scheme and that it does not seem possible to formally deduce space dimensionality from it. In the last analysis, therefore, one should resort to phenomenology to determine it, which, at the end, actually means to accept some kind of Anthropic argument.

Let us turn back to Whitrow's argument. In his important 1955 paper, he asseverates that for trying "*to isolate three-dimensional space as the only possibility for the world in which we find ourselves, we must now invoke some argument for showing why the number of dimensions cannot be less than three*". To do this, he adapted the well known topological result from knot theory, that we cannot make a knot in even-dimensional space, to the necessity of higher forms of animal life to have brains in which electrical pulse informations carried on by nerves could not interfere destructively, which excludes a twofold and other even-fold spaces. This argument automatically constrains space to have an odd dimensionality ≥ 3. Then, in the conclusion of this paper one can read:

> *"Despite various recent attempts to show that [space dimensionality] is either a necessary attribute of our conception of physical space or is partly conventional and partly contingent, the problem cannot be considered as finally solved. A new attempt to throw light on the question indicates that this fundamental topological property of the world may possibly be regarded as partly contingent and partly necessary, since it could be inferred as the unique natural concomitant of certain other contingent characteristics associated with the evolution of the higher forms of terrestrial life, in particular of Man, the formulator of the problem." [7.b].*

Following a different approach, based on the *stability problem* and the Uncertainty Principle, Barrow & Tipler [1] stressed that

> *"(...) it has been claimed that if we assume the structure of the laws of Physics to be independent of the dimension, stable atoms, chemistry and life can only exist in N < 4 dimensions.".*

And therefore they conclude, perhaps inspired on the aforementioned Whitrow's words, that "*the dimensionality of the Universe is a reason for the existence of chemistry and therefore, most probably, for chemists also.*".

There is no doubt that both conclusions above have an Achilles' heel: there is indeed no support to the hypothesis that the laws of Physics are in principle independent of the dimensions, except simplicity; an example of application of Ockham's maxima *Entia non sunt multiplicanda praeter necessitatem*. Probably, in general, they are not. Let us remember, for example, that the group structure of the Euclidean Group of rotations is different for various numbers of dimensions. This fact has led Hermann Weyl to consider, in 1949, that "*mathematical and physical laws may cease to be indifferent to the number of dimensions on some deeper level than has been touched by physics*" [12]. The result published by R. Mirman, in 1984, that standard assumptions about the basic principles of Quantum Mechanics are not compatible with space-times with dimensions different from (3 + 1), is also to be recalled [13]. In any case, it still remains the possibility that a particular physical system or dynamical process could have place in other dimensions but being described by a new mathematical law, in such a way that their main features and properties are maintained. The third objection could rise in the light of a 1999 paper for it contradicts all previous results based on the stability of atomic orbits, since the authors have claimed that there could have be a stable hydrogen atom in higher dimensions [14]. In addition they sustain that some spectroscopic experiment can be used to explain that our space is three-dimensional. However, such a result is based on very strong assumptions; for example, that "*the specific expression for the force between charged particles and the stability of atoms are of more basic physical importance than the validity of Gauss' law*" and as a consequence Maxwell's equations should be modified. Criticisms of those ideas will appear elsewhere. In any case, despite of them, the intrinsic limitations of being still an approach which depends on Anthropic assumption could not yet be avoided. To the best of our knowledge, this is always the case. Therefore, let us try to push it on by presenting now some remarks about the time (and space) "scale" of the arguments previously discussed.

The first is related to Ehrenfest's stability argument which is typically valid for distances of the order of the solar system and in a time scale large enough to make the evolution of life possible on Earth, as mentioned by Whitrow [7]. However, his argument about this subject [7.b] could be improved by stressing that it is not sufficient that the intensity of solar radiation on Earth's surface should not have fluctuated greatly for life still exist on Earth; actually, the fact that Sun's spectra of radiation did not fluctuate very much should also be required [9]. By other side, Tangherlini's work about the stability of H atoms is often invoked to suggest the validity of Chemistry in the same time scale as a necessary, although not sufficient, condition – at least Chemical Thermodynamics of irreversible process should be also valid. Thus, as pointed out in [9], "*the presence of atomic spectra in remote stars may also indicate[s] that space has had the same dimensionality at cosmic scale.*" The existence of such a cosmic constraint on space dimensionality is a very interesting consideration and this subject was treated in [15].

The second one is also related to the general idea that among a large number of possible universes, the actual Universe is the one that which contains intelligent life, or at least had some form of life in a very long time scale. We have quoted above what Withrow, Barrow and Tipler said about human life and how it imposes some constraints on the number of dimensions. Infallibly this query addresses us to Biochemistry. There is a nice chapter on this subject on Barrow and Tipler's book [1], where several relevant topics are discussed in details, and so will not be treated here. Among them we can quote the unique properties of carbon, hydrogen, oxygen and nitrogen, or whether or not it is possible to base life on elements other than these ones, and finally that those unique properties are probably necessary to guarantee the ecological stability required by highly-evolved life, although not sufficient. Our aim here is to introduce a new argument in favor of a stable scenario for space dimensionality for a time scale longer than that required for the existence of human or another kind of highly-evolved life on Earth, remembering that the usually accepted scales are: 2 Millions years ago the *homo erectus* have appeared, while the first *skeletons and easily recognizable fossils* range are of 600 Millions of years ago. This new argument is related to the methane structure as will be shown now.

Let us consider the famous experimental result published in 1959 by Harald C. Urey and Stanley Miller [16]. They showed to be possible, by means of an electrical discharge, to transform an admixture of gases consisting of methane, water, ammonia and hydrogen into a great number of organic compounds, among them some amino acids essential to life. Although it is not a *proof*, this result is widely considered as a strong evidence for the creation of life in a kind of primitive Earth atmosphere, quite different from that of the present days, composed of the four substances just mentioned. Accepting this means to accept that in a certain sense methane, which has the most simple formula among the organic compound (CH_4), is somehow related to the origin of amino acids that could build up primitive life. In addition, it is implicit in this reasoning that the atomic structure and chemical properties of the elements have not changed.

Based on X-ray spectroscopy and on the empirical fact that an isomer of methane was never found, the tetrahedral structure of carbon was established [17]. In other words, Nature seems to have chosen just one spatial disposal for methane atoms and also for all compounds of the type CH_3Y e CH_2YZ, with Y and Z being any group of atoms. This rules out any flat configuration for the simplest organic compound and requires, obviously, that the space in which it exists should be at least three dimensional.

So, to believe on Urey-Miller's experiment as a clue for the origin of amino acids essential to life, associated to an atmosphere possibly rich on methane, implicitly assumes that three is the minimum space dimensionality required by methane structure and for life to be developed this way. Putting this together with what was said above about the spectra of remote stars, a scenario where space dimensionality should be at least three for very large spatial and temporal scales seems plausible; much greater than that required by human life on Earth.

Remember that some authors believe the origin of life – probably thermophiles – occurs 3,500 Millions of years ago. Despite its speculative nature, this is a new constraint imposed not only on the number of dimensions but also on its stability throughout a very large space and time scale, obtained from a sort of modified strong Anthropic principle, namely, from the assumption that *the early Universe should necessarily contain amino acids*. A recent analysis [18] of the cosmic background radiation spectrum measured by COBE collaboration suggests that space dimensionality did not vary significantly in a huge temporal scale, once this background radiation is expected to be related to the Big Bang. This time scale can be safely put on the later epoch where the universe was about 3×10^5 yr old (red shift $z \simeq 10^3$).

In conclusion, we would like to say that physicists and philosophers should still pay attention to many epistemological difficulties concerning the problem of space dimensionality, among which we could emphasize a certain incompleteness in the majority of approaches to this problem so far as they consider physical events taking place only in space, not in *space–time*. Thus, the problem of the number of space dimensions and that of time dimensions are probably not independent. Finally, whether or not a deeper comprehension on the problem of space dimensionality is to be reached and, in particular, if it could be possible to go on discussing this problem without taking into account any kind of Anthropic argument as some stage of a particular reasoning are still good questions without good answers.

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Effects on UV spectrum of caffeine in aqueous solution by cluster and polarized continuum model

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Caffeine is the alkaloid with biologic activity more consumed worldwide. The concentration absorbed by the organism may produce diseases that may be from instability of the nervous system to cardiovascular problems. However, UV-Vis spectrophotometric determination is preferred because it is possible to obtain rapidly high accuracy and reproducibility from small samples using a relatively simple and inexpensive procedure. In this work, it is shown the absorption spectrum result of caffeine in aqueous solution by two methods: Cluster and Polarized Continuum Model (PCM). The Cluster model allowed a study of the hydrogen bridge, but it did not present the lowest spectroscopic satisfactory result due to effects of shield produced by water molecules. The PCM method, presented a highest relative error, but showed better spectroscopic result.

1. Introduction

Caffeine (CF) is an alkaloid with biological activity more consumed worldwide. The pharmacological actions produce many effects on human behavior. The greatest ingestion of caffeine is not from pharmacological medicines, but from food and drinks. The determination of the presence of caffeine in food or drinks is important to determine its concentration with complementary techniques. The high concentration of CF absorbed by the organism may produce diseases that may be from instability of the nervous system to cardiovascular problems. However, UV-Vis spectrophotometric determination is preferred because it is possible to obtain rapidly high accuracy and reproducibility from small samples using a relatively simple and inexpensive procedure when compared to chromatographic techniques. Another problem is that it can have the formation of complex of caffeine with other molecules that may increase its effects. The interactions between pyrene and CF were studied using absorption spectroscopy in aqueous solutions. However, the absorption spectrum was a superposition of three species: free pyrene and two pyrene-CF complexes [2]. Dinc et al [3] developed a new spectrophotometric method for simultaneous analysis of ternary mixture, without necessary separation. Goicoechea et al [4] applied partial least-squares multivariate calibration of spectrophotometric data, it has been applied to the determination of theophylline in blood serum. The electronic absorption spectrum was processed using a calibration design which includes a series of serum pools spiked with theophylline. Studies in the presence of other substances, the CF had shown that only the latter was able to interfere, as with other techniques. Wong et al [5] showed that the ultraviolet absorption spectra change for different methylxanthines, mainly in the wavelengths of maximum absorption by changing pH. Lopez-Martinez et al did experimental absorption spectrum of CAF in coffee and tea samples using UV-Vis spectrophotometry [6]. Though the continuum model of solvation is a rather simple approach, it is expected to provide good results when non-polar and/or non-coordinating solvents are considered. Solvent effects in polar media and in media where hydrogen-bonds or other specific interactions play a significant role may be much more difficult to represent. However, still in this case, the continuum model has been shown to describe UV/V spectra reasonably well [7]. Obtaining more accurate results is possible by incorporating into the calculations a discrete representation of the liquid together with a statistical treatment. Such types of approaches have also been developed in the literature [8]. In this work, are shown the first results about the spectrum absorption to the CF inside the aqueous medium in different procedures. In the first procedure, we used the method of polarization of continuum medium (PCM), in the second method it was used one water cluster, where the caffeine was immersed. The Cluster model allowed a study of the hydrogen bridge, but it did not present the lowest spectroscopic satisfactory result due to effects of shield produced by water molecules. The PCM method, presented a highest relative error, but showed better spectroscopic result.

2. Methodology

The CF molecule was optimized in vacuum and solution at level MP2/6-311G.

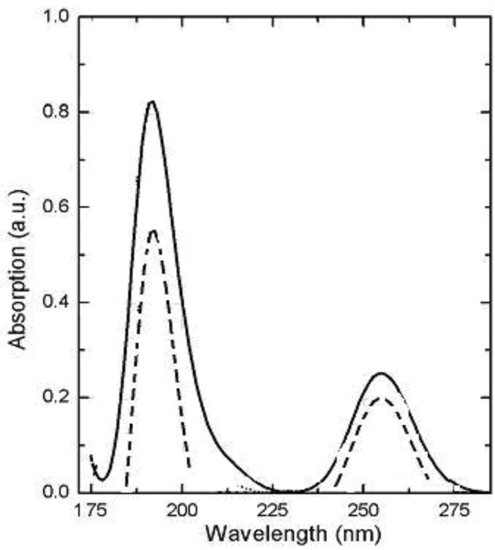
The absorption spectrums to both mediums were obtained by TD-DFT, at level B3LYP/6-311G. For these cases were used the Gaussian03 package program. [9] For the water cluster and CF, were used the optimized structure in vacuum. We have used 60 water molecules around water and CF. The system was optimized by PM3 method with the MOPAC 2007. [10]. The absorption spectrum was obtained using the ZINDO [11] program and it was compared with spectrum absorption obtained by functional density.

3. Results and Discussions

Figure 1 shows the CF absorption spectrum in vacuum and polarized medium, obtained by functional density method. There are two bands, located in ~194 nm and ~255 nm. Theses results show that CF does not have solvatocromism in aqueous solution. The CF shows two characteristic bands, where the absorption band is always with the same wavelength. This is not surprising as those absorption bands are connected with $\pi \rightarrow \pi^*$ electronic transitions that, typically, are not quite sensitive to medium effects. This results show value of 5.4% (~194 nm) and 6.6% (~255 nm) to relative error to the bands approaching to the experimental values, respectively. The CF UV absorption spectrum shows one pair of peaks at 205 nm and 273 nm with one characteristic peak between them. [6] Due to the substances varieties that have absorption in the UV range, many times it is necessary to use complementary methods to this analysis, from previous separation methods

Figure 1.

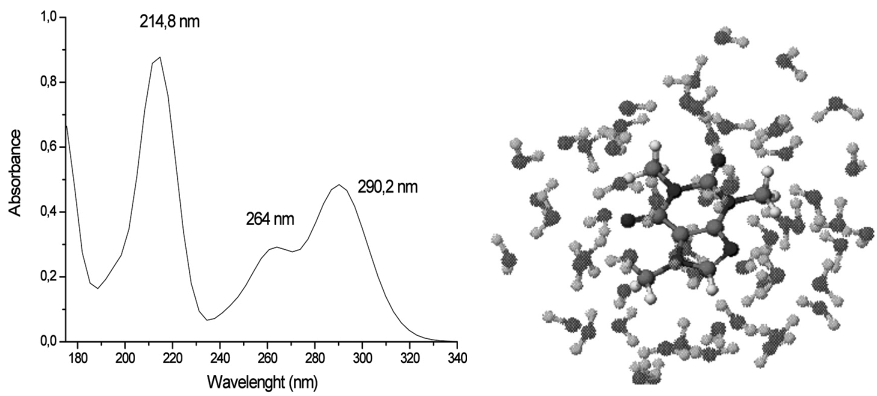
Absorption spectra by PCM method with the main absorption bands to caffeine on vacuum (–) and aqueous solution (---). For both cases, the values to absorption bands are identical.



The principal contribution to the least intense peak is of HOMO $\rightarrow$ LUMO (81%), which shows contribution of carbons, corresponding to one transition of $\pi \rightarrow \pi^*$. While to the most intense transition, the contribution is one transition set originated from H-3 –> LUMO (13%), H-2 –> LUMO (49%), HOMO –> L+1 (17%) and HOMO –> L+2 (15%). These are principally contributions of carbons and one low contribution of the oxygen atom, corresponding to the transition $\pi \rightarrow \pi^*$. The theoretical description showed results that satisfy the effects of the compounds in solution. The chosen of water is due to the great utilization of aqueous-CF solution as organic compounds solvent. The transition corresponding to the 240 nm peak correspond to one HOMO $\rightarrow$ LUMO+1 transition that is one transition originated from oxygen $\pi \rightarrow \pi^*$. The functional density does not describe the transition corresponding to this band.

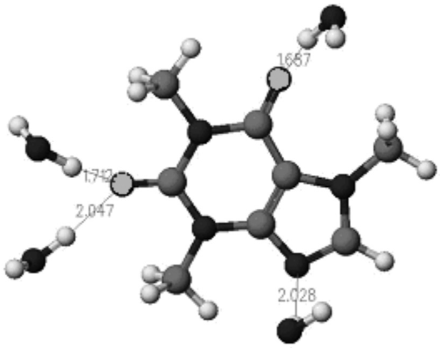
Figure 2 shows the absorption spectrum by the water, and two bands localized in ~214 nm and ~255 nm to the great and less intensity. The water cluster simulation around CF was done to try to understand the hydrogen bridge formations that influence the absorption spectrum for the medium described. This system simulation shows a water molecule interaction that is less found in polarized medium, but it could have a strong influence on the absorption spectrum. In figure 2, it is possible to observe that the formation of hydrogen bridges in this system influences the spectrum displacing it to the great wavelength, but this influence happens only where theses transitions are originated from regions under the van der Waals force. This system shows one shift (~264 nm) in the transition originated from oxygen atoms in relation to the experimental spectrum.

Figure 2.
Absorption spectrum by Cluster method (left) with main absorption bands and the spatial representation of the caffeine on aqueous system used.



In the case of CF, we have a quasi-hydrophilic molecule, because there are formations of four hydrogen bridges, principally with the oxygen. The figure 2 showed the hydrogen bridges formation with the oxygen and only Hydrogen Bridge formed with the carbon, to the distance of 2.028 Å (figure 3).

Figure 3.
Representation of the formation of hydrogen bridge with its respective distances.



The greatest divergence is shown in the transition due to the excitations of $\pi \rightarrow \pi^*$ originated from the oxygen atom. Once we are simulating the same solvent, it would be presumed that the dipole values showed approximated values. But, the influence of each medium with function of the total dipole of this system presents values of 5.1 D and 10.5 D for the PCM and cluster, respectively. The vacuum dipole moment is 5.4 D. For the PCM model, the polarizability sphere stays under the atoms, while for the cluster model, the solvent effects are greater where there is the hydrogen bridge formation, interfering in the electric dipole of the molecule. The difference to these values could be understood that for the cluster method there is low statistic for this method, i.e., from the average of different conformations theses values show more accurate results.

With both methods, the results present discordance with the experimental values. For the case studied by PCM method, the effects are hypsochromics (blue shift), while to the cluster method, the effects are bathochromics (red shift). The shift of the most intense peak in relation to the experimental value by functional density is 100 nm and to the least intense peak presents one shift of -18 nm. For the cluster method, the most intense peak shows one shift of +17 nm. The relative errors for the PCM method are 5.4% (~194 nm) and 6.6% (~255 nm) and the relative errors to the cluster method are 4.9% (~214 nm) and 6.2% (~290 nm) to the great and less intensity in both cases, respectively. Despite the differences found in both studies, the PCM method confirms the fact that the CF molecule does not show solvatocromism with respect to the results obtained in vacuum, but it has one relative greater error to the characteristic peaks. The cluster method showed a less relative error to the characteristic peaks. But, the cluster method showed a strong influence of the van der Waals and a strong batocromic effect.

4. Conclusions

Two methods were shown here, one model where the medium is polarized continuously and one cluster model. The cluster method allows a study of hydrogen bridge formations, but did not show satisfactory spectroscopic results due to the field effect in the CF molecule. The CF molecule did not show solvatocroism in the PCM method, opposite to the cluster method. Due to the formation of the hydrogen bridges with the oxygen atoms, it was clear the displacement of the transitions $\pi \rightarrow \pi^*$ originated from oxygen. This transition in relation to the experimental data showed a transition to the greatest wavelength. The PCM method, despite of showing a relative greater error, showed better results than the cluster method.

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Science, ethics and environment

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1. Introduction

It is a great pleasure for me to write an essay in honour of Professor José Maria Filardo Bassalo who devoted many years to a relevant and prominent activity in research, teaching and political academic engagement. His writings collected in *Crônicas da Física* (Vols. 1-6), in *Nascimentos da Física* (in 3 volumes) and in several other instances show his broad interests concerning physical sciences. However, the amplitude of his interests is not merely centred on physics and related disciplines. Bassalo has also written books on ethics and politics. Bassalo's work on environment is less evident from the readings of his papers. But, I am sure, contrarily to what at first sight seems to be, that the development of his country associated to a healthy environment and social justice is a theme appearing recurrently in Bassalo's thought. In this chapter the attention will be centred upon some aspects of Bassalo's work concerning ethics, science, politics and environment.

In this essay I would like to argue in order to articulate my defence of the three following theses:

(1) The distance that several scientists historically took from ethics was a tragedy for the further development of humankind;

(2) During the Second World War (1939-1945) and in the consequent period of the Cold War (1945-1989) several important attitudes and behaviours of Liberals, Nazis, Fascists and Socialists were practically undistinguished. All of them perpetrated hideous crimes against human dignity and human liberty and this fact constituted a very harmful ethical degradation that implied severe consequences for the geopolitical correlations of our present world. However, this does not mean that all of them are equal in all the senses of the term. Of course, the scientific community was not immune to these severe consequences;

(3) The geopolitics competition for weapons, for hegemony and for economic influence was harmful for the humankind and also for the environment of our planet. Of course, great progress in science and technology took place. However, the benefits reaching a limited part of countries has been accompanied by an enormous abyss among rich and poor peoples and also by a degradation of the earth ecosystems and biological diversity. This abyss has been increased in the world. I defend the thesis according to which the broadly adopted development model must be seriously revised in order to contemplate a very difficult combination of social equality, economic efficiency and environmental conservation.

In this essay I would like to discuss the three terms constituting the above title. The order of the terms appearing in the title is not arbitrary. *Science* has been frequently discussed or separately or in connection with technology, education and society. However, the connection between *science* and *ethics* has been scarcely and insufficiently explored and, in consequence of this, ethics seems to be very far from science. As a result of this distortion ethics becomes conceived as a mere externality that does not provide any intellectual insight for scientists or as a mere superfluous thing for the social prestige of them. Therefore, ethics is frequently conceived as a mere perfumery. With respect to environment the situation seems to be even more problematic. In fact, the western thinking has a clear historical difficulty to treat suitably *environment* as a conceptual category.

Our aim in this paper is to explore such difficulties.

Our point of interest in this paper is centred on the fact according to which the ethical and environmental dimensions involved in the scientific enterprise are frequently forgotten. It seems to be that for scientists, ethics consists merely in something superfluous, or something implicitly assumed as a necessity for the scientific practice. Regarding to environment we can say that this important and not well defined conceptual category has been considered by the overwhelming majority of scientists as merely an externality and not an essential dimension of reality. Surely, people talk about Hiroshima and Nagasaki bombs and their impact on the environment, but the focus is not centred on the environment itself. In fact, the focus is centred on the geopolitical consequences in the periods of Cold War and post 1989 when the centralized model of total state monopoly collapsed.

2. Economics far from ethics: a great tragedy

Amartya Sen (1933-), who received the Nobel Price of Economics in 1998, argued that the increasing distance that economics took from ethics constituted a harmful circumstance for the science of economics itself (Sen, 1999). Sen severely criticises the broadly known opinion according to which Adam Smith (1723-1790) – the father of modern economics- would have separated economics from ethics[1]. Sen considered that such a point of view does not correspond to a deep and critical reading of Smith's seminal work[2]. Besides this, Sen argued that if this divulgated thesis were true, then Adam Smith would be a schizophrenic man. As we know, Smith was professor of Moral Philosophy in Glashow and so it is evident that a genuine preoccupation with Moral Philosophy would not be coherently connected with an attitude involving an extreme and aggressive competition like, for example, one very close to *pleonexia*[3] (see, Monteiro et. al., 2006).

Of course, in the context of the desirable Heraclitus's tension between arc and lire, it is possible to argue that honest competitions are necessary and, in fact, they played a very positive role. However, exacerbated and aggressive competitions characterized by *pleonexia* play a harmful role and, in consequence of this fact, they must be refused. I emphasise that a combination between healthy

1 The more frequent Smith's quotation claiming this point of view is: "It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our own necessities but of their advantages." (Smith, 1978, p.7)

2 Sen argues that Smith also emphasized the central role played in economics by generosity and public politics of distribution of richness.

3 "The more specific sense that Aristotle gives to Justice, and from which the most familiar formulation derive, is that of refraining from pleonexia, that is, from gaining some advantage for oneself by seizing what belong to another, his property, his reward, his office, and the like, or by denying a person that which is due to him, the fulfillment of a promise, the repayment of a debt, the showing the proper respect, and so on." (Rawls, 2003, p. 9)

competition and synergetic cooperation is also necessary. In fact, a combination like this constitutes a very positive circumstance.

Sen (2000) also argued that the exercise of freedoms is one the most relevant factors in order to provide the development process. According to him all the circumstances avoiding the exercise of freedoms constitute the worst scenario to provide genuine development. Here, it is necessary to distinguish a mere economic growth process from the genuine development. The first one would be compatible with an authoritarian and dictatorial regime, but the second would not be compatible with the suppression of freedoms. The sense of the expression *exercise of freedoms* given by Sen is more comprehensive than a mere *free market* historically emphasized by liberal thought of all types (liberal, neo-liberal, etc). Besides this, as several authors have emphasized, the *free market* in the sense of a famous metaphor of an *invisible hand of market* constitutes, in fact, an idealization which does not occur in a real world. In fact, giant corporations control the "free markets" in evident contradiction with the presupposed theoretical regulatory role attributed to national States. Besides this, the giant States of the great powers have spurious and entangled relationship with giant economic corporations.

Indeed, the liberal thought of all kinds is immersed in an ocean of ambiguities and contradictions as Domenico Losurdo has argued in one of his splendid chapters (see, Losurdo, 1999). In this chapter Losurdo extensively documents the ambiguities and contradictions of the liberal thought (liberal, neo-liberal, etc.).

In the context of the tensions caused by war and also in the immediate post-war period, the contradictions become evident. In one of his books, Chomsky[4] (1999) documents that immediately after the Second World War the occupation forces began an operation to protect the Nazis and, at the same time, to persecute the forces that struggled against Nazis.

At first sight this attitude seems to be paradoxical, but five minutes of criticism are enough to permit us to conclude that this perverse and contradictory logic is inserted in a bad ethics ruled by a pragmatism based upon mere circumstances and geopolitical interests. According to this logic, to be friend or to be enemy is reduced to a mere circumstance. It is not difficult to understand the reason by which Gulags, Auschwitz, Guatanamos and many other examples present impressive similarities. All the principal countries on both sides of the last world confrontation perpetrated hideous crimes against human dignity and human liberty. All of them built concentration fields for enemies and great violations of human dignity were perpetrated.

4 See Chomsky, 1999, pp.111-134

3. Physics far from ethics: another great tragedy

Sen's thesis according to which the distance took by economics from ethics was harmful to economical sciences has several supporters. However, it is not evident the broad acceptation of the thesis according to which an analogous distance between physics and ethics was really harmful for the development of physics.

On the contrary, several physicists argue that in spite of the perpetration of hideous crimes against humankind occurred in the war period, the knowledge concerning physical sciences has been substantially increased during these times. This is equivalent to say that the worst extern developments can stimulate the development of an excellent physics. The examples adduced in favour of this argument are several like, the radar, the jet airplane, the rockets, the nuclear fission, etc.

I think that the above thesis can provide means for a dangerous conception and for a dangerous engagement. I can recognize that the tensions occurred during the war can have stimulated important developments, but this really occurred with a very high cost for the standard profile of ethical relationship among scientists and for the social responsibility of them. Indeed, I can say that neither the scientific community, and with much more reason, nor the scientific establishment, are immune to both: external and internal social pressures. Consequently the ethical standards of the practice of scientists were substantially altered by the irremovable social pressures suffered by scientists.

My privileged example is the *Manhattan Project*. This project is broadly considered as being the starting point of a new phase of science so called of *Big Science*. It is evident that during the Manhattan Project several relevant scientific developments occurred. However, the crucial point consists of knowing in what measure the advent of Big Science altered the ethical and scientific standards of the scientific practice and consequently in what measure the nature of the institution of science was altered.

It is possible to adduce that the alteration implied by the bird of Big Science led to an increasing in the submission of scientists to institutions responsible for the financial support of research. Furthermore, the bird of Big Science implied severe ethical consequences. Scientists were more susceptible to temptations and to venality than in the corresponding early times. The only possibilities to explain the reasons by which scientists accepted their engagement in research which is directly involved in destructive aims like napalm, super napalm, weapons of mass destruction and so on, are centred on the venality, on the prestige of their careers, and on fear of a persecution as a response to an eventual refusal to contribute.

It is perfectly possible to say that the ethical dimension is not the only dimension severely affected by the advent of Big Science. The cognitive dimension was too substantially affected by the new configuration and power of Big Science. The complex universities-industries-militaries reached to levels of entanglement which are unknown until those times.

In order to analyse these cognitive changes and repercussions, it is possible to adduce the criticism of the philosopher of science Karl Raimund Popper (1902-1994) against Kuhn's conception of *normal science*. Popper has argued that until 1939 all the scientific activity was engaged in a practice that could be called of *extraordinary science* in contraposition to *normal science* described by Kuhn, this last one requiring only an engagement in the dominant *paradigm* and a hard work to obtain results. In the context of the philosophy of science this interesting discussion is very important, but the focus here is only centred on Popper's claims according to which the period beginning at 1939 represents a starting point of a strong degradation with respect to the intellectual level of scientists. For a discussion of these points see Popper (1989, 1974, 2001), Kuhn (1996) and Bastos Filho (2000, 2001, 2006).

Surely, the Manhattan Project constituted a very special and singular example. The possibility of an atomic bomb being built by Nazis was a terrible ghost on the head of everybody. The project, in spite of being secret, could have a dissuasive purpose in the sense of once the objective is reached an eventual demonstration with international observers could be made.

Joseph Rotblat (1908-2005) was a scientist who participated of the Manhattan Project. He initially considered that the Project was necessary in order to prevent the possibility of the worst scenario to become a concrete reality: the monopoly of atomic weapons in hands of the Nazis. Indeed, one possibility like this could give rise to an unscrupulous use of these weapons with serious and unpredictable consequences for humankind. Rotblat himself has been moved by social responsibility convictions and accepted to work in the Project in this dramatic situation. The Project was directed by the general Leslie Groves (1896-1970) and so the military aims were a concrete reality. The great North American physicist Julius Robert Oppenheimer (1904-1967) was responsible for the scientific part of the Project.

At the end of 1944 when was evident that the Germans could not have had any possibility to build atomic weapons, Joseph Rotblat abandoned the Manhattan Project. According to Rotblat there was no ethical or moral reason to continue. His argument becomes still more emphatic with the defeat and capitulation of Nazis German in May 1945. Since 1939 Rotblat knew about the concrete possibility of an eventual construction of a nuclear weapon of great explosive power. It is interesting to quote here the following text:

> 1939 was the year in which two German scientists split the uranium atom, and set other scientists around the world on the pursuit of nuclear fission and the valuable energy it would release. Joseph Rotblat was among the first to realise that this reaction could be very fast and explosive, and could be used to make a massively powerful bomb.[5]

The continuation of this quotation expresses the feeling of Joseph Rotblat himself:

> As soon as I had this idea, I tried to push it out of my mind. But I had the feeling that other scientists might not have the same moral scruples. (ROTBLAT[6])

Since 1939 he knew about this possibility and so his participation in the Manhattan Project few years later was motivated by the dramatic situation that would constitute an eventual scenario of a monopoly of nuclear weapons in Nazis hands.

When the first nuclear test occurred in 16 July 1945 Germany was defeated but the USA was still in war against Japan. Japan was well prepared in conventional weapons, but without any possibility to construct atomic weapons. Independently of nuclear weapons, the USA superiority in terms of conventional weapons was a reality and as a consequence of this fact the victory of the USA against Japan was perfectly predictable. It was a mere question of time.

With the argument according to which the use of nuclear weapons against Japan could abbreviate the war and could save American lives from a bloody conventional confront the political establishment of the Truman government took the decision consisting of using nuclear weapons against cities in Japan. In my view this argument is unsatisfactory and hides the most probable motivation.

In first place the military establishment did not have special preoccupation with lives (including American lives) and in second place there was another geopolitical target to be reached: the power of Soviet Union and his influence in the post-war period.

5 http://www.ppu.org.uk/learn/infodocs/people/pp-rotblat.html

6 The same above electronic site

This explanation can be supported by many real facts. For example, in March 1944 Rotblat was shocked, at a private dinner at Chadwick's, to hear general Leslie Groves say "Of course, the real purpose in making the bomb was to subdue the Soviets"[7] (see also Cornwell, 2OO3, p. 374).

Other important facts corroborating the Pentagon Programme in order to subdue the Soviets were the operations in the immediate post-war aiming to protect the Nazis and to persecute the forces which were of crucial importance during the struggle against the Nazis (see Chomsky, 1999). All the efforts have been made by the occupation forces in order to avoid the growth of socialist influence in the Europe and in the world.

4. The Physicists and Ethics

I consider here four personalities who participate of the Manhattan Project: Joseph Rotblat (1908-2005), Julius Robert Oppenheimer (1904-1967), Edward Teller (1908-2003) and Luis Alvarez (1911-1988). Regarding to the reasons of Roblat, I have shortly commented above some of these. Rotblat becomes a pacifist and played an important role in the Pugwash Conferences and was one of eleven outstanding scientists which subscribed[8] the Einstein-Russell Manifest (see Souza Barros, 2005). Julius Robert Oppenheimer was the boss of the Project. After the war he was considered as a non reliable man. Edward Teller in a solemn circumstance once declared that Oppenheimer was a non reliable man in order to participate of the research of the hydrogen bomb. Edward Teller is known as the father of the American Hydrogen bomb. I would want to concentre my attention here on Luis Alvarez.

In his autobiographical book, Alvarez wrote:

> The atomic bombs that ended World War II, Little Boy and Fat Man, were delivered to Hiroshima and Nagasaki from an airfield on Tinian, a small island in the Mariana chain, between Guam and Saipan. An American physicist, thirty-four years old in 1945, I was there at the time and flew the first of the two historic missions, the leader of a small group responsible for monitoring the energy of the explosion. (ALVAREZ,, 1987)

The text displayed immediately above constitutes the first phrase of the first chapter entitled "Hiroshima Mission" of Alvarez's book entitled "Adventu-

7 http://en.wikipedia.org/wiki/Joseph_Rotblat

8 Max Born, Percy W. Bridgman, Albert Einstein, Leopold Infeld, Frederic Juliot-Curie, Hermann J. Muller, Linus Pauling, Cecil F. Powell, Joseph Rotblat, Bertrand Russell and Hideki Yukawa.

res of a Physicist". One direct interpretation of his text is enough to conclude that the so called "historical mission" is not something comparable to an adventure of a nice holiday trip. In this case, his adventure is reduced to something neither light nor admirable. On the contrary several thousand persons died in this "adventure". In other words his adventure is not comparable to the corresponding Indiana Jones adventures, last ones fiction products for entertainment. The bomb dropped on Hiroshima was a great tragedy for humankind and caused suffering for many people. I consider that the scope of monitoring the energy of the explosion is not any ethical defensible argument and does nor justify dropping bomb on a populated city. Evidently, an experiment of this kind could be made without killing human lives. Still worst is the argument according to which the choice of a previously non bombarded city as a target was motivated by providing an improved analysis of the impact of the destruction. The minimum to say is that this argument is cynical and criminal.

I assert with all my emphasis that the Hiroshima and Nagasaki events were perfectly avoidable and so no ethical reason could justify these two barbarities perpetrated against humankind and against the environment.

5. Irwin Bross

Concerning the residual radiation several experiments have been performed in the post-war period without ethical preoccupation. Several people in several situations have been submitted to these exposures. It is really impressive for the purpose of the present essay to read Irwin Bross's report on this subject. He wrote an essay for the International Congress *Scienza e Democrazia/Science and Democracy* entitled *On "Making History"*, also published in Italian with the title *Sul "Fare Storia"* (Bross, 2006). Let us read Bross's words:

> Of the approximately 222,000 Atomic Veterans, how many cancer deaths were caused by their exposures? A simple calculation in my book FIFTY YEARS provides a rough estimate. In a cohort of servicemen who were not Atomic Veterans about one-third of all deaths would be from cancer. In the data from the 1946 CROSSROADS test 68%, about two-thirds, died from cancer. The estimated proportion of excess cancer deaths, about 0.22, multiplied by 222,000 is about 50,000 excess cancer deaths. The excess deaths from all causes add to death toll. This demonstrates that the Pentagon "harmless" myth was a very deadly myth indeed! Nevertheless, despite the massive scientific evidence that now refutes the Big Lie, the Pentagon doctrine is still accepted by establishment scientists worldwide. (BROSS)

Following Irwin Bross (1921-2004) the scientific establishment on both sides of the Iron Curtain supported the thesis according to which the low-level ionizing radiation was "harmless". This fact shows that the dangerous entanglement involving military power, industry power and scientific establishment implied a great ethical problem in all parts of the world. Bross's words with respect to this subject are:

> The leaders of the scientific establishments on both sides of the Iron Curtain also saw eye-to-eye; both saw the wisdom of cooperating fully with the generals. Hence, the Pentagon doctrine that low-level ionizing radiation is "harmless" was not only quickly accepted by the British and French science establishments, it was immediately accepted by the Soviet science establishment as well. (BROSS)

Let us consider two of Bross's five conclusions:

> Pentagon takeover of the infrastructure of U.S. science and medicine has resulted in the end of the mid-20th century Golden Age. Scientific fraud and commercialism have led to the death of genuine science in the U.S.
>
> Preservationists trying to slow or stop the Hegemony juggernaut face a dangerous, persistent, and powerful adversary. At an intellectual level, preservationists can counter the propaganda about the wonders of U.S. science and medicine. They can disseminate the sad truth: 85% of the current "breakthroughs" reported in U.S. media – both in the mass media *and* in technical journals – are trivial, erroneous, or outright fraudulent.

I invite all to think on these above conclusions and upon the consequences of the war and the cold war.

6. Concluding Remarks

I arrive to the end of the comments of this essay in honour of Professor José Maria Filardo Bassalo. In writing this essay, my choice has been one centred on the ethical and political aspects of the scientific activity. The social responsibility of scientists was an important focus of my considerations. I think that we live in an open world which is susceptible to completely different scenarios of the future. Of course, the worst scenario would be the total destruction of human life on the surface of the earth. Evidently, every man, independently of being scientist or not, have the obligation of doing all their efforts in order to struggle against this possibility.

Fortunately there are several scenarios and we must work in the direction of the better scenario for the future. The contributions and the force of the civil society are essential in order to reach this aim.

Professor Bassalo is an intellectual from the Amazon region of Brazil. His activity at the Physics Department of the Federal University of Para during more than 40 years has been centred on the physical sciences, on the physical correlated disciplines and upon academic politics ruled by rigorous ethical principles. Starting from the development of physical sciences and from an education strongly rooted on a broad field of interests he was able to build in collaboration with his colleagues a Physics Department devoted to excellent research and excellent education.

His intellectual formation is sufficiently broad to conceive that the development of the scientific education in his region is strongly connected with the development of the immense possibilities that a generous and exuberant environment can provide. I am sure that Bassalo wants a development with Brazilian autonomy. This development involves the conservation of the rich ecosystems and rich biodiversity of his region and of the planet as a whole. This development also requires economic efficiency and social equality.

Finally, I would want to emphasize that a culture of peace is necessary. War is harmful for humankind and also to the environment. The impact caused to the environment by a war like this last one against Irak is incredible.

I finish this essay with a reference to the Kantian Ethics. On the contrary to the broadly accepted conception asserting that everything has price, Kant argued that there are beings for which we cannot find a monetary equivalence. Consequently, these beings have an inherent dignity and consequently these beings have not a price. Let us read Kantian words:

> In the kingdom of ends everything has either value or dignity. Whatever has a value can be replaced by something else which is *equivalent*; whatever, on the other hand, is above all value, and therefore admits of no equivalent, has a dignity (KANT, 1952, p. 274)
> *Autonomy* then is the basis of the dignity of human and every rational nature. (KANT, op. cit. p. 275)

Nature in the Amazon, comprehended its exuberant biodiversity and its wonderful ecosystem, the Amazon people and the Brazilian people have an inherent dignity and consequently have no price.

Bassalo is an outstanding example of a man of dignity that struggles for a better world.

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Synchrotron scalar radiation from a source orbiting a static chargeless black hole

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The radiation emitted by a scalar source rotating around a Schwarzschild black hole in both stable and unstable circular orbits is investigated. These results are compared with the ones obtained assuming the same source orbiting a central object due to a Newtonian gravitational force in Minkowski spacetime. The amount of radiation emitted from the source which is absorbed by the black hole is also computed. This work is dedicated to José Maria Filardo Bassalo.

The line element of a Schwarzschild black hole is given by [1]

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\theta^2 - r^2(\sin\theta)^2 d\varphi^2 \,, \tag{1}$$

where $f(r) = 1 - 2M/r$, and M is the black hole mass.

The positive-frequency modes associated to the timelike Killing field ∂_t, solutions to the Klein-Gordon equation in this black hole geometry, can be written as

$$u^n_{\omega l m}(x^\mu) = \sqrt{\frac{\omega}{\pi}}\,\frac{\psi^n_{\omega\, l}(r)}{r}\,Y_{lm}(\theta,\varphi)e^{-i\omega t} \quad (\omega > 0)\,. \tag{2}$$

There are two independent sets of solutions, here chosen to be incoming modes (*i*) from the past horizon $\mathcal{H}^-$ and (*ii*) from the past null infinity $\mathcal{J}^-$, labeled by n = $\rightarrow$ and n = $\leftarrow$, respectively.

In terms of the complete set of positive $\left(u^n_{\omega l m}\right)$ and negative-frequency modes $\left(u^{n\,*}_{\omega l m}\right)$, the scalar field $\left(\widehat{\Phi}\right)$ can be expanded as

$$\widehat{\Phi}(x^\mu) = \sum_{n=\rightarrow,\leftarrow} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \int_0^\infty d\omega$$
$$\times [u^n_{\omega lm}(x^\mu)\hat{a}^n_{\omega lm} + u^{n*}_{\omega lm}(x^\mu)\hat{a}^{n\dagger}_{\omega lm}] \,.$$

Let us consider a scalar source with $\theta = \pi/2$, r = R and constant angular velocity $\Omega > 0$ (as defined by asymptotic static observers), in uniform circular motion around the black hole, described by the current density

$$J(x^\mu) = \frac{q}{u^0\sqrt{-g}}\delta(r-R)\delta(\theta - \frac{\pi}{2})\delta(\varphi - \Omega t)\,, \tag{3}$$

where q is the coupling constant. The source has been normalized requiring that $\int d\sigma^{(3)} J(x^\mu) \;=\; q$, where $d\sigma^{(3)}$ is a three-volume element orthogonal to the four-velocity of the source, given by

$$u^\mu(\Omega, R) = (1,0,0,\Omega)/\left(f(R) - R^2\,\Omega^2\right)^{\frac{1}{2}} \,. \tag{4}$$

According to general relativity, stable circular orbits are allowed for $R > 6M$ and unstable ones for $6M > R > 3M$. For both stable and unstable circular orbits we have $R = (M/\Omega^2)^{1/3}$ [1].

The minimal coupling between the source and the scalar field is expressed through the interaction action

$$\widehat{S}_I = \int d^4x\,\sqrt{-g}\;J\widehat{\Phi}\,. \tag{5}$$

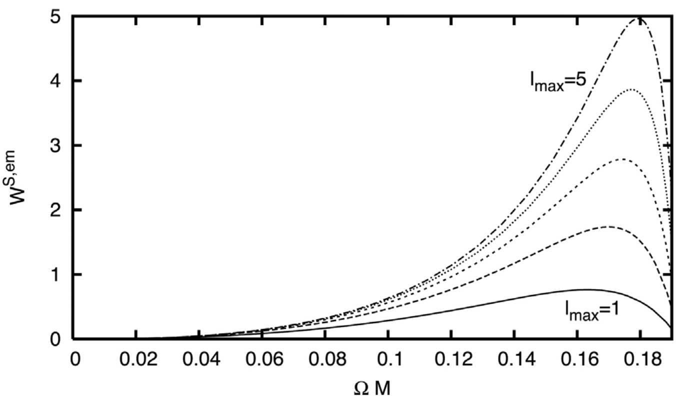


Figure 1: $W^{S,em}$ is plotted as a function of Ω, with the angular momentum summations carried on until l_{max} =1, 2, 3, 4, 5.

The power emitted by the rotating source with fixed values of the set (n, l, m) is [2, 3]

$$W_{lm}^{S,n,em} = \int_0^\infty d\omega\, \omega\, \frac{|\mathcal{A}_{\omega lm}^n|^2}{T}\,, \tag{6}$$

where $T \equiv 2\pi\delta(0) = \int_{-\infty}^{\infty} dt$ is the source total time according to asymptotic static observers [4], and $\mathcal{A}_{\omega lm}^n$ is the emission amplitude at tree level of one Boulware particle [5] with quantum numbers (n, ω, l, m).

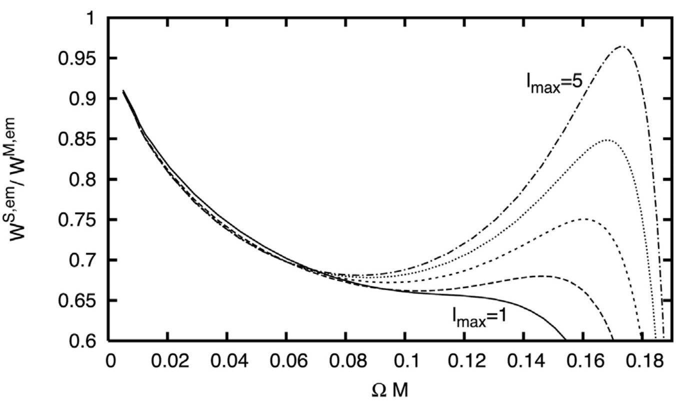
The total emitted power is given by

$$W^{S,em} = \sum_{n=\rightarrow,\leftarrow} \sum_{l=1}^{\infty} \sum_{m=1}^{l} 2q^2 m^2 \Omega^2 \,\times$$

$$\left[f(R) - R^2\Omega^2\right] \frac{|\psi_{\omega_0 l}^n(R)|^2}{R^2} \left|Y_{lm}\left(\frac{\pi}{2}, \Omega t\right)\right|^2 . \tag{7}$$

$W^{S,em}$ is plotted in Fig. 1 as a function of the angular velocity Ω of the rotating source, as measured by asymptotic static observers, with contributions from particles with angular momentum up to l_{max} =1, 2, 3, 4, 5. One sees that the contribution of larger values of the angular momentum $l = m$ become increasingly important as the rotating source gets nearer R =3M.

Figure 2:
The ratio $W^{S,em}/W^{M,em}$ is plotted as a function of Ω, with the angular momentum summations considered up to l_{max} =1, 2, 3, 4, 5.



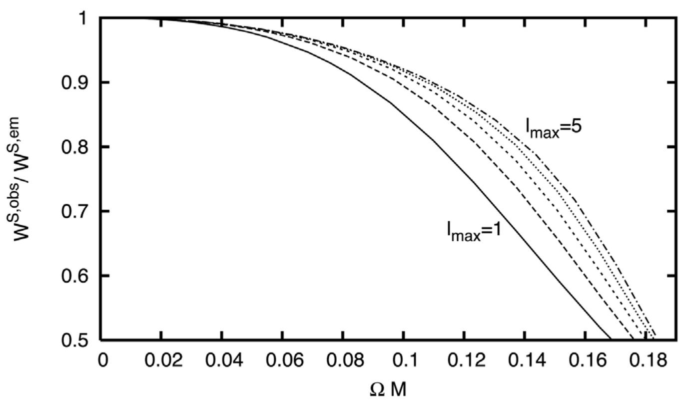
Let us now compare the emitted power of the source rotating around a Schwarzschild black hole with the emitted power obtained assuming a rotating source in Minkowski spacetime in orbit around a central object due to a Newtonian force, namely [2]

$$
\begin{aligned}
W^{M,em} &= \sum_{l=1}^{\infty} \sum_{m=1}^{l} 2q^2 m^2 \Omega^2 \gamma^{-2} \left| j_l \left[m \left(M\Omega \right)^{\frac{1}{3}} \right] \right|^2 \\
&\times \left| Y_{lm} \left(\frac{\pi}{2}, \Omega t \right) \right|^2 , \qquad (8)
\end{aligned}
$$

where $j_l(r)$ are the spherical Bessel functions [6], and γ is the Lorentz factor.

The ratio between the emitted power in Schwarzschild [$W^{S,em}$, given by Eq. (7)] and Minkowski [$W^{M,em}$, given by Eq. (8)] spacetimes is plotted in Fig. 2, considering angular momentum contributions up to l_{max} =1, 2, 3, 4, 5. This ratio tends to the unity as the charge rotates far away from the attractive center. This is compatible with the fact that Schwarzschild spacetime results should tend to Minkowski spacetime ones for wavelengths much larger than the black hole radius. For the last stable circular orbit at R =6M the ratio $W^{S,em}/W^{M,em}$ is about 0.7. As the source approaches R =3M (rotating around the black hole in unstable circular orbits according to general relativity), where higher values of the angular momentum have to be considered, this ratio is increased. In these ultrarelativistic

Figure 3:
The ratio $W^{S,obs}/W^{S,em}$, given by Eq. (9), and $W^{S,em}$, given by Eq. (7), is plotted as a function of Ω for geodesic (stable and unstable) circular orbits, considering angular momentum contributions up to l_{max} = 1,2,3,4,5. For stable circular orbits more than 95% of the emitted radiation reaches static observers at infinity. For unstable circular orbits, however, an increasing amount of radiation is absorbed by the black hole. The amount of absorption is more than 50% for Ω = 0.192/M, which is the value of the angular velocity corresponding to R≈3M.



orbits the emitted particles can be very energetic, with small wavelengths compared to the black hole radius. The closer to $R= 3M$ the rotating source is considered, the larger are the values of l that have to be taken into account in the angular momentum summations. The fall of the ratio $W^{S,em}/W^{M,em}$ in Fig. 2 close to $\Omega M=0.192$ is a consequence of the fact that the summations in the angular momentum have been performed up to a fixed value of l_{max}.

The radiation emitted by the swirling source, which is observed asymptotically, is given by

$$W^{S,obs} = \sum_{l=1}^{\infty} \sum_{m=1}^{l} \left[|\mathcal{T}^{\rightarrow}_{\omega_0 l}|^2 \, W^{S,\rightarrow,em}_{lm} + |\mathcal{R}^{\leftarrow}_{\omega_0 l}|^2 \, W^{S,\leftarrow,em}_{lm} \right] . \tag{9}$$

The ratio $W^{S,obs}/W^{S,em}$ is plotted in Fig.3 as a function of Ω for geodesic (stable and unstable) circular orbits, considering the summations in the angular momentum quantum numbers up to $l_{max} =1, 2, 3, 4, 5$. It can be seen that although only a small amount of the emitted power is absorbed by the black hole when the source is in stable circular orbits, this absorption raises up to more than 50% of the emitted radiation when the source approaches $R =3M$.

Acknowledgments

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On the existence periodic orbits in a class of Mechanical Hamiltonian Systems – An elementary mathematical analysis

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We present an illustrative application of the two famous Mathematical Theorems in Manifolds in R^N, namely: The Hadamard Theorem on the characterization of convex hypersurfaces in R^N and the well-known Brower fixed point Theorem – in order to show the existence of periodic orbits with an arbitrary given period for a class of Hamiltonian systems. This mathematical result sheds some light on the old mathematical question of existence of periodic orbits in the N-body problem in Classical Mechanics.

1 Elementary may be deep - T. Kato

One of the most fascinating study in Classical – Astronomical Mechanics is that one related to the inquiry on how energetic-positions configurations in a given mechanical dynamical system can lead to periodic systems trajectories for any given period *T*, a question of basic importance on the problem of the existence of planetary systems around stars in Dynamical Astronomy ([1]).

In this short note we intend to given a mathematical characterization of a class of *N*-particle Hamiltonian systems possesing intrinsically the above mentioned property of periodic trajectories for any given period *T*, a result which is obtained by an elementary and illustrative application of the famous Brower fixed point theorem.

As a consequence of this mathematical result we conjecture that the solution of existence of periodic orbits in Classical Mechanics of *N*-body systems moving through gravitation should be searched with the present status of our Mathematical Knowlegdment.

Let us, thus, start our note by considering a Hamiltonian function $H(x^i,p^i) \equiv H(z^i) \in C^\infty(R^{6N})$ of a system of *N*-particles set on motion in R^3. Let us suppose that the Hessian matrix determinant associated to the given Hamiltonian is a positive-definite matrix in the $(6N-1)$-dimensional energy-constant phase space $z^i = (x^i, p^i)$ of ourgiven Mechanical system

$$\left\| \frac{\partial^2 H}{\partial z^k \partial z^s} \right\|_{H(z^s)=E} > 0 \tag{1-a}$$

In the simple case of N-particles with unity mass and a positive two-body interaction with a potential $V(|x_i - x_j|^2)$, such that the set in R^n, $\{x^i | V(x^i) < E\}$ is a bounded set for every $E > 0$. Our condition eq. (1-a) takes a form involving only the configurations variables $(\{x^p\}_{p=1,\ldots,N}; x^p \in R^3)$. Namely

$$||M_{ks}(\{x^p\})|| > 0 \tag{1-b}$$

with

$$\begin{aligned} M_{ks}(\{x^p\}) = \sum_{i<j}^{N} [V''(|x_i - x_j|^2) 4(x_i - x_j)^2 (\delta_{is} - \delta_{js})(\delta_{ik} - \delta_{jk}) \\ + 2V'(|x_i - x_j|^2)(\delta_{ik} - \delta_{jk})(\delta_{is} - \delta_{js}))] \end{aligned} \tag{1-c}$$

The class of mechanical systems given by eq.(1-b)/eq.(1-c) is such that the energy-constant hypersurface $H(z^s) = E$ has always a positive Gaussian curvature and – as a straightforward consequence of the Hadamard Theorem – such energy-constant hypersurface is a convex set of R^{6N}, besides of obviously being a compact set. As a result of the above made remarks, we can see that our mechanical phase space is a convex-compact set of R^{6N}.

The motion equations of the particles of our given Hamiltonian system is given by

$$\begin{aligned} \frac{dx^i(t)}{dt} &= \frac{\partial H}{\partial p^i}(x^i(t), p^i(t)) \\ \frac{dp^i(t)}{dt} &= -\frac{\partial H}{\partial x^i}(x^i(t), p^i(t)) \end{aligned} \tag{2}$$

with the initial conditions belonging to our phase space of constant energy $H_E = \{(x^i, p^i); H(x^i, p^i) = E\}$, namely:

$$(x^i(t_0), p^i(t_0)) \in \mathcal{H}_E \tag{3}$$

The existence of periodic solutions of the system of ordinary differential equations eq.(2) – eq.(3); for any given period $T \in R^{+}$ is a consequence of the fact that the Poincaré recurrent application associated to the (global) solutions eq.(2) of the given mechanical system. Note that $H(x^i(t),p^i(t))= E$

$$P\colon \mathcal{H}_E \to \mathcal{H}_E \qquad (4)$$
$$(x^i(t_0), p^i(t_0)) \to (x^i(T), p^i(T))$$

has always a fixed point as a consequence of the application of the Brower fixed point theorem ([3]). This means that there exists always initials conditions in the phase space eq.(3) that produces a periodic trajectory of any period T in our considered class of mechanical N-particle systems.

As a physical consequence, we can see that the existence of planetary systems around stars may be not a rare physical astronomical event, but just a consequence of the mathematical description of the nature laws and the fundamental theorems of Differential Topology and Geometry in R^{N}.

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On Noncommutative Field Theories

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I review some aspects of noncommutative field theories which are nonlocal theories presenting some unusual properties. Among these properties, we discuss a peculiar mixture of scales, the ultraviolet/infrared mixing and the breaking of Lorentz symmetry.

In the recent years, a considerable interest has been devoted to the investigations of the properties of noncommutative field theories. The motivations behind these studies come from diverse areas as renormalization [1], quantum gravity [2] and strings in a magnetic background [3]. Whatever the motivation is [4], the basic idea is that position operators do not commmute but rather satisfy

$$[q^{\mu}, q^{\nu}] = i\Theta_{\mu\nu}, \tag{1}$$

where the antisymmetric matrix $\Theta_{\mu\nu}$ in general may depend on the coordinates; in the most studied situation to which I shall restric myself, it is a constant matrix. Because of the above commutation rules, ordinary functions of coordinates become nontrivial operators. However, using the so called Weyl correspondence, we may work with functions of commutating coordinates, x^{μ} endowed with the noncommutative Moyal product

$$\begin{gathered}\phi_1(x) * \phi_2(x) = \lim_{y \to x} \mathrm{e}^{\frac{i}{2}\Theta^{\mu\nu}\frac{\partial}{\partial y^{\mu}}\frac{\partial}{\partial x^{\nu}}} \phi_1(y)\phi_2(x) = \\ \phi_1(x)\phi_2(x) + \frac{i}{2}\Theta^{\mu\nu}\partial_{\mu}\phi_1(x)\partial_{\nu}\phi_2(x) + \ldots .\end{gathered} \tag{2}$$

From the above, one sees that these theories are highly nonlocal. This nonlocality implies in the occurrence of a peculiar mixture of scales, the so called Utraviolet/Infrared (UV/IR) mixing, by which some of the usual ultraviolet divergences are transmutated into infrared ones. Besides that, notice that a breaking of unitarity/causality will occur if the noncommutativity affects the time, i.e., if$\Theta_{0\iota} \neq 0$ [5]. This is due to the noncommutativity between the time ordering prescription and the exponentials containing time derivatives. For local theories, where the dependence on the time derivatives is polynomial, this noncommutativity implies into the appearance of contact terms, without any physical relevance. In the case one uses the Moyal product, which contains derivatives of arbitrary order the result is much more complicated and can not be disregarded. For this reason, in this work we evade this problem by keeping the time local, setting $\Theta_{0\iota}$=0.

In general, noncommutative field theories are constructed by replacing in the action of models the pointwise product of fields by the Moyal one, using

$$\begin{aligned}
&\int dx\phi_1(x) * \phi_2(x) * ... * \phi_n(x) = \\
&\int \prod \frac{d^4k_i}{(2\pi)^4}(2\pi)^4\delta(k_1+k_2+\ldots+k_n)\tilde{\phi}_1(k_1)\tilde{\phi}_2(k_2)\ldots\tilde{\phi}_n(k_n) \\
&\exp(-i\sum_{i<j} k_i \wedge k_j),
\end{aligned} \tag{3}$$

where $k_i \wedge k_j = \frac{1}{2}k_i^\mu k_j^\nu \Theta_{\mu\nu}$. Proceeding in this way, the Feynman rules of the perturbative series are modified so that the vertices acquire phase factors which depend on the integration variables. In some circumstances, these trigonometric factors cancel and the resulting contribution is said to be planar (unless by multiplicative constants, it is equal to the amplitude of the corresponding commutative model). In other situations, called nonplanar, the phases do not cancel and, due to their oscillating character, produce integrals that are ultraviolet finite but may diverge for small momenta. Whenever these infrared singularities are stronger than logarithimic, they become very dangerous, leading to the breakdown of most of usual perturbative schemes. An illustration of this possibility is provided by the integral

$$\int d^4k \frac{\cos(k_\mu\Theta^{\mu\nu}p_\nu)}{k^2-m^2} \approx \frac{1}{(\Theta p)^2}, \tag{4}$$

which occurs in the lowest order contribution to the self-energy of the basic field in the noncommutative φ^4 model. Clearly, insertions of this result in the amplitudes associated to large diagramas lead to nonintegrable singularities, i. e., to the breakdown of the perturbative series.

The investigation on the occurrence of dangerous singularities is therefore crucial for the caracterization of models that are consistent at least as far the UV/IR mixing is concerned. These studieslead to the conclusion that the best behaved models are the supersymmetric ones[6].

It is also worth to note that the mere separation of Feynman graphs into planar and nonplanar parts may bring difficulties to the renormalization process. Examples: the four dimensional $O(N)$ linear sigma model with $N>2$ [7] and the (2+1) dimensional Gross-Neveu [8,9] models. In both situations, there are two different structures which in the commutative case are made finite by the renormalization of a single parameter; however, the introduction of noncommutativity spoils this delicate balance bringing difficulties to the renormalization program. To see how this takes place, let us consider the Gross-Neveu model which in commutative setting is described by

$$\mathcal{L} = \frac{i}{2}\overline{\psi}\ \partial\!\!\!/\psi - \frac{\sigma}{2}(\overline{\psi}\psi) - \frac{N}{4g}\sigma^2, \tag{5}$$

where ψ_i, with i =1, 2,...,n are two-components Majorana fermions and σ is an auxiliary field introduced to implement most easily the $1/N$ expansion. Observe that, by integrating over the σ field one arrives at the four-fermion self-interaction, characteristic of the model. A basic feature of this model is that the coupling constant g plays a double role: On one hand, the gap equation

$$\frac{M}{2g} - \int \frac{d^Dk}{(2\pi)^D}\frac{Mi}{k^2 - M^2} = 0. \tag{6}$$

fixes g in terms of $M = \langle 0|\sigma|0\ \rangle$. Onthe other hand, the twopoint function of the auxiliary field: also depends on g as

$$F(p) \ = \ -\frac{iN}{2g} - N\int \frac{d^Dk}{(2\pi)^D}\frac{k\cdot(k+p)+M^2}{(k^2-M^2)[(k+p)^2-M^2]}. \tag{7}$$

Notice now that, as the integrals in (6) and (7) have the same ultraviolet divergence, they can be made finite by aconvenient choice of Δ in the reparametrization, $1/g \rightarrow 1/_{gR}+\Delta$. Nevertheless, in the noncommutative situation: the gap equation does not changes whereas a trigonometric factor $(\cos^2(k \wedge p) = \frac{1}{2}[1 + \cos(k \wedge p)])$ appears in the integrand of the integral in Eq. (7) and, as a consequence, the cancellation of the ultraviolet divergence no longer occurs.

In the literature there are some proposals to circumvent the problems posed by the UV/IR mixing; some of them are

1. Resummations to rearrange the perturbative series to get a better behaved expansion[10]. A shortcoming of this approach is that it is not a systematic procedure as there is no apriori small parameter to control the new series.
2. Enforce invariance under duality transformation which exchanges position and momentum [11]. For the noncommutative φ^4 model it wasproved that [12]

$$
\begin{aligned}
S(\varphi) \;=\; \int d^4x \left(\frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \varphi) * (\tilde{x}^\mu \varphi) + \frac{m^2}{2}\varphi * \varphi \right. \\
\left. - \frac{\lambda}{4!}(\varphi)^4_* \right); \qquad \tilde{x}_\mu = 2\Theta^{-1}_{\mu\nu} x^\nu
\end{aligned}
$$

leads to a well defined expansion, without the UV/IR mixing. A difficulty with this approach is the explicit breaking of translation invariance [13]. Also ageneralization to incorporate gauge invariance is still lacking.

3. Forgauge theories,it wasproposed[14] to add to the Lagrangian the term $\lambda(x)*\Theta_{\mu\nu}F^{\mu\nu}$, where $\lambda(x)$ is an auxiliary field which turns the gauge propagator transversal to $\bar{k}^\mu=\Theta^{\mu\nu}k_\nu$. Finiteness,inthe axialgauge, wasproved up to two loops[15].
4. The coherent state approach[16]. I exemplify this method by considering noncommutativity in 2+1 dimensions. Since $\Theta_{0i}= 0$, the only nonvanishing components of the $\Theta_{\mu\nu}$ matrix are $\Theta_{12} = -\Theta_{21} = \Theta$.

It begins by introducing complex coordinates $\hat{z} = q_1 + iq_2$ and $\hat{z}^\dagger = q_1 - iq_2$ which satisfy

$$[\hat{z}, \hat{z}^\dagger]=\Theta. \tag{8}$$

Defining a "vacuum" state by $\hat{z}|0>=0$, a coherent state $|\alpha>$ is an eigenstate of the "anihilation" operator $\hat{z}$:

$$\hat{z}|\alpha >= \alpha|\alpha >, \qquad \text{so that } |\alpha >= \exp\left(-\tfrac{1}{2}|\alpha|^2\right)\exp\left(\alpha\hat{z}^\dagger\right)\,|0>. \tag{9}$$

Now, given a field φ we may associate to it an operator Φ by

$$\Phi(q) = \int \frac{d^3k}{(2\pi)^3} \mathrm{e}^{-ikq}\tilde{\varphi}(k), \tag{10}$$

where $\tilde{\varphi}$ (k) denotes the Fourier transform of φ(x). Its expectation value

$$\psi(x) =< \alpha|\Phi|\alpha >= \int \frac{d^3k}{(2\pi)^3} \mathrm{e}^{-ikx-\frac{1}{4}\Theta|k|^2}\,\tilde{\varphi}(k) \tag{11}$$

defines a new field. If φ is a free scalar quantum field then the two point of ψ is

$$\begin{aligned} \Delta_F(x-y) \;&\equiv\; < 0|T\psi(x)\,\psi(y)|0 >= \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\mathrm{e}^{-ik_1x-ik_2y} \\ &\times\;\mathrm{e}^{-\frac{1}{4}\Theta(|k_1|^2+|k_2|^2)}(2\pi)^3\delta^3(k_1+k_2)\frac{i}{k_1^2-m^2} \\ &= \int \frac{d^3k}{(2\pi)^3}\mathrm{e}^{-ik(x-y)}\frac{i}{k^2-m^2}\mathrm{e}^{-\frac{1}{2}\Theta|k|^2} \end{aligned} \tag{12}$$

Using this formalism, we are in a position to propose the Lagrangian for the Gross-Neveu model as being[17]

$$\mathcal{L} = \bar{\psi}\frac{1}{2}(i\,\partial\!\!\!/ - M)\mathrm{e}^{-\frac{\theta}{2}\triangle}\psi - \frac{\sigma}{2}(\overline{\psi}\psi) - \frac{N}{4g}\sigma^2. \tag{13}$$

The resulting noncommutative theory is finite and free of te UV/IR mixing fornonvanishing Θ. In particular, the gap equation furnishes

$$\frac{1}{2g} - i\int \frac{d^3k}{(2\pi)^3}\frac{1}{k^2-M^2}e^{-\frac{1}{2}\Theta|\vec{k}|^2} = 0 \tag{14}$$

Interesting enough, the Θ →0 limit is not the commutative Gross-Neveu model, but an asymptotically free theory.

Lorentz invariance is another point of concern when treating noncommutative theories. Indeed, for noncommutative models Lorentz symmetry breaking is introduced from the very start by the stipulation that the $\Theta_{\mu\nu}$ matrix is constant. Nonetheless, Lorentz invariance is one of the most sucessful symmetries having survived a plethora of stringent tests. In this situation, we can envisage various possibilities:

1. In all energy scales Lorentz symmetry holds irrestricted so that the usual (canonical) non-commutative theories that we have been describing needs to be modified [18–20]. In this respect we recall that in Snyder's original idea [1] Lorentz symmetry was mantained by suggesting that the commutator between coordinates should be an operator. In fact he imposed

$$[q^{\mu}.q^{\nu}]= i\mathrm{a}\, M^{\mu\nu}, \text{ where } M^{\mu\nu} \text{ are the generators of the Lorentz group} \tag{15}$$

The contraction of Snyder's algebra [21] then leads to a noncommutative space where $\Theta^{\mu\nu}$ is a new dynamical variable:

$$[q^{\mu}.q^{\nu}]= i \qquad \hat{\Theta}^{\mu\nu}[\mathrm{q}^{\rho}, \hat{\Theta}^{\mu\nu}]=0 \qquad [\hat{\Theta}^{\mu\nu}, \Theta^{\rho\sigma}]=0 \tag{16}$$

According to a recent proposal one should the integrate over the Θ's variables [21, 22]. In this approach it is not clear how to introduce observables.

2. Lorentz symmetry is broken at very high energy scales, possibly of the order of Planck energy ($\approx 10^{19}$Gev) but its effect on our energy scale is strongly suppressed. For commutative but <u>nonsupersymetric theories</u> (Yukawa like models) large Lorentz symmetry violations occur at all scales although situation seems to be more favorable for supersymmetric models. Also canonical comutativity for nonsupersymmetric models turns out to be problematic: A smooth commutative limit seems to be required. The situation for noncommutative supersymmetric theories is exemplified by the Wess-Zumino model described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_m + \mathcal{L}_g \,, \tag{17}$$

where

$$
\begin{aligned}
\mathcal{L}_0 &= \frac{1}{2}A(-\partial^2)A + \frac{1}{2}B(-\partial^2)B + i\frac{1}{2}\overline{\psi}\ \partial\!\!\!/\psi + \frac{1}{2}F^2 + \frac{1}{2}G^2\,, \\
\mathcal{L}_m &= mFA + mGB - \frac{m}{2}\overline{\psi}\psi\,, \\
\mathcal{L}_g &= g(F*A*A - F*B*B + G*A*B + G*B*A \\
& -\overline{\psi}*\psi*A - \overline{\psi}*i\gamma_5\psi*B)\,.
\end{aligned} \tag{18}
$$

In this model the self-energy for the scalar field A contains the Lorentz breaking nonplanar part [23]

$$
\begin{aligned}
\Gamma^{(NP)}(p) &= \frac{g^2}{2\pi^2}\,p^2\int_0^1 dx\,x\,K_0\left(\sqrt{a^2\,p\circ p}\right)\,, \\
a^2 &\equiv m^2 - x(1-x)\,p^2 \qquad p\circ p = \left(\vec{p}\cdot\vec{\theta}\right)^2 - \vec{p}^{\,2}\,\vec{\theta}^{\,2}\,.
\end{aligned}
$$

To quantify the loss of Lorentz invariance one uses the parameter suggested in [19]

$$
\xi = \frac{\partial^2\Gamma^{(NP)}(p)}{\partial(p^0)^2} + \frac{1}{3}\sum_{j=1}^{3}\frac{\partial^2\Gamma^{(NP)}(p)}{\partial(p^j)^2}\,. \tag{19}
$$

which for the present situation gives [23]

$$
\xi = \frac{g^2}{3\pi^2}\,. \tag{20}
$$

independent of Θ. So, there is no supression and the breaking occurs at all energy scales.

3. At very short distances Lorentz symmetry is not violated but it is replaced by Twisted Lorentz symmetry [24,25]. It must be noticed that the (deformed) Moyal product

$$
\phi_1 * \phi_2 = m_\theta(\phi_1\otimes\phi_2) = m_0(\mathcal{F}\phi_1\otimes\phi_2), \tag{21}
$$

where

$$\mathcal{F} = \mathrm{e}^{-\frac{i}{2}\Theta_{\mu\nu}P_\mu \otimes P_\nu} \qquad P_\mu = -i\partial_\mu \tag{22}$$

and $m_0(\phi_1 \otimes \phi_2) = \phi_1\, \phi_2$ is the usual pointwise multiplication map, is not compatible with the usual coproduct given by $\Delta_0(\Lambda)=\Lambda\otimes\Lambda$ but requires the use of

$$\Delta_\theta(\Lambda) = \mathcal{F}^{-1}\Delta_0(\Lambda)\mathcal{F} \ . \tag{23}$$

Furthermore, it has been argued that a criterious application of twisted Lorentz symmetry leads, surplisingly, to models without the UV/IR mixing [26]. In this setting the usual commutation relations between creation and anihilation operators would be modified so that Pauli's principle would be violated (an opposite viewhasbeen sustainedin[27]).

From the above, we see that there are many avenues in which the study of noncommutative field theories may proceed. Some of them are very interesting routes. However, consistency and mathematical beauty are not the only requirements for physically relevant theories to succeed and we all wait for more physical insights that we hope will come in the next years.

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Conformal Special Relativity

A tribute to J.M. F. Bassalo on his retirement

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It is shown that the information loss/recovery theorem in black-holes based on the ADS/CFT correspondence is not consistent with the stability of the Schwarzschild or Reissner-Nordstrom black holes in the AdS_5 bulk space.

In 1974 S. Hawking proved his well known theorem on black hole evaporation: Virtual particle pairs are formed in the vicinity of a black hole. While one of the particles falls inside the black hole horizon, the other escapes to infinity, resulting in a thermal radiation of the black hole. During this process the correlation between the spin states of the particles pairs is lost, so that the unitarity of quantum mechanics does not hold [1]. More recently, Hawking presented a new version of his theorem, based on the conformal covariance of the Yang-Mills equations in Minkowski's space-time and using the ADS/CFT correspondence. Furthermore, it was assumed that the black holes in question are charged (like for example, the Reissner-Nordstrom black hole) embedded in the AdS_5 space. After the evaporation, they would leave a remnant subspace in which the information is stored and eventually could be partially retrieved [2]. The use of the ADS/CFT correspondence saves the quantum unitarity near a strong gravitational field.

To understand how this works, consider the well known property that the four-dimensional space-time AdS_4 is a hypersurface with negative constant curvature embedded in the five dimensional flat space M_5 (3,2), with maximal isometry SO (3,2) [3,4]. By simply adding one extra spatial dimension to each of these spaces, we obtain a five-dimensional AdS_5 embedded in M_6 (4,2), with maximal isometry SO(4,2).

Now, the group $SO(4,2)$ happens to be is isomorphic to the 15-parameter conformal group C_0 defined in Minkowski's space-time (M_4), also acting as a symmetry for Maxwell's equations and in fact of all Yang-Mills (Y.M.) fields. Therefore, this isomorphism establishes a correspondence between the conformal covariant Yang-Mills fields in M_4 and isometric invariant Yang-Mills fields in a four-dimensional subspace of the AdS_5 space. The correspondence between a conformal field theory in M_4 and an isometric field theory in a submanifold of the 5-dimensional anti-deSitter space-time AdS_5 can be summarised in the diagram

$$\begin{array}{ccccccc} M_4 & \overset{conform}{\Longleftarrow} & C_o & \overset{isomorphic}{\sim} & S0(4,2) & \overset{isometric}{\Longrightarrow} & AdS_5 \\ \updownarrow & & & & & & \updownarrow \\ Y.M.\ in\ M_4 & \leftarrow & & corresponds\ to & & \rightarrow & Y.M.\ in\ AdS_5 \end{array}$$

Since all Yang-Mills fields are quantisable, this correspondence also implies in the existence of a unitary quantum theory of fields in the vicinity of a four-dimensional black hole embedded in the AdS_5.

However, the black hole eventually evaporates, so that the geometry of the embedded space-time necessarily changes, meaning that the embedding cannot be rigid, but rather dynamical. This means that the integrability equations for the embedding of a black hole into the AdS_5 bulk must hold along the process of evaporation.

The AdS_5 is a space with constant Riemann curvature $\Lambda/6$, which is characterised by the Riemann tensor

$$^5\mathcal{R}_{ABCD} = \frac{\Lambda}{6}(\mathcal{G}_{AC}\mathcal{G}_{BD} - \mathcal{G}_{AD}\mathcal{G}_{BC}), \quad A, B = 1...5$$

Applying this to the integrability conditions for a four-dimensional space-time embedded in the AdS_5 , we obtain the Gauss-Codazzi equations involving the metric $g_{\mu\nu}$ and the extrinsic curvature $k_{\mu\nu}$ [3]:

$$R_{\alpha\beta\gamma\delta} = (k_{\alpha\gamma}k_{\beta\delta} - k_{\alpha\delta}k_{\beta\gamma}) + \frac{\Lambda}{6}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) \qquad (1)$$

$$k_{\alpha[\beta;\gamma]} = 0, \qquad (2)$$

From the Einstein-Hilbert principle applied to the AdS_5 geometry, plus the above conditions we obtain the gravitational equations for the embedded space-time [5]

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} - Q_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi G T_{\alpha\beta} \tag{3}$$

where we notice the presence of an extrinsic curvature term

$$Q_{\alpha\beta} = (k^{\rho}{}_{\alpha}k_{\rho\beta} - hk_{\alpha\beta}) - \frac{1}{2}(K^2 - h^2)\ g_{\alpha\beta}$$

with $h = g^{\mu\nu}k_{\mu\nu}$ and $K^2 = k^{\mu\nu}k_{\mu\nu}$.

However, not all four-dimensional solutions of Einstein's equations satisfy (1), (2) and (3) without being constrained by the AdS_5 geometry. The simplest example is given by Minkowski's space-time M_4 itself, regarded as a subspace embedded in the AdS_5 . In this case, the left hand side of (1) vanishes. Using (2) we obtain the general solution for the extrinsic curvature $k_{\mu\nu} = \sqrt{\Lambda/6}\ \eta_{\mu\nu}$. The presence of such nonzero extrinsic curvature $k_{\mu\nu}$ means that although M_4 is flat, it is warped, like for example a cylinder, a cone or a helicoid (all being flat spaces according to Riemann). The consequence is that the translational symmetry of the Poincaré group does not hold in all directions of such warped M_4 (The Minkowski flat plane is not characterised as such by the vanishing of the Riemann curvature alone. It relies on the Poincaré symmetry to be truly a plane).

The other example is given by black holes. Assuming that the four-dimensional subspace embedded in AdS_5 is a Schwarzschild or a Reissner-Nordstrom black hole, their metric and extrinsic curvatures must also satisfy (1) and (2). For any spherically symmetric diagonal metric, the general solution of (2) is like $k_{\mu\nu} = \alpha_0 g_{\mu\nu}$, where α_0 is a function of r. Replacing this solution in in (3), we obtain the Schwarzschild-anti-deSitter solution

$$ds^2{=}(1{-}\frac{2m}{r} + (3\alpha_0^2{-}\Lambda)r^2)^{-1}dr^2 + r^2d\omega^2 - (1{-}\frac{2m}{r} + (3\alpha_0^2{-}\Lambda)r^2)dt^2$$

and the Reissner-Nordstrom-anti-de Sitter solution

$$ds^2{=}(1{-}\frac{2m}{r}{+}\frac{q}{r^2}{+}(3\alpha_0^2{-}\Lambda)r^2)^{-1}dr^2{+}\ r^2d\omega^2{-}\ (1{-}\frac{2m}{r}{+}\frac{q}{r^2}\ {+}(3\alpha_0^2{-}\Lambda)r^2)dt^2$$

As it can be seen, due to the presence of the r^2 term in these metrics, the gravitational field grow indefinitely, creating an unstable horizon. This instability is a consequence of geometry of the AdS_5 bulk which imposes restrictions to the geometry of the embedded geometries. Since Λ is locally insignificant it may be eliminated, but we have no experimental evidence on the behavior of $k_{\mu\nu}$ near a black hole, except that it must not be zero.

A possible solution for the above instability is to consider that the physical space is the whole flat space M_6 (4, 2) instead of only its hypersurface AdS_5 . In fact, both black-holes are known to be embedded in M_6 (4, 2) without restrictions to its extrinsic geometry [3, 4]. The six dimensional flat plane M_6 (4, 2) replaces M_4 in what could be called Special Conformal Relativity. M_6 (4, 2) has the same group of isometries as the AdS_5 , so that all arguments of the ADS/CFT correspondence which depend only on the Lie group properties (therefore excluding supersymmetry),can be extended without loss of generality to this six dimensional flat space. Thus, by the same argument used in the ADS/CFT correspondence, the quantum Unitarity of theYang-Mills fields is maintained in M_6 (4, 2), with the advantage that the there is horizon instability. However we need to learn how to do physics in a space-time with two times.

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Geometrical description of spin-2 fields

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We show that the torsion of a Cartan geometry can be associated to two spin-2 fields represented bysymmetric second-order tensors $A_{\mu\nu}$ and $B_{\mu\nu}$. We use the generalized Hilbert-Einstein Lagrangian R to generate a dynamics for the Cartan space-time thus conciliating the possibility of having torsion in a geometrical framework in which the metric is given by the Minkowski form $\eta_{\mu\nu}$. The main reason for this is that in the generalized Einstein dynamics the energies associated to the fields $A_{\mu\nu}$ and $B_{\mu\nu}$ have opposite signs and cancels out each other without producing a Riemannian curvature. This fact allows us to introduce the notion of torsion vacuum state.

1. Introductory comments

Let us suppose, that one faces the following question: "how to associate a symmetric second-order tensor $\varphi_{\mu\nu}$, that represents a spin-2 field, into a geometrical framework? "The lesson we had learned from the last century led almost univocally to a prompt answer: one should add this field to the Minkowski metric tensor of the flat background spacetime and generate a curved Riemannian geometry:

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \varphi_{\mu\nu}. \tag{1}$$

Then, the one who asks for a dynamics within this geometrical scheme is led almost naturally to the Hilbert-Einstein action concerning the curvature scalar of the associated Riemannian geometry:

$$S = \int \sqrt{-g}\, R\, d^4x \tag{2}$$

This kind of geometrisation is the basis of Einsteins General Relativity (GR). We are interested in answering the question: is such formulation the unique way to geometrize a spin-2 field? In the present paper we will present an alternative approach to the answer of the above question that apparently has not beem noticed before. Let us stress from the beginning that we are not considering an alternative to describe gravitational effects but rather to examine the consequences of having more than one spin-2 field besides the one hidden in the metric tensor. The motivation of this paper is precisely this: to report an alternative form to integrate spin-2 field in geometry. We shall see that one must consider a structure — the so-called Cartan geometry [1] — which, besides the metric, contains two symmetric second-order tensors $A_{\mu\nu}$ and $B_{\mu\nu}$ hidden in the antisymmetric torsion $\tau^{\alpha}_{\mu\nu}$ displayed by this geometry.

The idea that torsion can be associated to fundamental fields has been examined previously (see for instance [2]) in an attempt to incorporate in torsion definition the concept of a manifestation of an antisymmetric field typical of string theories. In the present paper we intend to present another way to consider this problem by incorporating two spin-2 tensors into the Cartan geometrical scheme.

In the present paper, we will deal with the simplest structure that besides the torsion there exists a metric tensorgiven by the Minkowski form $\eta_{\mu\nu}$ in order to show the net properties of such an identification more clearly. We use the curvature scalar as a natural generalization of Hilbert-Einstein action. We shall see that the existence of a non-null curvature is a consequence only of the presence of the affine connection represented by the torsion tensor, once the Riemannian part of the curvature – which depends on the metric tensor and on the associated Christoffel symbol – vanishes. One could distinguish the Einstein sector only by interpreting torsion acting as a source of the associated Riemann curvature. This characteristics, for the case of fields $A_{\mu\nu}$ and $B_{\mu\nu}$ associated to the torsion, may at first seem to cause an apparent difficulty. The reason is the following. If we adopt the point of view in which torsion can be thought as containing these two fields, then one could ask for the reasons why the energy associated to the torsion fields does not cause a Riemannian curvature, as required by the equivalence principle and GR. The solution of this question comes after realizing that the energies of each spin-2 fields associated to the dynamics generated by the generalized Hilbert-Einstein action, have opposite signs and cancel out each other in its fundamental state, as will be shown in this paper.

For the sake of definiteness we have limited our present analysis to the case in which such fields are sterile in the sense that they undergo only gravitational interaction. We have postponed the analysis of other possible interactions for a future work.

A. Synopsis

In the present paper we will deal with Cartan geometry. In section II we present a set of definitions and equations which are needed to understand the nomenclature we have adopted. In Section III we deal with the particular case of restricted Cartan geometry, in which the degrees of freedom of the torsion field are reduced from 24 to 10. We present the affine connection and the associated curvature tensor. We use the curvature scalar of such a restricted case in order to produce a dynamics for torsion. We show that this dynamics coincides with the standard dynamics for a spin-2 field, that is, the Einstein linearized case. In the next section we generalize this formalism and deal with two spin-2 fields, which correspond to 20 degrees of freedom. We use the same structure of the curvature scalar to produce a dynamics for torsion and we show that it is nothing but the same dynamics for two non-interacting spin-2 fields. An unexpected result then comes out: the spin-2 fields have opposite energies. We finish the present paper with some conclusions and comments on the possibility of associating such a description of spacetime torsion by two sterile spin-2 fields a possible relation with the cosmological constant. We have added an appendix containing propertiesof thespin-2 field description in terms of Fierz variables.

2. Some mathematical machinery

We consider a four dimensional spacetime endowed with a Minkowskian metric tensor $\eta_{\mu\nu}$ and a non-symmetric affine connection $\Gamma^{\alpha}{}_{\mu\nu}$ that defines a covariant derivative. This structure is usually called Cartan space. For an arbitrary vector W^{μ}:

$$W^{\mu}{}_{;\nu} \equiv W^{\mu}{}_{,\nu} + \Gamma^{\mu}_{\nu\alpha} W^{\alpha}. \tag{3}$$

The torsion tensor $\tau^{\alpha}_{\mu\nu}$ is given by

$$\tau^{\alpha}_{\mu\nu} \equiv \frac{1}{2} \left(\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu} \right). \tag{4}$$

It is worthwhile to decompose torsion tensor into its irreducible components by setting:

$$\tau^{\alpha}_{\mu\nu} = L^{\alpha}_{\mu\nu} + \frac{1}{3} \left(\delta^{\alpha}_{\mu} \, \tau_{\nu} - \delta^{\alpha}_{\nu} \, \tau_{\mu} \right) - \frac{1}{3} \, \eta_{\mu\nu}{}^{\alpha\lambda} \, \tau^{*}_{\lambda} \, . \tag{5}$$

Note that $\eta^{\alpha}{}_{\mu\nu\lambda}$ is the completely antisymmetric Levi-Civita symbol. The dual of any antisymmetric tensor $W_{\mu\nu}$ is defined by $W_{\overset{*}{\mu\nu}} \equiv \frac{1}{2}\eta^{\alpha\beta}{}_{\mu\nu}W_{\alpha\beta}$. The dual of the Levi-Civita object allows the introduction of the tensor

$$g_{\alpha\beta\mu\nu} \equiv \eta_{\alpha\mu}\,\eta_{\beta\nu} - \eta_{\alpha\nu}\,\eta_{\beta\mu} \tag{6}$$

since we have the equality

$$g_{\alpha\beta\mu\nu} = -\,\overset{*}{\eta}_{\alpha\beta\mu\nu}. \tag{7}$$

The quantity $\tau_\mu = \tau^\alpha{}_{\alpha\mu}$ is the trace while $\overset{*}{\tau}_\mu = \tau^{\alpha *}{}_{\alpha\mu}$ is the pseudo-trace. Note that the condition of metricity is satisfied, that is

$$g_{\mu\nu;\alpha} = 0. \tag{8}$$

3. Restricted Cartan geometry (RCG)

In this section we deal with the simple case of a pseudotraceless Cartan geometry (PTCG), thatis, we set:

$$\overset{*}{\tau}_\alpha = 0. \tag{9}$$

This condition decreases the degrees of freedom of the torsion from 24 to 20.

It is useful to represent torsion by an associated quantity $F_{\mu\nu}{}^\alpha$ defined by the combination

$$\tau^\alpha{}_{\mu\nu} - g^{\alpha\epsilon}{}_{\mu\nu}\,\tau_\epsilon = F_{\mu\nu}{}^\alpha. \tag{10}$$

It follows that this tensor $F_{\mu\nu\alpha}$ is anti-symmetric in the first two indices:

$$F_{\mu\nu\alpha} = -\,F_{\mu\nu\alpha}. \tag{11}$$

Besides, since the torsion tensor has no pseudo-trace it follows the additional symmetry

$$\overset{*}{F}{}^\alpha{}_{\beta\alpha} = 0 \tag{12}$$

or, equivalently,

$$F_{\mu\nu\alpha} + F_{\nu\alpha\mu} + F_{\alpha\mu\nu} = 0. \tag{13}$$

Thus, $F_{\mu\nu\alpha}$ has 20 degrees of freedom, like the torsion without the pseudo-trace. Following Fierz [3], a very appealing representation can be given by writing $F_{\alpha\mu\nu}$ in terms of two symmetric second-order tensors $A_{\mu\nu}$ and $B_{\mu\nu}$. In this session we limit our analysis to the case in which one of these tensors, say $B_{\mu\nu}$, vanishes. We postpone the exam of the full case for the next session. We set

$$2\,F_{\alpha\beta\mu} = A_{\mu[\alpha,\beta]} + (A_{,\alpha} - A_{\alpha}{}^{\epsilon}{}_{,\epsilon})\;\eta_{\beta\mu} - (A_{,\beta} - A_{\beta}{}^{\epsilon}{}_{,\epsilon})\;\eta_{\alpha\mu}, \tag{14}$$

where $A = A_{\mu\nu}\,\eta^{\mu\nu}$. The trace F_{α} is defined by

$$F_{\alpha} = F_{\alpha\mu\nu}\eta^{\mu\nu} \tag{15}$$

that is,

$$F_{\alpha} = A_{,\alpha} - A_{\alpha}{}^{\epsilon}{}_{,\epsilon}. \tag{16}$$

Then one can equivalently write equation (14) in the concise form

$$2\,F_{\alpha\beta\mu} = A_{\mu[\alpha,\beta]} + F_{[\alpha}\eta_{\beta]\mu}, \tag{17}$$

where we are using the anti-symmetrization symbol [] like

$$[x, y] \equiv xy - yx. \tag{18}$$

We use an analogous form for the symmetrization symbol ().

$$(x, y) \equiv xy + yx. \tag{19}$$

In order to analyze the curvature properties of this RCG let us remind that the important quantity is not torsion itself but the so called contortion, $K_{\alpha\beta\mu}$, defined by

$$\Gamma^{\epsilon}_{\mu\nu} = \{^{\epsilon}_{\mu\nu}\} + K^{\epsilon}{}_{\mu\nu}\,. \tag{20}$$

Once the metric tensor is Minkowskian, the Christoffel symbols can be vanished by a convenient choice of the coordinate system. Using equation (10) the contortionis written as

$$K^{\epsilon}{}_{\mu\nu} = 2F^{\epsilon}{}_{\nu\mu} + F_{\nu}\,\delta^{\epsilon}_{\mu} - F^{\epsilon}\,\eta_{\mu\nu} \tag{21}$$

or, in terms of the field $A_{\mu\nu}$:

$$K_{\epsilon\mu\nu} = A_{\epsilon\mu,\nu} - A_{\mu\nu,\epsilon}, \tag{22}$$

from which follows the antisymmetry $K_{\varepsilon\mu\nu} + K_{\nu\varepsilon\mu} = 0$.

A. Curvature tensor in RCG

The curvature in an affine geometry is defined by

$$R^{\alpha}{}_{\sigma\beta\lambda} = \Gamma^{\alpha}_{\beta\sigma,\lambda} - \Gamma^{\alpha}_{\lambda\sigma,\beta} + \Gamma^{\alpha}_{\lambda\rho}\,\Gamma^{\rho}_{\beta\sigma} - \Gamma^{\rho}_{\lambda\sigma}\,\Gamma^{\alpha}_{\beta\rho}. \tag{23}$$

Since the metric tensor is Minkowskian this tensor contains only the contributions of the torsion. The contracted curvature tensor is then given by

$$R_{\mu\nu} = K^{\alpha}{}_{\alpha\mu,\nu} - K^{\alpha}{}_{\nu\mu,\alpha} + K^{\alpha}{}_{\nu\rho}\,K^{\rho}{}_{\alpha\mu} - K^{\alpha}{}_{\alpha\rho}\,K^{\rho}{}_{\nu\mu}, \tag{24}$$

Using equation (22) this curvature tensor can be rewritten as

$$R_{\mu\nu} = \Box A_{\mu\nu} - A^{\alpha}{}_{(\mu,\nu)\alpha} + A_{,\mu\nu} + [KK]_{\mu\nu} \tag{25}$$

where

$$[KK]_{\mu\nu} \equiv K^{\alpha}{}_{\nu\rho}\,K^{\rho}{}_{\alpha\mu} - K^{\alpha}{}_{\alpha\rho}\,K^{\rho}{}_{\nu\mu} \tag{26}$$

and $\Box \equiv \eta^{\mu\nu}\,\partial_{\mu}\,\partial_{\nu}$. Then usingthe decompositionintermsof the field $A_{\mu\nu}$ as above this can be re-written as

$$\begin{aligned} [KK]_{\mu\nu} &\equiv (A_{\nu\alpha,\rho} - A_{\nu\rho,\alpha})\,(A^{\alpha\rho}{}_{,\mu} - A_{\mu}{}^{\alpha,\rho}) \\ &- (A_{,\alpha} - A_{\alpha}{}^{\epsilon}{}_{,\epsilon})\,\eta^{\alpha\rho}\,(A_{\nu\rho,\mu} - A_{\nu\mu,\rho})\,. \end{aligned} \tag{27}$$

From this expression it follows immediately that the trace $[KK] \equiv [KK]_{\mu\nu}\,\eta^{\mu\nu}$, is

$$[KK] = -2U \tag{28}$$

where Y is the invariant

$$U \equiv F_{\alpha\beta\mu}\, F^{\alpha\beta\mu} - F_\alpha\, F^\alpha. \tag{29}$$

Then, for the curvature scalar in RCG we obtain

$$R = 2\,\Box A - 2\,A^{\alpha\beta}{}_{,\alpha\beta} - 2\,U. \tag{30}$$

or, using the equation (16)

$$R = 2\,\left(F^{\alpha}{}_{,\alpha} - U\right). \tag{31}$$

B. Dynamics in RCG

It seems natural to examine the dynamical torsion by choosing the curvature scalar for the Lagrangian function. From what we have shown above this yields,

$$S = \int R\, d^4x = -2 \int U\, d^4x \tag{32}$$

up to a total divergence.

This dynamics is precisely the standard one proposed by Fierz[4] to describeaspin-2 field and corresponds to the linear limit of Einstein General Relativity. Indeed, from the above Lagrangian[7] by varying $A_{\mu\nu}$ it follows:

$$\delta S = -\int 2F^{\alpha(\mu\nu)}{}_{,\alpha}\, \delta A_{\mu\nu}\, d^4x\ . \tag{33}$$

Since we have the identity

$$F^{\alpha\mu\nu}{}_{,\alpha} = \frac{1}{2}\, F^{\alpha(\mu\nu)}{}_{,\alpha} = -\,\frac{1}{2}\, G^{L}{}_{\mu\nu}, \tag{34}$$

the equation of motion can be re-written as

$$\delta S = 2 \int G^{L}{}_{\mu\nu}\, \delta A^{\mu\nu}\, d^4x, \tag{35}$$

where

$$G^L{}_{\mu\nu} \equiv \Box A_{\mu\nu} - A^{\epsilon}{}_{(\mu,\nu),\epsilon} + A_{,\mu\nu} - \eta_{\mu\nu} \left(\Box A - A^{\alpha\beta}{}_{,\alpha\beta}\right), \tag{36}$$

where the label L means the linear part of the Einstein equation for GR. This accomplishes the proof of the following two assertions:

- Restricted torsion (10 degrees of freedom) describes a symmetric second-order tensor $A_{\mu\nu}$;
- The dynamics generated by the curvature scalar (Hilbert-Einstein action) yields the standard linear equation for the spin-2 field $A_{\mu\nu}$.

The next step is to go beyond this property to include other degrees of freedom for torsion. We shall prove now that it will appear a second spin-2 field when we increase the above mentioned degrees of freedom of the torsion variables from 10 to 20.

4. Generalized Cartan Geometry

The generalization of the above formulation starts by modifying the representation of the torsion as in equation (10) by the following one:

$$\tau^{\alpha}{}_{\mu\nu} - g^{\alpha\epsilon}{}_{\mu\nu}\, \tau_{\epsilon} = F_{\mu\nu}{}^{\alpha} + \eta_{\mu\nu}{}^{\alpha\epsilon}\, \Sigma_{\epsilon}, \tag{37}$$

where the free variables Σ_{ε} are independent of $F_{\alpha\mu\nu}$ and are introduced in order to complete the total of 24 degrees of freedom needed for $\tau_{\alpha\mu\nu}$. Conditions on these variables can reduce from 24 to 20 the degrees of freedom of $\tau_{\alpha\mu\nu}$.

The trace and the pseudo-trace are given, respectively, by

$$\tau_{\alpha} = \frac{1}{2}\, F_{\alpha} \tag{38}$$

$$\tau^{*}_{\alpha} = -F^{*}_{\alpha\beta}{}^{\beta} - 3\,\Sigma_{\alpha}. \tag{39}$$

Since we are dealing with $F_{\varepsilon\nu\mu}$ that has 20 independent components, we use the Fierz decomposition in terms of two symmetric second-order tensors $A_{\mu\nu}$ and $B_{\mu\nu}$:

$$F_{\alpha\beta\mu} = A_{\alpha\beta\mu} + \frac{1}{2}\, \eta_{\alpha\beta}{}^{\rho\sigma}\, B_{\rho\sigma\mu}, \tag{40}$$

where

$$2\,A_{\alpha\beta\mu} \equiv A_{\mu[\alpha,\beta]} - g_{\alpha\beta\mu\epsilon}\,A^{\epsilon} \tag{41}$$

$$2\,B_{\alpha\beta\mu} \equiv B_{\mu[\alpha,\beta]} - g_{\alpha\beta\mu\epsilon}\,B^{\epsilon} \tag{42}$$

$$A_{\lambda} \equiv A_{,\lambda} - A_{\lambda}{}^{\mu}{}_{,\mu} \tag{43}$$

$$B_{\lambda} \equiv B_{,\lambda} - B_{\lambda}{}^{\mu}{}_{,\mu}, \tag{44}$$

with $A \equiv A_{\mu\nu}\eta^{\mu\nu}$ and $B \equiv B_{\mu\nu}\eta^{\mu\nu}$. We impose an additional condition for each field $A_{\alpha\beta\mu}$ and $B_{\alpha\beta\mu}$ in order to allow them to represent a single spin-2 field:

$$A^{*}_{\alpha(\mu\nu),\beta}\,\eta^{\alpha\beta} = 0, \tag{45}$$

and do the same for field $B_{\alpha\beta\mu}$. The trace and the pseudo-trace of the tensor $F_{\alpha\beta\mu}$ are, respectively,

$$F_{\alpha\beta}{}^{\beta} = A_{\alpha} \tag{46}$$

$$F^{*}_{\alpha\beta}{}^{\beta} = -\,B_{\alpha}. \tag{47}$$

Note that the pseudotrace F^{*}_{α} is not zero and the cyclic symmetry for $F_{\alpha\beta\mu}$ is not validinthis general case. Consequently, for the contortion, expression(21) becomes

$$K_{\epsilon\mu\nu} = F_{\epsilon\mu\nu} + F_{\epsilon\nu\mu} + F_{\mu\nu\epsilon} + g_{\epsilon\nu\mu\lambda}\,F^{\lambda} - \eta_{\epsilon\nu\mu\lambda}\,\Sigma^{\lambda}. \tag{48}$$

The curvature scalar is given by

$$R = K^{\alpha}{}_{\alpha\mu,\nu}\eta^{\mu\nu} - K^{\alpha}{}_{\mu\nu,\alpha}\,\eta^{\mu\nu} + [KK]. \tag{49}$$

Let us evaluate [KK]. From the above decomposition equation(48) we find:

$$K^{\alpha}{}_{\alpha\mu} = F_{\mu} \tag{50}$$

$$K_{\mu\alpha\beta}\,\eta^{\alpha\beta} = -\,F_{\mu} \tag{51}$$

$$K_{\alpha\mu\rho}\,K^{\rho\alpha\mu} \;=\; 2F_{\alpha\rho\mu}F^{\mu\alpha\rho} - F_{\mu\alpha\rho}F^{\mu\alpha\rho} + 2\,A^{\alpha}\,A_{\alpha} \qquad (52)$$
$$-4\,F^{*}_{\alpha}\Sigma^{\alpha} - 6\,\Sigma_{\alpha}\,\Sigma^{\alpha}.$$

Collecting all this yields

$$[KK] \;=\; -2A_{\alpha\beta\mu}\,A^{\alpha\beta\mu} + 2B_{\alpha\beta\mu}\,B^{\alpha\beta\mu} - 2B_{\alpha}\,B^{\alpha}$$
$$+\,2A_{\alpha}\,A^{\alpha} \;+\; 4B_{\alpha}\,\Sigma^{\alpha} - 6\Sigma_{\alpha}\,\Sigma^{\alpha} - 4A^{\alpha\beta\mu}\,B^{*}_{\alpha\beta\mu}. \qquad (53)$$

Using the condition (45) and the cyclic relation (valid for $A^{\alpha\beta\mu}$ $\beta^{\alpha\beta\mu}$ and $B^{\alpha\beta\mu}$)$A^{*\alpha\beta\mu}\,\eta_{\beta\mu}$= 0 the interaction term can be reduced to a pure divergence term by the transformation

$$A^{\alpha\beta\mu}\,B^{*}_{\alpha\beta\mu} = (A^{*\alpha\beta\mu}B_{\alpha\mu})_{,\beta} + \frac{1}{2}A^{*}_{\alpha(\mu\nu),\beta}\,\eta^{\alpha\beta}B^{\mu\nu}. \qquad (54)$$

At this point we restrict the degrees of freedom of the torsion in order to deal only with two spin-2 fields, that is, we impose a condition on Σ_{μ} by setting Σ_{α}= $^{2}\text{-}_{3}$ B_{α} yielding

$$[KK] \;=\; -2A_{\alpha\beta\mu}\,A^{\alpha\beta\mu} + 2B_{\alpha\beta\mu}\,B^{\alpha\beta\mu}$$
$$-\;\;2\,B_{\alpha}\,B^{\alpha} + 2\,A_{\alpha}\,A^{\alpha}. \qquad (55)$$

Then, for the curvature scalar we find

$$R = \text{ div } - 2U[A_{\alpha\beta}] + 2U[B_{\alpha\beta}]. \qquad (56)$$

where the functional Y is given by equation (29). The important point to stress here concerns the relative signs of $U[A_{\alpha\beta}]$ and $U[B_{\alpha\beta}]$, a property of Cartan geometry in the two spin-2 representation. We will come back to this later on.

Then the dynamics

$$S = \int R d^4x \qquad (57)$$

provides, for independent variation of the variables $A_{\mu\nu}$ and $B_{\mu\nu}$, equations of two independent non-interacting spin-2 fields:

$$G^{L}{}_{\mu\nu}(A) \;\equiv\; \Box\,A_{\mu\nu} - A^{\epsilon}{}_{(\mu,\nu)\,,\epsilon} + A_{,\mu\nu}$$
$$-\;\;\eta_{\mu\nu}\,\left(\Box A - A^{\alpha\beta}{}_{,\alpha\beta}\right) = 0. \qquad (58)$$

$$G^{L}{}_{\mu\nu}(B) \equiv \Box B_{\mu\nu} - B^{\epsilon}{}_{(\mu,\nu),\epsilon} + B_{,\mu\nu} - \eta_{\mu\nu}\left(\Box B - B^{\alpha\beta}{}_{,\alpha\beta}\right) = 0. \tag{59}$$

It is worth to say that the curvature R of the Cartan geometry, within the framework we are developing here, yields linear equations of motion for the associated spin2 fields. This property of linearity is typical for models in which the torsion is associated to fundamental fields. The association of spin-2 to the metric, as in GR, leads inevitably to a nonlinear theory.

5. Generalization to curved Riemannian backgrounfl

All the above analysis was carried out for the case in which the metric tensor has no Riemannian curvature.

This means that the metric tensor is identified with the Minkowskian geometry. The general case in which the Riemannian geometry is not flat is straightforward and causes no difficulty. The attentive reader could be concerned at this point with the known difficulty of coupling spin-2 fields with an arbitrary Riemannian geometry. The uses of the Fierz frame provide the answer to overcome such difficulty as shown recently [6]. In this case the total connection is given by

$$\Gamma^{\alpha}_{\mu\nu} = \{^{\alpha}_{\mu\nu}\} + K^{\alpha}_{\mu\nu} \tag{60}$$

The Christoffel symbol $\{^{\alpha}_{\mu\nu}\}$ is constructed in terms of the metric tensor $\gamma\nu$and its derivatives; $K^{\alpha}_{\mu\nu}$ is the contortion defined previously. The curvature tensor contains an additional term:

$$R_{\mu\nu} = \hat{R}_{\mu\nu} + K^{\alpha}{}_{\alpha\mu;\nu} - K^{\alpha}{}_{\nu\mu;\alpha} + K^{\alpha}{}_{\nu\rho}\, K^{\rho}{}_{\alpha\mu} - K^{\alpha}{}_{\alpha\rho}\, K^{\rho}{}_{\nu\mu}, \tag{61}$$

where $\hat{R}_{\mu\nu}$ is the contracted curvature tensor of the Riemannian sector of the curvature. The covariant derivative; must be taken in this sector. The complete dynamics in such general case is provided by the scalar R:

$$R = \text{ div } + \hat{R} - 2U[A_{\alpha\beta}] + 2U[B_{\alpha\beta}]. \tag{62}$$

We have postponed the complete analysis of this general case for other work.

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6. Conclusion and some comments

We have shown that there are two non-equivalent ways to incorporate spin-2 fields into the geometric structure of spacetime:

- The common metric tensor implementation;
- Torsion terms.

The first is the standard one proposed by Einstein in his General Relativity. It consists in making a change on Minkowski background defining an associated metric tensor by the exact expression

$$g_{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}.$$

In order to obtain from the above expression the corresponding inverse $g^{\mu\nu}$ defined by

$$g^{\mu\nu}\, g_{\nu\alpha} = \delta^{\mu}_{\alpha}$$

one is led automatically to an infinite series

$$g^{\mu\nu} = \eta^{\mu\nu} - \varphi^{\mu\nu} + \varphi_{\mu\alpha}\, \varphi^{\alpha\nu} + \cdots$$

These definitions lead to the inevitability of a nonlinear approach of the Einstein proposal[8].

The second possibility consists in the relation of symmetric second-order tensors to torsion and is described in the present paper. We have examined here the case of torsion that has 20 degrees of freedom in order to deal with the exact number to represent two symmetric second-order tensors. We divide this exam into two situations: First, we have eliminated pseudotorsion and one of these symmetric second-order tensors, dealing only with 10 degrees of freedom. Second we have dealt with 20 degrees of freedom by eliminating a certain combination of trace and pseudo-trace. In order to deal with the complete case of 24 quantities one hasto consider in equation (37) the vector Σ_{μ} as an independent quantity. Let us remark that one can deal with alternative representations, e.g.,

$$\tau^{\alpha}{}_{\mu\nu} = F_{\mu\nu}{}^{\alpha} + \lambda\, g^{\alpha\epsilon}{}_{\mu\nu}\, M^{\epsilon} + \sigma\, \eta^{\alpha\epsilon}{}_{\mu\nu}\, M_{\epsilon}$$

in which λ and σ are constants and M is independent of $F_{\alpha\beta\mu}$. We will come back to this extra field elsewhere. The main resultrelates the dynamics of torsion, obtained from the scalar of curvature R to the equation of two non-interacting spin-2 fields. Two main results then come out: the equation of motion for each one of the spin-2 fields is linear and the fields' energy distribution has opposite signs. This allows us to recognize the possibility of the existence of a situation in which the energies of both fields cancel out each other generating a sortof *torsion vacuum state* (TVS). Then, two questions naturally appear:

- Is such TVS stable?
- How to conciliate the presence of these two spin-2 tensors with a non-Minkowskian metric tensor?

These two questions are possibly correlated. Indeed, the compatibility of the description of spin-2 field in a Riemannian curved background requires that the Riemannian sector must be an Einstein space, that is the metric tensoris such that $R_{\mu\nu} = \Lambda g_{\mu\nu}$. Should this cosmological constant be related to the existence of these two spin-2 tensors hidden in the torsion tensor when it is no more in its TVS? We intend to analyze these questions elsewhere.

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[7] See the appendix for further properties of this description of a spin-2 field in terms of the three-index tensor $F^{\alpha\beta\mu}$.

[8] Note that the Einstein path starts from an a priori geometrical description. The best way to understand the equivalence between the field theoretical description and its corresponding geometric formulation is to follow the work of Gupta, Deser, Feynman and others. See [5] for details and further references.

7. Appendix

A. Some properties of $F_{\alpha\beta\mu}$

Following Fierz (op.cit.) the important object to deal with is a spin-2 field represented by $F_{\alpha\beta\mu}$ that satisfy the following properties

$$F_{\alpha\beta\mu} + F_{\beta\alpha\mu} = 0 \tag{63}$$

$$F^{*}_{\alpha\mu\beta}\,\eta^{\alpha\beta} = 0 \tag{64}$$

or, equivalently

$$F_{\alpha\mu\nu} + F_{\mu\nu\alpha} + F_{\nu\alpha\mu} = 0. \tag{65}$$

Such an object has 20 degrees of freedom. To reduce these variables to just 10 to represent a single spin-2 field one imposes the additional requirement

$$F^{*}_{\alpha(\mu\nu),\beta}\,\eta^{\alpha\beta} = 0. \tag{66}$$

which can be rewritten as 1

$$F_{\alpha\beta}{}^{\lambda}{}_{,\mu} + F_{\beta\mu}{}^{\lambda}{}_{,\alpha} + F_{\mu\alpha}{}^{\lambda}{}_{,\beta} - \frac{1}{2}\delta^{\lambda}_{\alpha}(F_{\mu,\beta} - F_{\beta,\mu}) - \frac{1}{2}\delta^{\lambda}_{\mu}(F_{\beta,\alpha} - F_{\alpha,\beta}) - \frac{1}{2}\delta^{\lambda}_{\beta}(F_{\alpha,\mu} - F_{\mu,\alpha}) = 0. \tag{67}$$

As a direct consequence of the definitions it obeys

$$F^{\alpha\beta\mu}{}_{,\mu} = 0\ . \tag{68}$$

Then it follows, in complete analogy with thespin-1 case, that if $F_{\alpha\beta\mu}$ satisfies equation (66) then there is a symmetric second-order tensor $A_{\mu\nu}$ such that $F_{\alpha\beta\mu}$ can be written in the form

$$2F_{\alpha\beta\mu} = A_{\mu\alpha,\beta} - A_{\mu\beta,\alpha} + F_{\alpha}\gamma_{\beta\mu} - F_{\beta}\gamma_{\alpha\mu} \tag{69}$$

where $F_{\alpha} \equiv A_{,\alpha} - A_{\alpha}{}^{\beta}{}_{,\beta}$, and $F_{\alpha} \equiv F_{\alpha\lambda}{}^{\lambda}$. Note the following properties

$$
\begin{aligned}
F^{\alpha(\mu\nu)}{}_{,\alpha} &= [F^{\alpha\mu\nu} + F^{\alpha\nu\mu}]_{,\alpha} \\
= & \; [F^{\alpha\mu\nu} - F^{\nu\mu\alpha} - F^{\mu\alpha\nu}]_{,\alpha} = \\
& \; [F^{\alpha\mu\nu} - F^{\mu\alpha\nu}]_{,\alpha} - F^{\nu\mu\alpha}{}_{,\alpha} \\
= & \; [F^{\alpha\mu\nu} + F^{\alpha\mu\nu}]_{,\alpha} = 2F^{\alpha\mu\nu}{}_{,\alpha}.
\end{aligned} \tag{70}
$$

The collection of all these result syields precisely the fact that the linear Einstein operator $\overset{L}{G}_{\mu\nu}$ can be written as a divergence:

$$
F^{\alpha}{}_{(\mu\nu),\alpha} = -\overset{L}{G}_{\mu\nu} \tag{71}
$$

QED.

B. Dynamics

One can construct three invariants with $F_{\alpha\beta\mu}$:

$$
\begin{aligned}
I &\equiv F_{\alpha\beta\mu} F^{\alpha\beta\mu} \\
II &\equiv F_{\mu} F^{\mu} \\
III &\equiv F_{\alpha\beta\mu} \overset{*}{F}{}^{\alpha\beta\mu}.
\end{aligned} \tag{72}
$$

The standard Fierz dynamics for spin-2 is provided by the Lagrangian

$$
L = I - II + \lambda III, \tag{73}
$$

for an arbitrary value of λ. The reason of such an arbitrary value is linked to the fact that $\int III \, d^4x$ is a topological invariant. In fact,

$$
\begin{aligned}
III &= \int \overset{*}{F}_{\mu\alpha\beta} \, F^{\mu\alpha\beta} d^4V \\
&= \int F^{\alpha\beta\mu} \left[\eta_{\alpha\beta}{}^{\rho\sigma} \, A_{\mu\rho,\sigma} + F^{\rho} \, \eta_{\alpha\beta\rho\mu} \right] dV
\end{aligned} \tag{74}
$$

Since

$$
F^{\alpha\beta\mu} \, \eta_{\alpha\beta\rho\mu} = 0
$$

then

$$III = \int [F^{\alpha\beta\mu}\, \eta_{\alpha\beta}{}^{\rho\sigma}\, A_{\mu\rho}]_{,\sigma} - \int F^{\alpha\beta\mu,\sigma}\, \eta_{\alpha\beta\rho\sigma}\, A_{\mu\rho}. \tag{75}$$

According to equation (66) the second term vanishes and it remains:

$$III = \int \partial_\sigma (\eta^{\alpha\beta\rho\sigma} F_{\alpha\beta\mu} A^\mu{}_\rho) \ . \tag{76}$$

Or, in the other way around III can be rewritten as

$$III = \left(\overset{*}{F}_{\rho\sigma\beta}\ A^{\beta\rho} \right)_{,\epsilon} \eta^{\sigma\epsilon}. \tag{77}$$

Calling

$$K_\sigma \equiv -\ \overset{*}{F}_{\sigma\alpha\beta}\ A^{\alpha\beta} \tag{78}$$

it follows

$$III = K^\mu{}_{,\mu}. \tag{79}$$

Thus, as far as theory is linear the dynamics for a spin-2 field is given by

$$S = \int (I - II) dV \tag{80}$$

and varying $A_{\mu\nu}$ we have

$$\delta S = \frac{1}{2} \int \overset{(L)}{G}_{\mu\nu}\ \delta A^{\mu\nu}\, dV \ . \tag{81}$$

Let us then make a further remark concerning the gauge property of this dynamics. Such property is of crucial importance for the case of nonlinear theory. Although we do not deal with such a case in the present paper we mention it in this appendix only for completeness. $\overset{(L)}{G}_{\mu\nu}$ is invariant by the gauge transformation

$$A_{\mu\nu} \rightarrow \tilde{A}_{\mu\nu} = A_{\mu\nu} + \Lambda_{\mu,\nu} + \Lambda_{\nu,\mu} \tag{82}$$

However, the field $F_{\alpha\beta\mu}$ is not invariant in this map. Indeed, we have

$$\delta F_{\alpha\beta\mu} \equiv \tilde{F}_{\alpha\beta\mu} - F_{\alpha\beta\mu} = \frac{1}{2} X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda} \tag{83}$$

$$X_{\alpha\beta\mu}{}^{\lambda} \equiv (\Lambda_{\alpha,\beta} - \Lambda_{\beta,\alpha})\delta^{\lambda}_{\mu} + \tag{84}$$
$$[\Lambda^{\sigma}{}_{,\sigma}\delta^{\lambda}_{\alpha} - \Lambda_{\alpha}{}^{,\lambda}]\gamma_{\beta\mu} - [\Lambda^{\sigma}{}_{,\sigma}\delta^{\lambda}_{\beta} - \Lambda_{\beta}{}^{,\lambda}]\gamma_{\alpha\mu} \tag{85}$$

and then it follows that

$$2\delta F_{\alpha} = X_{\alpha}{}^{\lambda}{}_{,\lambda} \tag{86}$$

where

$$X_{\alpha}{}^{\lambda} \equiv X_{\alpha\beta}{}^{\beta\lambda}.$$

As a consequence of this transformation the invariants I e II change accordingly

$$I' = I + F^{\alpha\beta\mu} X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda} + (X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda})^2 \tag{87}$$

where

$$(X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda})^2 \equiv X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda}\, X^{\alpha\beta\mu\sigma}{}_{,\sigma}.$$

For the other invariant:

$$II' = II + F^{\alpha} X_{\alpha}{}^{\lambda}{}_{,\lambda} + \frac{1}{4}(X^{\lambda}_{\alpha,\lambda})^2 \tag{88}$$

We remark that $X_{\alpha\beta\mu}{}^{\lambda}$ is **not** cyclic in the variables ($\alpha\beta\mu$)but the quantity

$$X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda} \tag{89}$$

has such cyclic property:

$$X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda} + X_{\beta\mu\alpha}{}^{\lambda}{}_{,\lambda} + X_{\mu\alpha\beta}{}^{\lambda}{}_{,\lambda} = 0 \ . \tag{90}$$

Besides

$$X^{\alpha\beta\mu\lambda}{}_{,\lambda,\alpha} = 0 \tag{91}$$

$$X^{\alpha\beta\mu\lambda}{}_{,\lambda,\mu} = 0 \tag{92}$$

and then

$$X^{\alpha\lambda}{}_{,\alpha\lambda} = 0\cdot \tag{93}$$

Thus

$$\begin{aligned}\delta I = I' - I &= [A^{\mu\alpha,\beta} + F^{\alpha}\,\eta^{\mu\beta}]\,X^{\alpha\beta\mu\lambda}{}_{,\lambda} \\ &+ \frac{1}{4}(X^{\alpha\beta\mu\,\lambda}{}_{,\lambda})^2\end{aligned} \tag{94}$$

$$\delta I = A^{\mu\alpha,\beta} X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda} + F^{\alpha} X_{\alpha}{}^{\lambda}{}_{,\lambda} + \frac{1}{4}(X^{\alpha\beta\mu\lambda}{}_{,\lambda})^2 \tag{95}$$

and

$$\delta II = F_{\alpha} X^{\alpha\lambda}{}_{,\lambda} + \frac{1}{4}(X^{\alpha\beta\mu\lambda}{}_{,\lambda})^2. \tag{96}$$

Then

$$\delta(I - II) = A_{\mu\alpha,\beta} X^{\alpha\beta\mu\lambda}{}_{,\lambda} + \frac{1}{4}(X^{\alpha\beta\mu\lambda}{}_{,\lambda})^2 - \frac{1}{4}(X^{\alpha\lambda}{}_{,\lambda})^2, \tag{97}$$

where

$$\int A_{\mu\alpha,\beta} X^{\alpha\beta\mu\lambda}{}_{,\lambda} = \int div - 2\int A_{\mu\alpha} X^{\alpha\beta\mu\lambda}{}_{,\lambda,\beta} \tag{98}$$

and note that the second term does not contribute for the dynamics. We can write

$$\begin{aligned}X_{\alpha\beta\mu}{}^{\lambda}{}_{,\lambda} &= \Lambda_{\alpha,\beta,\mu} - \Lambda_{\beta,\alpha,\mu} + (\Lambda^{\varepsilon}{}_{,\varepsilon,\alpha} - \Box\Lambda_{\alpha})\gamma_{\beta\mu} \\ &- (\Lambda^{\varepsilon}{}_{,\varepsilon,\beta} - \Box\Lambda_{\beta})\gamma_{\alpha\mu}\end{aligned} \tag{99}$$

Since

$$X_{\alpha\beta}{}^{\beta\lambda}{}_{,\lambda} \equiv X_{\alpha}{}^{\lambda}{}_{,\lambda} = 2[-\Box\Lambda_{\alpha} + \Lambda^{\beta}{}_{,\beta,\alpha}] \tag{100}$$

it follows that

$$\left(X^{\alpha\beta\mu\lambda}{}_{,\lambda}\right)^2 = (\Lambda_{\alpha,\beta,\mu} - \Lambda_{\beta,\alpha,\mu})^2 + \frac{1}{2}\left(X^{\alpha\lambda}{}_{,\lambda}\right)^2. \qquad (101)$$

Then, up to total divergence, we have:

$$\begin{aligned}
&\int (\Lambda_{\alpha,\beta,\mu} - \Lambda_{\beta,\alpha,\mu})^2 = \\
&\int 2\Lambda_{\alpha,\beta,\mu}\Lambda^{\alpha,\beta,\mu} - 2\int \Lambda^{\alpha,\beta,\mu}\Lambda_{\beta,\alpha,\mu} = \\
&\int 2\Lambda_\alpha(\Box\Box\Lambda_\alpha) - 2\int \Lambda^\alpha\Box\Lambda^\beta{}_{,\beta,\alpha} = \\
&\int 2\Lambda_\alpha\Box[\Box\Lambda^\alpha - \Lambda^\beta{}_{,\beta,\alpha}] = \\
&-\int \Lambda^\alpha\Box X_\alpha{}^\lambda{}_{,\lambda}
\end{aligned}$$

and, on the other hand,

$$\frac{1}{2}\int X_\alpha{}^\lambda{}_{,\lambda}X^{\alpha\beta}{}_{,\beta} = \int X_\alpha{}^\lambda{}_{,\lambda}[-\Box\Lambda_\alpha + \Lambda^\beta{}_{,\beta,\alpha}] =$$

$$-\int \Box(X_\alpha{}^\lambda{}_{,\lambda})\Lambda_\alpha + \int X^{\alpha\lambda}{}_{,\lambda,\alpha,\beta}\,\Lambda^\beta$$

Note that the second term does not contribute as well. Then, finally,

$$\begin{aligned}
&\int \delta(I - II) = -\frac{1}{2}\int \Lambda^\alpha\Box X_\alpha{}^\lambda{}_{,\lambda} - \frac{1}{4}\int (X_{\alpha^\lambda,\lambda})^2 = \\
&= -\frac{1}{2}\int \Lambda^\alpha\Box X_\alpha{}^\lambda{}_{,\lambda} - \frac{1}{4}(-2)\int \Lambda^\alpha\Box X_\alpha{}^\lambda{}_{,\lambda} = 0
\end{aligned}$$

This shows that the transformation

$$F_{\alpha\beta\mu} \to F_{\alpha\beta\mu} + X_{\alpha\beta\mu}{}^\lambda{}_{,\lambda}$$

for $X^{\alpha\beta\mu\lambda}{}_{,\lambda}$ given above, leaves invariant the dynamics.

Quantum statistics: The indistinguishability principle and the permutation group theory

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Há cerca de duas décadas passadas [1-3] mostramos matematicamente para sistemas 3-dim que, além de Bósons e Férmions, poderia existir na natureza um outro tipo de partículas que denominamos de Gentileons. Os nossos resultados foram obtidos rigorosamente dentro do contexto da mecânica quântica não-relativística e da teoria do grupo de permutações. Entretanto, os nossos artigos são difíceis de serem entendidos por físicos que não estão familiarizados com a teoria do grupo de permutações. Assim, no presente artigo nós vamos mostrar em detalhes, passo a passo, como obter as nossas previsões. Procedemos desse modo para permitir que estudantes de graduação de física possam entender claramente nossos cálculos com um conhecimento básico de teoria de grupos.

About two decades ago we have shown mathematically for 3-dim systems that,[1-3] besides Bosons and Fermions, it could exist a third kind of particles in nature that was named Gentileons. Our results have been obtained rigorously within the framework of the non-relativistic quantum mechanics and the permutation group theory. However, these papers are somewhat intricate for physicists not familiarized with the permutation group theory. In the present paper we show in details, step by step, how to obtain our theoretical predictions. This was done in order to permit a clear understanding of our approach by graduate students with a basic knowledge of group theory.

1. Introduction

In preceding papers[1-8] we have performed a detailed analysis of the problem of the indistinguishability of N identical particles in quantum mechanics. It was shown rigorously for 3-dim non-relativistic systems, according to the postulates of quantum mechanics and the principle of the indistinguishability, that besides *Bosons* and *Fermions* it could mathematically exist another kind of particles that we have called *Gentileons*. This analysis was performed using the irreducible representations of the *permutation group* (*symmetry group)* S_N in the Hilbert space. However, our first papers on the subject,[1-3] that were taken as a point of departure to investigate the existence of a new kind of particles (gentileons) is somewhat intricate and complex from the mathematical point of view. We have used the group theory shown in the books of Weyl,[9] Hamermesh[10] and Rutherford.[11] These papers are somewhat intricate for physicists not familiarized with the *permutation group theory* and its representations in the Hilbert space. So, in the present paper we intend deduce our main results adopting a more simple and didactic mathematical approach. We will present our calculations in such way that graduate students with a basic knowledge of group theory would be able to understand our predictions.

In Section 2 is analyzed the problem of the indistinguishability of identical particles in quantum mechanics.

In Section 3 we see how to connect the permutation of the particles with the eigenfunctions of the energy operator H using the *Permutation Group.*

In Section 4 is shown in details the calculation of the energy eigenfunctions of a system with N = 3 particles.

In Section 5 are given the essential results for the general case of systems with N identical particles.

In Section 6 are presented the Summary and Conclusions.

2. The Indistinguishability of Identical Particles in Quantum Mechanics.

Identical particles cannot be distinguished by means of any inherent property, since otherwise they would not be identical in all respects. In classical mechanics, identical particles do not lose their "individuality", despite the identity of their physical properties: the particles at some instant can be "numbered" and we can follow the subsequent motion of each of these in its paths. So, at any instant the particles can be identified.

In quantum mechanics,[12-14] the situation is completely different since, due to the uncertainty relations, the concept of path of a particle ceases to have any meaning. Hence, by localizing and numbering the particles at some instant, we make no progress towards identifying them at subsequent instants: if we localize one of the particles, we cannot say which of the particles has arrived at this point. This is true, for instance, for electrons in a single atom, for neutrons in a single nucleus or for particles which interact with each other to an appreciable extent. However, electrons of different atoms or neutrons of different nucleus, to good approximation, are regarded as distinguishable because they are well separated from each other.

Thus, in quantum mechanics, there is in principle no possibility of separately following during the motion each one of the similar particles and thereby distinguishing them. That is, in quantum mechanics identical particles entirely lose their individuality, resulting in the complete indistinguishability of these particles. This fact is called **"Principle of Indistinguishability of Identical Particles"** and plays a fundamental role in the quantum-mechanics of identical particles.[12-14]

Let us consider an isolated system with total energy E composed by a constant number N of particles that is described by the non-relativistic quantum mechanics. If H is the Hamiltonian operator of the system, the energy eigenfunction Ψ, obeys the equation $H\Psi = E\,\Psi$. The operator H and Ψ are functions of $\mathbf{x}_1,s_1,\ldots,\mathbf{x}_N,s_N$, where $\mathbf{x}_j$ and s_j denote the position coordinate and the spin orientation, respectively of the j^{th} particle. We abbreviate the pair $(\mathbf{x}_j,s_j)$ by a single number *j* and call 1,2,...,N a particle configuration. The set of all configurations will be called the *configuration space* $\varepsilon^{(N)}$. So, we have simply H = H(1,2,…,N) and $\Psi = \Psi(1,2,\ldots N)$. These quantum states Ψ form a Hilbert space $L_2(\varepsilon^{(N)})$ of all square integrable functions[1-3] over $\varepsilon^{(N)}$.

Let us define by P_i the "permutation operator"(i = 1,2,..., N!) which generate all possible permutations of the N particles in the space $\varepsilon^{(N)}$. Since the particles are identical the physical properties of the system must be invariant by permutations. In next section we show how to use the *Permutation Group S_N* to describe the N-particles quantum system.

3. The Permutation Group and its Representations in the Configuration and Hilbert Spaces.

As seen above, P_i is the "permutation operator"(i = 1,2,...N!) which generate all possible permutations of the N particles in the space $\varepsilon^{(N)}$. The permutations P_i of the labels 1,2,..,N constitute a symmetry group[9-11,16-19] S_N of order n =N!.

Because of the identity of the particles, H and Ψ obtained by merely permuting the particles must be equivalent physically, that is, $[P_i,H] = 0$ and $= |P_i \Psi|^2 = |\Psi_i|^2 = |\Psi|^2$. This implies that the permutations are unitary transformations and that the energy E has a N! degenerate energy spectrum. We assume that all the functions $\{\Psi_i\}_{i=1,2..,n}$ are different and orthonormal. To each operator P_i of the group S_N we can associate, in a one-to-one correspondence, an unitary operator $U(P_i)$ in the $L_2(\varepsilon^{(N)})$.[14,15]

Now, let us put n = N! and indicate by $\{\Psi_k\}_{k=1,2..,n}$ the set of n- degenerate energy eigenfunctions, where $\Psi_k = U(P_k)\Psi$. It is evident that any linear combination of the functions Ψ_k is also a solution of the wave equation $H\Psi = E\Psi$. In addition, since $[U(P), H] = 0$ we see that $H\,U(P)\,\Psi_k = U(P)\,H\,\Psi_k = U(P)\,E\,\Psi_k = E\,U(P)\Psi_k$. This means that if Ψ_k is an eigenfunction of H, $U(P)\,\Psi_k$ is also an eigenfunction of H. Hence, it must be equal to linear combinations of the degenerate eigenvectors, which is [14,15]

$$U(P)\Psi_k = \Sigma_{\,j=1\ldots n}\, \Psi_k D_{jk}(P), \tag{2.1}$$

where the $D_{jk}(P)$ are complex coefficients which depend on the group element.

According to Eq.(2.1) the n degenerate eigenfunctions of H thus span an n-dimensional subspace of the state-vector space of the system, and the operations of the group transform any vector which lies entirely in this subspace into another vector lying entirely in the same subspace, i.e., the symmetry operations leave the subspace invariant.

Repeated application of the symmetry operations gives [14,15]

$$U(Q)U(P)\Psi_k = \Sigma_{j=1\ldots n} U(Q)\Psi_k D_{jk}(P) = \Sigma_{j=1\ldots n} \Sigma_{i=1\ldots n} \Psi_i D_{ik}(Q)D_{jk}(P), \quad (2.2)$$

and also

$$U(QP)\ \Psi_k = \Sigma_{i=1\ldots n} \Psi_i D_{ik}(QP)\ . \quad (2.3)$$

Since U(QP) = U(Q)U(P) the left-hand sides of the Eqs.(2.2) and (2.3) are identical. Hence, comparing the right-hand sides of these same equations we get the basic equation:

$$D_{ik}(QP) = \Sigma_{j=1\ldots n} D_{ij}(Q)D_{jk}(P). \quad (2.4)$$

So, the permutation group S_N named "symmetry group" of the system, defined in the configuration space $\varepsilon^{(N)}$, induces a group of unitary linear transformations U in the n-dimensional linear Hilbert space $L_2(\varepsilon^{(N)})$. We have shown (see Eqs.(2.1)-(2.3)) that the unitary operations defined by U can be translated into a matrix equations by introducing a complete set of basis vectors in the n-dimensional vector space of Ψ. This Hilbert space $L_2(\varepsilon^{(N)})$ is named "representation space". The set of n x n square matrices D form a group of *dimension* (*degree*) n equal to the order of S_N. The complete set of matrices D are said to form a "n-dimensional unitary representation of S_N".

The eigenfunctions $\{\Psi_i\}_{i=1,2..,n}$ are all different and orthonormal since they are solutions of the same Schrödinger equation. These functions can be used[11-19] with the Young Shapes, to determine the *irreducible representations* of the group S_N in the configuration space $\varepsilon^{(N)}$ and the Hilbert space $L_2(\varepsilon^{(N)})$. To do this the *basis functions of the irreducible representations* using the Young Shapes are constructed taking $\{\Psi_i\}_{i=1,2..,n}$ as an orthogonal unit basis. It is important to note that choosing this particular basis functions we are simultaneously determining the irreducible representations of S_N and eigenfunctions of the operator H which is given by linear combinations and permutations of the $\{\Psi_i\}_{i=1,2..,n}$ .This method will be used in the Appendix I to determine the irreducible representations and the energy eigenfunctions for the trivial case of N =2 and for the simplest non-trivial case of N = 3.

In Section 4 using the method presented in Appendix I will be constructed the energy eigenfunctions of a system with N = 3 particles.

4. Systems with N = 3 Particles.

We will assume that a typical eigenfunction of energy E of the particles is written as $\Psi = \Psi(1,2,3) = u(1)v(2)w(3)$, where the single-particle functions (u,v,w) in the product are all different and orthogonal. According to our analysis in the Appendix I the 6-dimensional Hilbert space $L_2(\varepsilon^{(3)})$ spanned by the orthonormal unit vector basis (u,v,w) is composed by two 1-dim subspaces, h([3]) and h([1^3]), and one 4-dim subspace h([2,1]).

First let us consider the two *1-dim subspaces* in the Hilbert space which are represented by following eigenfunctions ϕ_s and ϕ_a:

$$\phi_s = [u(1)v(2)w(3) + u(1)v(3)w(2) + u(2)v(1)w(3) + u(2)v(3)w(1)$$

$$+ u(3)v(1)w(3) + u(3)v(2)w(1)]/\sqrt{6}, \quad (2.5)$$

which is *completely symmetric*, associated to the *horizontal Young shape* [3],

$$(2)\ \phi_a = [u(1)v(2)w(3) - u(1)v(3)w(2) - u(2)v(1)w(3) + u(2)v(3)w(1)$$

$$+ u(3)v(1)w(2) - u(3)v(2)w(1)]/\sqrt{6}, \quad (2.6)$$

completely antisymmetric, associated to the *vertical Young shape* [1^3].

The 4-dim subspace h([2,1]), associated to the *intermediate Young shape* [2,1] is represented by the state function Y([2,1]). This subspace h([2,1]) breaks up into two 2-dim subspaces, h_+([2,1]) and h_-([2,1]), that are spanned by the basis vectors $\{Y_1,Y_2\}$, $\{Y_3,Y_4\}$ and represented by the wavefunctions Y_+([2,1]) and Y_+([2,1]), respectively. The state functions Y([2,1]), Y_+([2,1]) and Y_+([2,1]) are given respectively, by:

$$Y([2,1]) = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}, \qquad Y_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \quad \text{and} \quad Y_- = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}, \quad (2.7)$$

where

$$Y_1 = [u(1)v(2)w(3) + u(2)v(1)w(3) - u(2)v(3)w(1) - u(3)v(2)w(1)]\ /\sqrt{4},$$

$$Y_2 = [u(1)v(2)w(3) + 2u(1)v(3)w(2) - u(2)v(1)w(3) + u(2)v(3)w(1) - 2u(3)v(1)w(2) - u(3)v(2)w(1)] /\sqrt{12},$$

$$Y_3 = [- u(1)v(2)w(3) + 2u(1)v(3)w(2) - u(2)v(1)w(3) - u(2)v(3)w(1) + 2u(3)v(1)w(2) - u(3)v(2)w(1)] /\sqrt{12}$$

and

$$Y_4 = [u(1)v(2)w(3) - u(2)v(1)w(3) - u(2)v(3)w(1) + u(3)v(2)w(1)] /\sqrt{4}.$$

As shown in the Appendix I, the functions $Y_+([2,1])$ and $Y_+([2,1])$ have equal symmetry permutation properties, that is, $P_i\ Y_\pm = D^{(2)(}P_i)\ Y_\pm$ where the (2x2) matrices $D^{(2)}(P_i)$ are unitary irreducible representations of the S_3 in the $\varepsilon^{(3)}$ and in the $L_2(\varepsilon^{(3)})$. The vectors Y associated with the 4-dim space h([2,1]) or with the two 2-dim subspaces, $h_+([2,1])$ and $h_-([2,1])$, will be indicated in what follows simply by Y([2,1]).

As well known,[12-15] the totally symmetric function ϕ_s defined by Eq.(2.5) describes the *Bosons* and the completely anti-symmetric function ϕ_a given by Eq.(2.8) describes the *Fermions*. When two Fermions occupy the same state we verify $\phi_a = 0$ which implies that two fermions are forbidden to occupy the same state. This kind of restriction does not exist for Bosons since $\phi_a \neq 0$ when three Bosons occupy the same state.

We see from Eq.(2.7) that $Y_\pm \neq 0$ when 1 or 2 particles occupy the same state, however $Y_\pm = 0$ when 3 particles occupy the same state.

From these results we see that the functions Y([2,1]) must represent particles which are different from *Bosons* or *Fermions*. These new kind of particles was called *Gentileons.* [3] This name was adopted in honor to the Italian physicist G.Gentile Jr . About six decades ago [20-22] he invented, without any quantum-mechanical or another type of justification, a parastatistics within a thermodynamical context. He obtained a statistical distribution function for a system of N weakly interacting particles assuming that the quantum states of an individual particle can be occupied by an arbitrary finite number *d* of particles. The Fermi and the Bose statistics are particular cases of this parastatistics for *d* = 1 and *d* = ∞ , respectively. A recent detailed analysis of the *d*-dimensional ideal gas parastatistics was performed by Vieira and Tsallis.[23]

Our analysis which gives support, within the framework of quantum mechanics and group theory, to the mathematical existence of news states Y([2,1])

associated with the intermediate Young shape [2,1], justifies, in a certain sense, the Gentile's hypothesis.

5. Systems Composed by N Identical Particles. The Statistical Principle.

In the Appendix I and in Section 3 we have studied in details the cases of systems composed by two and three particles. We have shown how to obtain the irreducible representations of the S_2 and S_3 in the configuration spaces $\varepsilon^{(2)}$ and $\varepsilon^{(3)}$ and in the Hilbert spaces $L_2(\varepsilon^{(2)})$ and $L_2(\varepsilon^{(3)})$. We have also constructed for these cases the eigenfunctions of the Hamiltonian operator.

In this Section we will give only the main results for the N-particles systems that have studied in details in preceding papers.[1-3]

We have shown[1-3] that the dimensions $f(\alpha)$ of the irreducible $f(\alpha) \times f(\alpha)$ square matrices assume the values $1^2, 2^2, \ldots, (N-1)^2$ and to each irreducible representation (α) is associated a subspace $h(\alpha)$ in the Hilbert space $L_2(\varepsilon^{(N)})$ with dimension $f(\alpha)$.

There are only two 1-dimensional ($f(\alpha) = 1$) irreducible representations given by the partitions $(\alpha) = [N]$ and $(\alpha) = [1^N]$. The first case is described by *horizontal shape* with N spaces. In the second case we have a *vertical shape* with N rows. The wavefunctions associated to them are, respectively:

$$\varphi_s = \frac{1}{\sqrt{N!}} \sum_{i=1}^{n} \Psi_i$$

and (2.8)

$$\varphi_a = \frac{1}{\sqrt{N!}} \sum_{i=1}^{n} \delta_{Pi} \Psi_i$$

where $\delta_{Pi} = \pm 1$, if P_i is even or odd permutation.

The remaining representations have dimensions $f(\alpha)$ going from 2^2 up to $(N-1)^2$ and are described by the various *intermediate shapes*.[9-11,17-19] To each shape (α) there is an irreducible representation described by $f(\alpha) \times f(\alpha)$ square matrices $D_{ik}^{(\alpha)}$ with dimension $f(\alpha)$. The *tableaux* with the same *shape* (α) have equivalent representations and the different *shapes* cannot have equivalent representations. There is a one-to-one correspondence between each shape (α) and the irreducible matrices $D_{ik}^{(\alpha)}$.

To each *shape* (α) is associated a sub space $h(\alpha) \in L_2(\varepsilon^{(N)})$ with dimension

$\tau = f(\alpha)$ spanned by the unit basis $\{Y_i\}_{i=1,2,\ldots,\tau}$. In this subspace $h(\alpha)$ the energy eigenfunction $Y(\alpha)$ is given by

$$Y(\alpha) = \frac{1}{\sqrt{\tau}}\begin{pmatrix} Y_1(\alpha) \\ Y_2(\alpha) \\ \vdots \\ Y_\tau(\alpha) \end{pmatrix} \tag{2.9}$$

where the functions $\{Y_i\}_{i=1,2,\ldots,\tau}$, that are construct applying the Young operators to the functions $\{\Psi_i\}_{i=1,2..,n}$, obey the condition $< Y_i \mid Y_n > = \delta_{in}$.

Under the permutations $Y(\alpha) \in h(\alpha)$ is changed into $X(\alpha) \in h(\alpha)$ given by $X(\alpha) = U(P_i)\, Y(\alpha)$, where $U(P_i)$ is an unitary operator. This permutation operator can also be represented by an unitary matrix $T(\alpha)$**:**$X(\alpha) = T(\alpha)\, Y(\alpha)$. Since the subspaces $h(\alpha)$ are equivalence classes[9-11,19] different subspaces have different symmetry properties which are defined by the matrix $T(\alpha)$. This means that if $T(\alpha) \in h(\alpha)$ and $T(\beta) \in h(\beta)$, results $T(\alpha) \neq T(\beta)$ if $\alpha \neq \beta$.

Since $T(\alpha)^+T(\alpha) = 1$ the square modulus of $Y(\alpha)$ is permutation invariant, that is,$|Y|^2 = Y(\alpha)^+Y(\alpha) = X(\alpha)^+X(\alpha) = |X|^2$. So, the function $|\Phi(\alpha)|^2 = Y(\alpha)^+Y(\alpha) = \Sigma_i\, |Y_i|^2$ can be interpreted as the probability density function.

We note that for the 1-dim cases the symmetry properties of the state function $Y(\alpha)$ are very simple because $T = \pm 1$, whereas for the multi-dimensional $h(\alpha)$ the symmetry properties are not so evident because they are defined by a matrix $T(\alpha)$ which has τ^2 components. Moreover, the occupation number of the states by particles is not fermionic or bosonic.

To obtain the energy eigenfunction our basic hypothesis was that $[U(P_i), H] = 0$. Consequently, $[U(P_i), S(t)] = 0$, where $S(t)$ is the time evolution operator for the system.

The expectation values of an arbitrary Hermitean operator $A = A(1,2,\ldots, N)$ for the energy state-vectors $Y(\alpha)$ and $X(\alpha)$ are defined by $< A_y > = < Y(\alpha)|A\, |Y(\alpha) > = (1/\tau)\Sigma_i < Y_i(\alpha)|A|Y_i(\alpha) >$ and $< A_x > = < X(\alpha)|A\, |X(\alpha) > = (1/\tau)\Sigma_i < X_i(\alpha)|A|X_i(\alpha) >$, respectively. Since $X(\alpha) = T(\alpha)\, Y(\alpha)$ we see that $< A_x > = < X(\alpha) |A\, |X(\alpha) > = < Y(\alpha) \mid T(\alpha)^+ A\, T(\alpha)\, |Y(\alpha) > = < Y(\alpha)\, |A\, |Y(\alpha) > = < A_y >$, implying that $[U(P_i), A(t)] = 0$. Moreover, if $U(P_i)$ commutes with $S(t)$ the relation $[U(P_i), A(t)] = [U(P_i), S^+(t)A(t)S(t)] = 0$ is satisfied. This means that $< A_y(t) > = < A_x(t) >$ at any instant of the time. This expresses the fact that since the particles are identical, any permutations of them does not lead to any observable effect. This conclusion is in agreement with the postulate of indistinguishability.[12-15]

The occupation number of the states and the symmetries properties of the quantum energy eigenstates Y(α) associated with the intermediate Young shapes are completely different from the vertical (fermionic) and horizontal (bosonic) shapes. This lead us to propose the following statement which is taken as a principle (*Statistical Principle):"Bosons, fermions and gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively".*

6. Summary and Conclusions.

We have shown that for non-relativistic 3-dim systems besides Bosons and Fermions it can exist mathematically a new kind of particles, named Gentileons. Our theoretical analysis was done didactically using the basic group theory adopted in the graduate physics course.

Using the permutation group theory we studied in details the trivial case of systems formed by 2 particles and the simplest but non-trivial case of systems formed by 3 particles. For the general N-particles systems only a brief review of the main results obtained preceding papers.[1-8] have been presented.

According to the amazing mathematical properties[2-8] of the intermediate representations of the permutation group theory the gentilionic systems cannot coalesce, gentileons are always confined in these systems and cannot appear as a free particle.

Based on these exotic properties we have conjectured[2-8] that quarks could be gentileons since we could explain, from first principles, quark confinement and conservation of the baryonic number.

Let us suppose that only Bosons and Fermions could exist in nature. In this case there remains the problem to discover the selection rules which forbid the existence of gentileons.

Finally, we must note that besides the "gentileons" there are other particles for 3-dim systems that do not obey the Fermionic or Bosonic statistics, predicted by different theoretical approaches. We would like to mention first the "parabosons" and "parafermions" predicted by Green.[24, 3] A detailed analysis of the Green Parastatistics can be seen, for instance, in the book of Ohnuki and Kamefuchi.[25] For 2-dim systems are predicted the "anyons" 26 and for 1-dim systems, the "exclusons".27 The "anyons" and "exclusons" are quasiparticles that cannot be found asymptotically free in the nature, like the gentileons. Recently, according to Camino et al.[28] the existence of anyons ("Laughlin particle" with fractionary charge) has been confirmed in the context of the fractional quantum Hall effect. More information about this experimental confirmation are given by Lindley.[29]

APPENDIX I - Representations of the S_N Group in the Configuration Space $\varepsilon^{(N)}$ and in the Hilbert Space $L_2(\varepsilon^{(N)})$.

We give here the basic ideas[16] concerning the representations of the S_N group in the configuration space $\varepsilon^{(N)}$. More detailed and complete analysis about this subject can be found in many books.[9-11, 17-19]

If we can set up a homomorphic mapping

$$P_i : D^{(\mu)}(P_i) \tag{I.1}$$

between the elements $P_1, P_2, \ldots, P_n$ of the group S_N and a set of square (μ x μ) matrices $D^{(\mu)}(P_1), D^{(\mu)}(P_2), \ldots, D^{(\mu)}(P_n)$ ($n = N!$) such that

$$D^{(\mu)}(P_i)D^{(\mu)}(P_j) = D^{(\mu)}(P_i\, P_j), \tag{I.2}$$

then the matrices $D^{(\mu)}(P_1), D^{(\mu)}(P_2), \ldots, D^{(\mu)}(P_n)$ are said to be a μ-dimensional matrix representation of the group S_N in the configuration space $\varepsilon^{(N)}$. If the homomorphic mapping of S_N on $D(P_i)$ reduces to an isomorphism the representation is said to be *faithful*.

In general all matrices $D^{(\mu)}(P_i)$ of a μ-dimensional representation can be brought simultaneously to the form

$$D^{(\mu)}(P_i) = \begin{pmatrix} D^{(k)}(P_i) & A(P_i) \\ 0 & D^{(m)}(P_i) \end{pmatrix} \tag{I.3}$$

where $D^{(k)}(P_i)$ and $D^{(m)}(P_i)$ are diagonal blocks with $k + m = \mu$. When, by a similarity transformation, all matrices $D^{(\mu)}(P_i)$ can be put in a diagonal form, that is, when $A(P_i) = 0$, the representation is named *reducible* . If the matrices cannot be written in a diagonal block structure the representation is said to be *irreducible*.

Let us consider, for instance, the simplest but non trivial case of the permutation group S_3 and define $P_1 = I$ = identity = (123), P_2 = (213), P_3 = (132), P_4 = (321), P_5 = (312) and P_6 = (231). We can show[16] that the S_3 has two 1-dimensional irreducible representations ($D_1^{(1)}$ and $D_2^{(1)}$) and only one 2-dimensional irreducible representation ($D^{(2)}(P_i)$).

For the two 1-dimensional representations the matrices $D^{(1)}(P_i)$ are given by:

$$D_1^{(1)}(P_i) = 1 \text{ (i = 1, 2 ,…, 6);} \tag{I.4}$$

$$\begin{aligned} &D_2^{(1)}(P_i) = 1 \text{ (i = 1, 5 and 6)} \\ \text{and} \quad & \\ &D_2^{(1)}(P_i) = -1 \text{ (i = 2, 3 and 4),} \end{aligned} \tag{I.5}$$

which are homomorphic representations.

For the 2-dimensional representation the matrices $D^{(2)}(P_i)$ are given by:

$$D^{(2)}(P_1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad D^{(2)}(P_2) = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}, \quad D^{(2)}(P_3) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$D^{(2)}(P_4) = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}, \; D^{(2)}(P_5) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \text{ and } D^{(2)}(P_6) = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}, \tag{I.6}$$

which is a faithful representation. Since the matrices shown in Eq.(6) are all orthogonal this irreducible representation is called *orthogonal*.

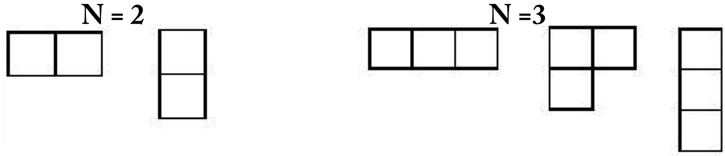
There are an infinite number of representations of a given group. We have obtained above the irreducible representations of the S_3 using the multiplication properties of the permutations P_i. Other two irreducible representations of S_3 can be obtained, for instance, taking into account (1)rotations of vectors in a 3-dim Euclidean space and (2)rotations of an equilateral triangle in the (x,y) plane.[18]

Determination of the SN representations by the Young Shapes

In the general case the determination of the S_N representations is performed by using more powerful methods developed by Young and Frobenius.[9-11,17-19] They consider the *substitutional expression* $\Pi = a_1 P_1 + a_2 P_2 + \ldots + a_n P_n$, where $P_1, P_2, \ldots, P_n$ are the n distinct permutations of the S_N and $a_1, a_2, \ldots, a_n$ are numerical coefficients, and take into account the partitions of number N. Any partition of the number N denoted by $[\alpha_1, \alpha_2, \ldots, \alpha_k]$, where $\alpha_1 + \alpha_2 + \ldots + \alpha_k = N$, with $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_k$ will be represented simply by (α), when no confusion is likely to arise. The first works[17] using this approach have been done, around 1900, independently by Frobenius and by Young, that was a country clergyman. To each partition (α) of N is constructed a *shape*, named *Young Shape*, denoted by α, having α_1

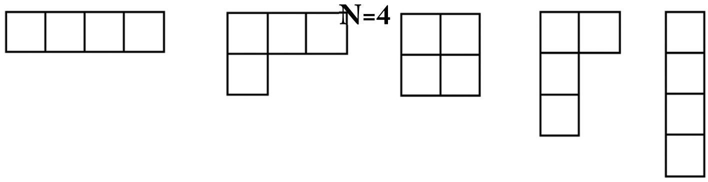
spaces in the first row, α_2 in the second row and so on.[9-11,17-19] By the *shape* we mean the empty box, i.e., the contour without the numbers. We show below all possible *shapes* associated with N =2, 3 and 4 particles.

N = 2 **N =3**



Shapes [2] $[1^2]$ [3] [2,1] $[1^3]$

N=4



Shapes [4] [3,1] [2,2] [2,3] $[1^4]$

The N numbers 1, 2 ,…, N are arranged in the spaces of the shape α in N! = n ways. Each such arrangement is called a *tableau* T and there are N! *tableaux* with the same shape. The tableau T, for a given shape, is called *standard tableau* if the numbers increase in every row of T from left to right and in every column of T downwards.

The tableaux are constructed as follows: insert the numbers 1, 2, 3,…, N into the shape in any order to give a Young tableau. Once the tableau has been fixed, we consider two types of permutations.[9] *Horizontal permutations* p are permutations which interchange only numbers in the same row. *Vertical permutations* q interchange only numbers in the same column. So, we define the *Young operator* by YO = P Q where the quantities P and Q are given by:

$$P = \Sigma_p\, p \text{ (“symmetrizer”) and } Q = \Sigma_q\, \delta_q\, q \text{ (antisymmetrizer”)}, \qquad (I.7)$$

where the sums are over the horizontal and vertical permutations and δ_q is the parity of the vertical permutation q. The tableaux are obtained by the application of the Young operators on the initial standard tableau.

Note that if the arranged numbers increase in every row of T from left to right and in every column of T downwards, the tableau, for a given shape, is called *standard tableaux*.

Let us indicate by $T_1^{\alpha}, T_2^{\alpha},\ldots, T^{\alpha}_n$ the different *tableaux* of the same shape α generated by the permutations defined by the operator Y. Any permutation applied to a tableau of shape α will produce another tableau of the same shape α.

Denoting by P_{ik}^{α} the permutations which changes T_k^{α} into T_i^{α}, we have $T_i^{\alpha} = P_{ik}^{\alpha} T_k^{\alpha}$. The matrices D_{ik} of an irreducible representation of degree f of S_N is calculated from the formula[10]

$$e_{ii} Pe_{kk} = D_{ik}\, e_{ik}\ ,$$

where e_{ik} (i,k = 1,2,..,f) are unit basis which satisfies the equations $e_{ij}e_{jk} = e_{ik}$ and $e_{ij}e_{hk} = 0$ $(h \neq j)$. The parameter f, named *degree* of the irreducible representation, gives the dimension of the irreducible matrices.

The elements D_{ik} of the (f x f) irreducible matrices can be determined adopting three different units e_{ik}: (1) *natural*, (2) *semi-normal* and (3) *orthogonal*. Note that the values found for the D_{ik} components depend on the choice of the unit basis.[9-11,17-19] Of course these three irreducible representations are equivalent.

Let us present a brief review of the fundamental properties of the irreducible representations of the S_N in the configuration space $\varepsilon^{(N)}$:

(1) To each partition (α) there is an irreducible representation described by square matrices $D_{ik}^{(\alpha)}$ with f(α) dimension. So, the *tableaux* with the same *shape* (α) have equivalent representations and the different *shapes* cannot have equivalent representations. There is a one-to-one correspondence between each shape (α) and the irreducible matrices $D_{ik}^{(\alpha)}$.

(2) The dimensions f(α) of the irreducible square matrices assume the values $1^2, 2^2,\ldots, (N-1)^2$.

(3) There are only two 1- dimensional irreducible representations given by the partitions $(\alpha) = [N]$ and $(\alpha) = [1^N]$. The first case is described by horizontal *shape* with N spaces. In the second case we have a vertical *shape* with N rows. The remaining representations have dimensions going from 2^2 up to $(N-1)^2$ and are described by the various shapes occupied by 3, 4,…, N particles, respectively.[9-11,17-19]

Systems with N =2 and N =3 particles: Determination of the Basis Functions of their Irreducible Representations, their Energy Eigenvalues and their

Irreducible Representations in the Configuration and in the Hilbert spaces.

We will show how to determine the irreducible representations for the trivial case **N = 2** and the simplest but non-trivial case of **N =3** using the Young operators. This is done constructing the *basis functions of the irreducible representations*[11,19] using orthogonal unit basis. We will take as the unit basis the n =N! degenerate energy orthonormal eigenfunctions $\{\Psi_i\}_{i=1,2..,n}$ which span an n-dimensional Hilbert space $L_2(\varepsilon^{(N)})$.

We will divide the process used to determine[11,19] the irreducible representations in three parts **a**, **b** and **c.**

(a)Construction of the Young operators

Following the recipes to construct the Young operators Y = PQ, defined by the Eq.(I.7) we obtain the following operators Y, associated with respective shapes: [11,19]

N =2

$$\begin{aligned} &\text{shape } [2] : YO[2] = [I + P(1,2)]/2 \\ &\text{shape } [1^2]: YO[1^2] = [I - P(1,2)]/2. \end{aligned} \tag{I.8}$$

N =3

shape[3]:

$YO[3] = [\Sigma_i P_i]/6 = [I + P(132)+ P(213)+P(231)+P(312)+P(321)]/6,$

shape $[1^3]$:

$YO[1^3]: = [\Sigma_i \delta_i P_i]/6 = [I - P(132) - P(213) + P(231)+P(312) - P(321)]/6$

shape [2,1] :

$$\begin{aligned} &YO_{11}[2,1] = [I + P(213) - P(231) - P(321)]/\sqrt{4} \\ &YO_{12}[2,1] = [P(132) - P(213) + P(231)/2 - P(312)]/\sqrt{4} \\ &YO_{21}[2,1] = [P(132) - P(231) + P(312) - P(321)]/\sqrt{4} \\ &YO_{22}[2,1] = [I - P(213) - P(312) + P(321)]/\sqrt{4} \end{aligned} \tag{I.9}$$

Let us indicate by $e_1 = \Psi(1,2)$ and $e_2 = P(1,2)\Psi(1,2)$ the unit vector basis of the 2-dimension Hilbert space $L_2(\varepsilon^{(2)})$. Similarly, by $e_1 = \Psi(1,2,3)$, $e_2 = \Psi(1,3,2)$, $e_3 = \Psi(2,1,3)$, $e_4 = \Psi(2,3,1)$, $e_5 = \Psi(3,1,2)$ and $e_6 = \Psi(3,2,1)$ the unit vector basis of the 6-dimension Hilbert space $L_2(\varepsilon^{(3)})$ obtained by the permutations $P_i e_1$ ($i = 1,2,..,6$).

(b)Construction of the Basis Functions and the Energy Eigenfunctions.

To construct the *basis functions for the various irreducible representations*[9,19] of the S_2 and S_3 we apply the Young operators YO defined by Eqs.(I.8) and (I.9) to the function $\Psi = \Psi(1,2)$ and $\Psi(1,2,3)$ respectively. In these conditions we obtain:

For **N =2** the completely symmetric ϕ_s and anti-symmetric ϕ_a normalized eigenfunctions of the two 1-dim subspaces are written as:

$$\phi_s = (e_1 + e_2)/\sqrt{2} \text{ and } \phi_a = (e_1 - e_2)/\sqrt{2} \qquad \text{(I.10)}$$

For **N= 3** we have the following eigenvectors:

$$\begin{aligned} &\text{Shape [3]: } \phi_s = (e_1 + e_2 + e_3 + e_4 + e_5 + e_6)/\sqrt{6} \\ &\text{Shape } [1^3]\text{: } \phi_a = (e_1 - e_2 - e_3 - e_4 + e_5 + e_6)/\sqrt{6} \end{aligned} \qquad \text{(I.11)}$$

Shape [2,1]:

$$Y_{11} = (e_1 + e_3 - e_4 - e_6)/\sqrt{4}$$
$$Y_{12} = (e_2 - e_3 + e_4 - e_5)/\sqrt{4}$$
(I.12a)
$$Y_{21} = (e_2 - e_4 + e_5 - e_6)/\sqrt{4}$$
$$Y_{22} = (e_1 - e_3 - e_5 + e_6)/\sqrt{4}.$$

For **N =3** the unit vector basis $\{e_i\}_{i=1,2...,6}$ spans a 6-dimensional Hilbert space which is composed by two 1-dimensional subspaces, h([3]) and h([1^3]), and one 4-dimensional subspace h([2,1]). Since the functions Y_{rs}(r,s= 1,2,3,4) form a set of linearly independent functions in h([2,1]) we can construct by an orthonormalization process the base-vectors $\{Y_i\}_{i=1,..,4}$ of the subspace h([2,1]) that are given by:

$$\begin{aligned} Y_1 &= (e_1 + e_3 - e_4 - e_6)/\sqrt{4} \\ Y_2 &= (e_1 + 2e_2 - e_3 + e_4 - 2e_5 - e_6)/\sqrt{12} \\ Y_3 &= (-e_1 + 2e_2 - e_3 - e_4 + 2e_5 - e_6)/\sqrt{12} \end{aligned} \qquad \text{(I.12b)}$$

$Y_4 = (e_1 - e_3 - e_4 + e_6)/\sqrt{4}$

In these conditions the subspace h([2,1]) is spanned by the orthonormal vectors $\{Y_i\}_{i=1,2,\ldots 4}$. and the eigenstate Y([2,1]) associated to this subspace is written as:

$$Y([2,1]) = \frac{1}{\sqrt{4}}\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \qquad \text{(I.13)}$$

where functions Y_+ and Y_- are defined by

$$Y_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \quad \text{and} \quad Y_- = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}. \qquad \text{(I.14)}$$

As can be easily verified, the functions ϕ_s, ϕ_a and $\{Y_i\}_{i=1,\ldots,4}$ are orthonormal, that is, $<f_n|f_m> = \delta_{nm}$, where n,m = s,a, 1,2,3 and 4. From these orthonormal properties we can easily verify that

$$|< Y| Y >|^2 = (|Y_1|^2 + |Y_2|^2 + |Y_3|^2 + |Y_4|^2)/4 \text{ and that}$$

$$|<Y_+|Y_+>|^2 = (|Y_1|^2 + |Y_2|^2)/2 = (|Y_3|^2 + |Y_4|^2)/2. = |<Y_-|Y_->|^2$$

From the Eqs.(I.13)-(I.14) we see that the 4-dim subspace h([2,1]), which corresponds to the *intermediate Young shape* [2,1], breaks up into two 2-dim subspaces, $h_+([2,1])$ and $h_-([2,1])$, that are spanned by the basis vectors $\{Y_1,Y_2\}$ and $\{Y_3,Y_4\}$, respectively. To these subspaces are associated the wavefunctions $Y_+([2,1])$ and $Y_-([2,1])$ defined by Eq.(I.14). There is no linear transformation S which connects the vectors Y_+ and Y_- .

Note that the above functions ϕ_s and ϕ_a defined by Eqs.(I.10) are the energy eigenfunctions for the system with N = 2 particles. Similarly, the functions ϕ_s, ϕ_a and $\{Y_i\}_{i=1,\ldots,4}$ seen in the Eqs.(I.11)-(I.14) are the energy eigenfunctions of the system with N = 3 particles.

(c)Calculation of Irreducible Representations of the S_2 and S_3 Groups

Finally, to calculate the *irreducible representations of the S_2 and S_3 groups* associated with the corresponding shapes it is necessary to apply the permutation operators P_i to the energy wavefunctions given by the Eqs.(I.10) and (A.11) and Eqs.(I.13) and (A.14).

N = 2 and **3**:

Horizontal shapes [2] and [3]: $P_i\ \phi_s = (+1)\ \phi_s$, that is, D[2] = D[3] = +1.

Vertical shapes [1^2] and [1^3]: $P_i\ \phi_a = (\pm 1)\ \phi_a$, that is, D[$1^2$] = D[$1^3$] = ±1,

showing that all the irreducible representations are 1-dimensional. To the shapes [2] and [3] are associated the matrix $D^{(1)} = 1$. To the shapes [1^2] and [1^3] are associated the matrix $D^{(1)} = \pm 1$.

N =3, *intermediate shape* [2,1].

Applying the permutation operators P_i to Y_+ and Y defined by the Eqs.(I.14) and taking into account that $P_i\ e_j = e_m$, where i, j, m = 1,2,3,…,6, we can show that

$$
\begin{aligned}
&P(123)\ Y_\pm = P_1 Y_\pm = I\ Y_\pm \\
&P(132)\ Y_\pm = P_2 Y_\pm = D^{(2)}(P_2)\ Y_\pm \\
&P(213)\ Y_\pm = P_3 Y_\pm = D^{(2)}(P_3)\ Y_\pm \\
&P(321)\ Y_\pm = P_4\ Y_\pm = D^{(2)}(P_4)\ Y_\pm \\
&P(231)\ Y_\pm = P_6\ Y_\pm = D^{(2)}(P_6)\ Y_\pm \\
&P(312)\ Y_\pm = P_5\ Y_\pm = D^{(2)}(P_5)\ Y_\pm
\end{aligned}
\qquad (I.15)
$$

where $D^{(2)}(P_i)$ (i=1,2,…,6) are the same 2x2 matrices of the 2-dimensional irreducible representation of the S_3 given by Eq.(A.6). This implies that the 4x4 representation matrices associated with the shape [2,1] are broken into 2x2 irreducible matrices $D^{(2)}(P_i)$. These irreducible representations are equivalent. In this way the 4x4 representation matrices in the 4-dimensional subspace h([2,1]) can be written as the direct sum of two 2x2 equal irreducible matrices.

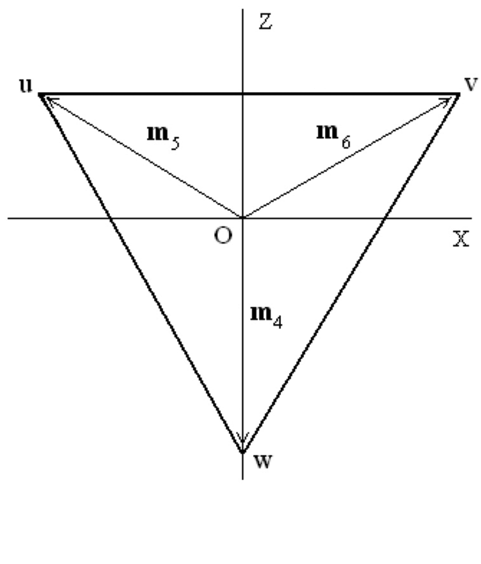
As pointed out above, adopting the particular unit vector basis $\{\Psi_i\}_{i=1,2,\dots,6}$ which are eigenvalues of the Hamiltonian H we have simultaneously determined the irreducible representations of the S_3 in configuration space $\varepsilon^{(3)}$ and in the Hilbert space $L_2(\varepsilon^{(3)})$ and constructed the eigenfunctions ϕ_s, ϕ_a, Y_+ and $Y\_$ of the energy operator H. The 6-dim Hilbert space $L_2(\varepsilon^{(3)})$ which is spanned by the basis vectors $\{\Psi_i\}_{i=1,2..,6}$ is formed by tree subspaces $h(\alpha)$. Two of them, $h([3])$ and $h([1^3])$, are 1- dimensional. The 4-dim subspace $h[(2,1])$ which is spanned by the unit basis vectors $\{Y_i\}_{i=1...4}$ is composed by two 2-dim subspaces, $h_+([2,1])$ and $h_-([2,1])$, spanned by the unit vectors $\{Y_1,Y_2\}$ and $\{Y_3,Y_4\}$, respectively.

APPENDIX II – Permutations in the $\varepsilon^{(3)}$ and the Rotations of an Equilateral Triangle in an Euclidean Space E_3.

It will be shown in this Section that the permutations operations P_i on the state $Y([2,1])$ can be interpreted as rotations of an equilateral triangle in the Euclidean space E_3. To show this we will assume that in the E_3 the states u,v and w can occupy the vertices of an equilateral triangle taken in the plane (x,z) plane, as seen in Fig.1. The unit vectors along the x, y and z axes are indicated by **i**, **j** and **k**. In Fig.1 the unit vectors $\mathbf{m}_4$, $\mathbf{m}_5$ and $\mathbf{m}_6$ are given by $\mathbf{m}_4 = -\mathbf{k}$, $\mathbf{m}_5 = -(\sqrt{3}/2)\,\mathbf{i} + (1/2)\,\mathbf{k}$ and $\mathbf{m}_6 = (\sqrt{3}/2)\,\mathbf{i} + (1/2)\,\mathbf{k}$, respectively.

We represent by Y(123) the initial state whose particles 1, 2 and 3 occupy the vertices u, v and w, respectively. As is shown in details elsewhere[5,7] the irreducible matrices $D^{(2)}(P_i)$ associated with the permutations $P_i\, Y = D^{(2)}(P_i)\, Y$ can be represented by unitary operators $U = \exp[i\, \mathbf{j}.\sigma(\theta/2)]$ and $V = i \exp[i\, \mathbf{m_i}.\sigma(\phi/2)]$; $\theta = \pm 2\pi/3$ are rotations angles around the unit vector **j**, $\phi = \pm \pi$ are rotations angles around the unit vectors $\mathbf{m}_4$, $\mathbf{m}_5$ and $\mathbf{m}_6$ and σ are the Pauli matrices.

Figure 1.
The equilateral triangle in the Euclidean space (x,y,z) with vertices occupie



From these results we see that: (a) the eigenvectors Y([2,1]] are spinors and (b) the permutation operators P_i in $\varepsilon^{(3)}$ are represented by linear unitary operators, U and V, in the Hilbert space $L_2(\varepsilon^{(3)})$.

According to a preceding paper,[3] we have called AS_3 the algebra of the symmetric group S_3 .This algebra is spanned by 6 vectors, the irreducible matrices $\{D^{(2)}(P_i)\}_{i= 1,2,..,6}$ that before[3] have been indicated by $\{\eta_i\}_{i= 1,2,..,6}$. We It was shown that associated to this algebra there is an algebraic invariant $K_{inv} = \eta_4 + \eta_5 + \eta_6 = (\mathbf{m}_4 + \mathbf{m}_5 + \mathbf{m}_6)\,\sigma = 0$. From this equality results that K_{inv} can be represented geometrically in the (x,z) plane by the vector $\mathbf{M}$ identically equal to zero $\mathbf{M} = \mathbf{m}_4 + \mathbf{m}_5 + \mathbf{m}_6 = 0$. Usually, for continuous groups, we define the *Casimir invariants* which commute with all of the generators (in our case the generators are η_4 and η_6) and are, therefore, invariants under all group transformations. These simultaneously diagonalized invariants are the conserved quantum operators associated with the symmetry group. In our discrete case we use the same idea. So, the operator K_{inv} which corresponds to the genuine gentilionic representation of the AS_3 is identified with a quantum operator which gives a new conserved quantum number related to the S_3. Assuming that quarks are gentileons,[3-8] and that the states u, v and w are the three SU(3) color states we have interpreted the constant of motion $K_{inv} = 0$ as a *color charge conservation* which would imply consequently in *quark confinement*. In this case the *AS_3 Casimir* $K_{inv} = 0$ was called *color Casimir*.

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On the Today Understanding of the Concepts and Theory of Quantum Physics. A Philosophical Reflection

Contribution to the José Maria Bassalo Festschrift, 2008.

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Dedicace

I offer this essay to José Maria Bassalo, of the Universidade Federal do Pará in Belém, a pioneer of Physics in the north of Brazil who acted with the purpose of making this science be known, understood and practised in fundamental research, through enthusiasm and reflection, for the benefit of all and of mankind. In memory also of our common experience and engagement in the first Universidade de Brasília[1], *"universidade interrompida"*[2], *in the year 1965, as a testimony of respect and friendship.*

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1 Paty [2003b].

2 Salmeron [1998].

1. Introduction

Quantum physics has been developed since more than one century, corresponding to a still growing knowledge of the properties of matter in a large variety of fields. Generally speaking, it corresponds to the domain of the constitution of matter that escapes, in the small dimensions, the direct apprehension by human senses, and can be attained at experimentally through apparatuses that provide only indirectly informations about physical phenomena and systems. The elaboration of Quantum Theory has started with the purpose of providing a theoretical description and explanation of this domain, with the usual conception of what a physical theory is. This elaboration has been performed in various stages, from the introduction of the quantum of action (Planck, 1900), the energy quantification of light extended to atomic properties (Einstein, 1905, 1908), the wave-particle duality for the light radiation (Einstein, 1909, 1916), the atomic structure and energy levels (Bohr, 1913), the first, "semi-classical", Theory of Quanta (Einstein, 1916), the particle-wave duality for matter elements (de Broglie, 1923), the indistinguishability of identical quantum systems (Bose, Einstein, Pauli, Fermi, Dirac, 1924-1925), the formulation of Wave Mechanics (Schrödinger, 1926), that of Quantum Mechanics (Born, Heisenberg, Jordan, Dirac, 1926-1927), the elaboration of Quantum Field Theory (various authors, beginning in 1927, and still developing nowadays)[1], …

This listing is not exhaustive, although we have tried to mention the most significant advances from the conceptual and theoretical point of view up to the advent of what is usually called "Quantum Theory", i.e. Quantum Mechanics and the dynamical theories elaborated on its basis. At the culminating stage of all these successful endeavours, reached with Quantum Mechanics, the obtained theoretical system seemed so unusual in all respects, and at the same time exceptionally powerful, that it appeared at first sight that one should, while keeping it, leave up the commonly admitted conception of what a physical theory is. If one considered the kind of mathematical quantities it involved, they did not look like the expression of physical concepts able to be directly put in correspondence with entities that could be considered unambiguously as physical. The connection of the theoretical scheme with the phenomena given in experiments was far from obvious, although a correspondence could be imagined with the help of indirect and somewhat artificial rules of use. As Einstein told, physicists had obtained an abstract operative machinery the (physical) meaning of which they did not know. Given the powerfulness of the "mathematical scheme", from which no escape seemed thinkable, it was commonly admitted by those who were satisfied with it, that an appropriate "interpretation" was to be supplemented to what was then considered by the same as a mere mathematical "formalism".

Such was the state of things when Quantum Mechanics was firmly established. It gave rise to further developments, theoretically and experimentally, that are well known, and at the same time, to a strong epistemological debate among the main protagonists of the elaborations of Quantum Physics, a debate that is also rather known, at least in its outlines[2]. Our intention, in the following, is not to enter in details in these questions, but to reflect physically and philosophically on how the *understanding* of Quantum Physics and specifically of Quantum Theory has been modified since that pioneer period.

We shall proceed by mentioning first, but very briefly, as a kind of dish entry ("hors d'œuvre"), the specific link that Quantum Physics maintains with probability and statistics, which has often, for many, summarized the essence of the peculiar situation, and that seems also to be intrinsically related with the problem of observation and experimentation in their connexion with that of the theoretical description. We shall continue by stating some general considera-

1 For an overall presentation, see, for instance, Paty [2003], chapters 5, 6, 7.

2 Jammer [1974], Beller [1983].

tions about theory and physical meaning. Then, focalizing on Quantum Theory, which has been traditionally seen as "a formalism" endowed with an "interpretation", we shall try to analyse and clarify what is called "interpretation", and put forward the intellectual need for a meaningful physical theory with only an *inner interpretation*, that would correspond to an immanent description of the physical domain of quantum phenomena and systems considered for themselves, i.e. as independent self-consistent entities.

We shall sketch what such an *inner interpretation* of Quantum Theory could be (and possibly show that it already is effective). For that purpose, we shall question the epistemological status of the "principle of superposition" (we shall indicate how it has already effectively gained its status from a "formal" to a "physical" principle). We shall conclude by some simple reflection on "decoherence", a kind of phenomena strongly linked with the superposition principle (for coherent quantum systems) as exhibiting the frontier between the classical and the quantum domains and evidencing the legitimacy to characterize intellectually the second one independently of the first.

2. On the link between Quantum Physics and probability

The use of statistics to collect, from experiments, informations about the properties of quantum systems has been inherent to Quantum Physics since its beginnings (with the first contributions of Planck and of Einstein), in continuity with what was the case in Molecular and Atomic Physics, historically related with the Thermodynamics of molecular gases (Boltzmann) and Statistical Mechanics (Gibbs), this being related to the huge difference between the atomic and the macroscopic dimensions, that can be symbolized by the number of Avogadro[3].

The statistical character of the quantum observations has been conceptualized in Quantum Theory through a specific use of probability, namely, the "Born's probabilistic interpretation" of the wave function or state vector, with the introduction of a new type of notion, that of an "amplitude of probability", whose conceptual status has remained for a long time ambiguous. Such a concept – if it is to stand as such - is not a mathematical one, as it does not appear in the mathematical theory of probability. It therefore must be a physical one, the meaning of which is precisely given by "Born's interpretation", which can be seen effectively, not so as an "interpretation" than as merely a definition : it defines the physical meaning of the state vector characteristic of the quantum system by

3 Boltzmann [1909], Gibbs [1902]. Avogadro number is 6,2. 10^{23} molecules per mole.

relating it to the probability of the considered state of the system, relatively to the other possible or existing states[4].

Incidentally, we may notice that conceptualizing the "probabilistic amplitude" as a *physical theoretical concept* has a precedent with probability itself, with Boltzmann who, in his first papers on the theory of gases (1866-1871), before his statistical thermodynamics interpretation[5], defined a probability function as affected to each individual particle, and therefore as having a theoretical meaning for this latter, and not only for the set of particles for which it provides the statistical distribution[6]. However this first thought had remained unperceived and forgotten, the statistical or frequency conception receiving all the attention.

Concerning the probabilistic aspect of the quantum descriptions[7], probability has been for many years univoquely associated with statistics, casting doubt on the possibility for Quantum Theory to describe individual systems or events. But we nowadays know (since three decades) that individual quantum system is a physically meaningful entity, for one knows how to produce such systems, without destroying their quantum character. Individual quantum systems (or « particles") are responsible individually for quantum phenomena (such as interference through diffraction grids), even if one identifies such phenomena in experiments through the use of statistical distributions (by performing a number of similar experiments on identical individual quantum systems)[8].

From this consideration, one can estimate that a positive response has been given by this way to Einstein's requirement for a "complete" Quantum Theory, that it should describe individual physical systems[9]. Einstein had a second demand related to this first one : that such characterizable individual physical systems be locally separated from other ones with which they had been linked but are no more interacting (in the usual meaning of the term). But on this, which was a matter of "interpretation" at that time, the response has been negative, for it has been established theoretically and experimentally that quantum systems having been correlated in the past remain always correlated, whatever be their mutual distance in space, up to the moment when they are destroyed or when

4 Born's interpretation of the state function Ψ is that its squared modulus gives the probability of the corresponding state.

5 Expressed by Boltzmann's equation or principle, relating the entropy S with the molecular configuration probability W for the states : $S = k \text{ Log } W$ (k being Boltzmann constant as it had be named by Max Planck). See Boltzmann [1909].

6 Aurani [1992], chapter 1.

7 For insights on the role of probabilities in Quantum Physics (on which there exists an enormous litterature), let us mention in particular Mugur-Schächter [1992].

8 Notably : Pflegor & Mandel [1967], Grangier [1986]. See : Paty [1999, 2001], [2003a], chapter 4.

9 For instance : Einstein [1948, 1949]. See Paty [1995 ; to be publ., b].

their initial conditions are redefined[10]. These two properties of quantum systems now established as physical facts (individuality, non locality) have contributed, with other ones, some of which more recent we shall refer to also, to shift the problem of the theoretical form versus its physical interpretation such as it was traditionally considered towards a more satisfactory one.

It is useful, at this place, to consider what is meant exactly by the notion of "Physical Theory", in general, and in the case of Quantum Theory, in order to clarify the kind of knowledge Quantum Physics is, with respect to the question of its understanding.

3. About theory and physical meaning.

Any new knowledge, be it experimental or theoretical, is doomed to suffer transformations of its understanding along the course of time, for at the moment of its apparition or presentation, the minds are not yet ready or prepared to integrate it in their own actual internal knowledge system, organized and structured around what they knew up to then. When the new piece of knowledge is received, it induces in the individual minds a process of understanding, which moves and transforms the relationships existing between the elements of the mental representation corresponding to the relevant field – and, indeed, it may as well entail significant modifications of the whole view of the world one had built previously[11].

There is a difference in this respect, concerning Physics, between the kind of "understanding" involved by what we call "experimental" or "theoretical" knowledge. Experimental data can eventually be just registered and taken into account for pragmatic purposes, without further considerations, although it is difficult to think that rough data, coming in last instance from the experience of the senses, are not, at a stage or another, incorporated in a process of reflection that corresponds to a somewhat conceptual and theoretical concern, even if at a rudimentary level. In science, "experimental knowledge" is always strongly linked to mental reconstructions (such as concepts and relations of concepts) and theoretical representations (rationally structured bodies of concepts related together through general propositions such as physical principles) of a given domain of knowledge (the very domain, indeed, where the theoretical ideas had

10 Such property of «quantum local-non-separability» has been evidenced through Bell's theorem on locality (Bell [1964, 1966] and experiments on quantum distant correlations (in particular Aspect [1981]). See Paty [1988], chapter 6 ; [1986].

11 On the processes of understanding and of knowledge transformations, see Paty [2004, 2005b].

been previously thought of and elaborated, and where they are currently tested). Actually it would be difficult, if not impossible, to conceive of experimental knowledge without guiding ideas, borne out of the brains, and that integrate these data in the mind, all these being related together inside a meaningful picture. In nowadays physics, such an integrated combination of "thought elements of the world" given through experimental knowledge always takes the form of a theory, being it a fundamental theory or merely a series of theoretical models.

Speaking about "theoretical knowledge", it would seem at first view that it immediately entails its own understanding, if the paper of the theory is, precisely, to connect in a rationally satisfactory way the elements of knowledge, the old ones and the new ones, the empirical ones and the conceptual and theoretical ones, that have been brought to our minds from our senses, whatever be the process of such elaborations (inferences, mental creations, etc.). Is it not through the obtaining of such a rational arrangement - that is called a "theory" - what is meant when one speaks of "understanding"? For, the relevant concepts and propositions being rationally linked together, one can think to be able to get all of them only from some given ones, either in an analytic or in a synthetic way : one could grasp the whole from the elements whose meaning is precisely given by the system of the mutual relations of the elements that are integrated into the whole system. If this were the case, then the word (and the concept) of "theory" would mean "organization of a rational understanding" concerning the domain of the knowledge in question. In this case, the intellectual structure (the architectonics, to employ a Kantian expression) of the thought, which ensures the rational intelligibility, would coincide with the very form taken by the theoretical overall constituted by the elements of knowledge and their mutual relations.

Physics, such as it has developed historically since xviith century, suits rather well this consideration, at least as soon as each of its particular theories has acquired a relatively stable formulation : analytical mechanics, wave optics, electromagnetic field theory, thermodynamics, the theory of relativity, special and general. For all these henceforth "classical" physical theories, the theory constitutes so to speak the proper *instance of thought* according to which phenomena are conceived and understood.

However, this "internal auto-signification" is not complete: some elements of the theory may have a meaning that escape the theoretical structure and is given from the outer. Such were in classical physical theories the concepts of absolute space and time, or the instantaneous attraction at-a-distance. Of that order are, as well, parameters whose numerical values are only fixed empirically, by experiment (masses, fundamental constants of interaction, etc.). "Incompleteness" and unachievement[12] of theories calls for the modification of these by eliminating (as far as it is possible) their arbitrary (for the theoretical point of view) part. The degree of such elimination may be considered as the measure of the progress of successive theories relatively to each other (for the same domain). As it is formulated here from the point of view of the theory, it is an internal criterion. It corresponds, actually, to a better adequation of the theoretical description to the phenomena it aims to describe, which are given in experiment, bearing on the external world (in the considered domain)[13].

That reserve being done (which preserves the possibility of their modification), for all the mentioned domains, henceforth considered as "classical" ones, the specific physical theory is the proper instance of thought by reference to which the phenomena are conceived and understood. The theoretical relationships are closer and straighter and correspond to some kind of necessity that covers better than previously the necessity proper to natural phenomena.

Actually this way of considering a physical theory, which had been commonly shared by most physicists since Fresnel and Ampère and up to Maxwell, Boltzmann, Duhem, Einstein, Schrödinger and others, has not totally been admitted by a number of thinkers, scientists or philosophers, even of the XX^th^ century, as one can observe for instance from the debates which have accompanied the reception of the theory of general relativity. Some, in an empiricist vein, minimized the specificity of the theory, giving the effective meaning to the mathematical structure of the theory on one side and to the experimental data on the other. Such was, for instance, Hans Reichenbach's conception of the relationship between Geometry and Physics in General Relativity[14]. The Physical Theory was effectively seen as merely a "Practical Geometry" endowed with relations of coordination between the geometrical concepts and the data of experience. On the other side, Albert Einstein conceived such a "Practical Geometry" (which he called equivalently "Physical Geometry") as only the starting point of the elaboration of the theory, and already at that stage it could be formulated in a less empirical way : the primary (elementary) *physical concepts* are thought of and defined with the help of mathematical quantities (for example, geometrical ones such as distances) endowed with relations of coordination

12 On these notions, see Paty [1988b].

13 Methodological auto-reference is, actually, a controlled thought substitute for adequation to external physical reality which cannot be qualified outside symbolic representation : see Paty [1988a], [1992].

14 See Paty [1993], chapter 7.

with corresponding quantities considered for physical bodies. But once these *thought elements*, expressed mathematically and prepared for a physical purpose, are considered, *they need something else to be truly physical*, which is not of the order of definitions, neither correlation or interpretation, but which is the submission to constraints that are entailed by *they being considered as physical*, that is as obeying physical laws (and principles) in all the relevant domains. Therefore these primarily loosely defined concepts become wholly physical through they being built up into a more refined physical conceptual structure that organizes the physical constraints (as, for example, general physical properties, formulated as physical principles, such as the principle of general relativity and the principle of equivalence in the case of the General Theory of Relativity[15]).

To sum up, the Physical Theory, in this elaboration, has not been let in the first stage of its form, that of mathematical quantities endowed with relations of coordination with thought physical elements, and it has been transformed by the intertwining of the exactly formulated relations just mentioned, becoming in that way a genuine conceptual structure. As an effect, we understand better how the physical concepts, expressed by mathematical quantities yielding their mutual relationships, receive their content, or physical meaning, from the whole theoretical structure. In other words, according to this conception, one can reinforce indeed what has been stated before, i. e., that the physical theory, elaborated in such a way, is the proper instance of intelligibility. Such was indeed the conception shared by most physicists (and theoretical physicists) about classical as well as relativistic physics[16]. In this conception of a physical theory, its meaning is not given from an external interpretation, but comes from its very inside. The physical interpretation is, so to speak, *immanent* to the theory itself.

4. Quantum Theory as formalism and interpretation

Curiously enough and rather striking, for there is not any direct and obvious relation between the debate just evoked and the one that has developed in the domain of quantum physics, one observes in the latter an opposition between two conceptions of the meaning of physical theory, that has occupied the minds

15 The principle of general relativity states that the laws of physical phenomena are invariant under general transformations (i.e. with accelerations) of the coordinaates or reference system. The principle of equivalence of the inertial mass and the gravitational mass (i.e. $m_I = m_G$) states as well the equivalence of a uniformly accelerated motion and a uniform field of gravitation (see Einstein [1916] ; Paty [1993], chapter 4 & 5).

16 We can include as well most mathematicians of the field of mathematical physics. One can evoke the names of : Albert Einstein, Hermann Weyl ; Max Born and Erwin Schrödinger when they wrote on the theory of relativity, Paul A. Dirac, David Hilbert and others ; and, later on, most physicists of atomic, nuclear and quantum fields physics.

of quantum physicists, which is germane to the one above, despite important differences. Initially, the quantum physicists were motivated by searching for a theory that could account for the phenomena of the quantum domain and provide the intelligibility of it, in the modality of the "*genuine theory conception*" as we may call that one illustrated above and that was in favour up to then. Experiments and theoretical reasoning having yielded a number of unclassical characteristics of quantum systems and phenomena, Einstein and Max Born did express (around 1924) their conviction that one should formulate a "Quantum Mechanics" as a theory of these,[17] and it was in the spirit of the kind of "genuine theory" that we just considered. Only when the theory of Quantum Mechanics was obtained (in 1926-1927) through the works of Erwin Schrödinger, Werner Heisenberg, Max Born, Pascual Jordan, Paul Dirac, and formulated in an "axiomatic" form a short time later, with Dirac himself, John von Neumann, David Hibert[18], did the "genuine theory conception" become no longer accepted, in the name of the invocated necessity, mainly argued by Niels Bohr[19], to provide an external *interpretation* of the "*quantum mathematical formalism*". Henceforth, "quantum theory" was to be identified as a "mathematical formalism" supplemented with an appropriate "interpretation", and the word "theory" lose its previous admitted meaning when associated with "quantum". To the "interpretation" was referred the physical and cognitive meaning, which the so-called "mathematical formalism" was considered unable to provide.

The situation of the theoretical status of Quantum Theory, as it stood at the time of its advent and as it remained for several decades, can be considered as somehow paradoxical. For Quantum Mechanics, and more generally Quantum Theory (including the Dynamical Quantum Theories that were built in the quantum mechanical frame, such as S (for scattering)-Matrix Theory and Quantum Field Theories), with its highly mathematical formalisation, showed an up-to-then unequalled efficiency to give account of the phenomena of the "unseen" and new atomic domain, seemed at the same time to need an *interpretation* if one wanted to get the exact physical meaning of its statements. The formalized theory alone was considered insufficient for that purpose. The quantities used in the theoretical formalism and in the equations (as operators, instead of functions of numerical variables) had lost the kind of mathematical form and the direct physical meaning

17 See Einstein and Born Correspondence : Einstein & Born [1969].

18 Hilbert, von Neumann & Nordheim [1927], Dirac [1930], von Neumann [1932].

19 Bohr [1958].

they usually had in previous physical theories and it was needed to formulate another one, unavoidably not so direct, this being done by enunciating "rules" ; in these rules, observation was given a privileged status, being made the reference for the theoretical entities or concepts ; probabilities and statistics were uttered to be at the root of the theoretical description; etc..

The problem of the interpretation of Quantum Mechanics and Quantum Theory[20] appeared to be a rather complex one, as it called not only for limited or "localized" physical interpretations of mathematical quantities (as one had in the past, for example with differential, "infinitesimal", variables), but also for more general considerations which implied philosophical conceptions about nature and about the knowledge of it. We shall not enter it in the details and variety of the proposed positions, which have also evolved with time. We shall focalize on some aspects of the *intelligibility* of quantum knowledge, put forward as the motivation to look for interpretations.

In the doctrine decreed by Niels Bohr under the name of "point of view" (or sometime, equivalently, "philosophy") of "complementarity"[21], the only truly *physical* concepts that could be asserted were the concepts of classical physics, as it was these that could give account of the results of observations and measurements of quantum phenomena, which were necessarily performed with macroscopic devices. Complementary concepts (particle and wave, position and impulsion, etc.), with limitations of use with respect to their classical meaning, taken successively, succeeded to give a full account of quantum events. Furthermore, in this interpretation, the theoretical scheme is not intended to describe something like an object, but only the overall non dissociable system of such object and the observation apparatus that makes us know it : in other words, there is no object in the proper meaning, but at best an "object being observed".

This conception of the peculiarity of the quantum domain of phenomena, known as the "Copenhagen interpretation, or complementary interpretation", and as the "orthodox interpretation" as well, has dominated for various decades the debate on the interpretation of Quantum Mechanics. Its denial of the possibility to speak physically of "real physical systems" existing in Nature independently of their observation, challenged all his life long by Einstein among some others[22], had obvious philosophical implications. Was one obliged in order

20 For historical studies, see : Jammer [1974], Beller [1983], …

21 Bohr [1958], Rosenfeld [1955, 1963].

22 As expressed in particular in Einstein [1948, 1949], Einstein & Born [1969]. See Paty [1995 ; to be publ., b]

to think “quantum-mechanically”, to share a “philosophy of complementarity”, which is a particular form of observationalist philosophy, and to deny to physicists and to anyone referring to Quantum Physics, the right to adopt consistently a realist conception of Nature ?

In the evoked context, those who thought that philosophical freedom was possible – and even necessary – to think physics deeply, and argued for alternative interpretations, were qualified as heretics. “Interpretation” was a way to attend the demand of understanding the new elements of knowledge that had appeared and needed to be made intelligible. But was that one, which was dominant, the only possible one ?

5. The new way of understanding : minimizing the external interpretation

One could ask oneself whether there was not a simpler and more “economic” way of understanding the new physics than to overthrow the longstanding general conceptions of the relations between science and philosophy, and if it would not be better to try to understand the changes by inquiring first the proper *physical meaning* of the theoretical entities, considering the possibilities to rearrange the conceptual and theoretical elements of the system of Quantum Physics in such a way that they become “immediately” understandable.

Physical understanding and philosophical conceptions are linked together in the mind’s activity and its thought processes, and they have always been linked, as it can be seen in the history of science and of philosophy. Philosophical categories inform somehow the understanding of the physical concepts and participate to their rational organization in the mind.

Actually, most of the further developments of Quantum Physics that occurred since its foundation have been little – if not at all – dependent on the debate on interpretation. What has made these developments possible has been the ways physicists have *thought* the physical problems met with by making use of the available theory or theoretical scheme. As a result, they made their minds so as to *think* more and more directly the intellectual tools they operated (the elements of the so-called “formalism”), integrating them in a kind of *synthetic mind picture* that they were able to connect immediately with physical phenomena that could be considered in the laboratory. No classical image was needed, and no reference to classical concepts, unless in the case of particular explicit approximations. They were performing, by the daily use of such procedure, which became an habit of thinking (like a “second nature" for intellection), what can rightly be called a "proper quantum physical thinking"[23].

The only kind of interpretation that they needed to understand what they were doing was restricted to the "inner interpretation" of the elements of the so-called "mathematical formalism", as were called the mathematical properties of these elements which yielded their mutual relationships (such as, for instance, the superposition principle for the state function), and their connexion with physically meaning entities that can lead to observation in duly prepared experiments. Before the experiment be performed or even designed, such physical meaning can be attributed to some element of the *thought physical phenomenon (or system)* under consideration.

The signification of what has just been said is that *physical meaning* is not any more restricted to classical observable entities. Physical systems, or their states, can be (and indeed are) described or represented synthetically in the mind, through the thought process, by vectors of Hilbert space, associated with (physical) properties provided by the dynamical variables which have the form of operators on these vectors. The representative state vectors are (and are thought as) linear superposition of eigenstate vectors, each one of these being associated with the corresponding eigenvalue of the dynamical variable (operator).

The other "rules" of the so-called formalism (those which define the probability amplitude, etc.) contribute to complete the relationing of the theoretical representation so obtained with the physical phenomenon up to its experimental detection and measurement. In this perspective, the "postulate of reduction" (of the state vector onto one of its components) might well be considered as nothing more than a rule of using the classical experimental device to "measure" the quantum system.

6. About the "Quantum measurement problem".

Let us propose at this point a brief reflection about the problem of measurement of quantum systems, a problem which has generated, during eight decades, mountains of articles and floods of hard thinking, including highly imaginative pictures, sometimes very strange ones (such as the interaction of consciousness with quantum entities which would cause the reduction on the observed state, according to Eugene Wigner and Fritz London, Edmond Bauer[24]).

23 On the idea of a «proper Quantum Theory», see Lévy-Leblond [1977].

24 Wigner [1961] ; London & Bauer [1939] (see Omnès [2002].

We take this problem as it seems to stand now, in the perspective sketched above, without recalling even the main positions about it, if not to mention that in Bohr's view there was no problem of measurement, for our knowledge of the quantum phenomena can only be given from classical concepts and macroscopic measurement devices, and measurement bears on the non dissociable aggregation of the measuring and the measured systems. Opposite to that view is the idea that entities at the quantum level can be thought of, and are known by us through their measurement in a macroscopic apparatus, with which they interact (through the interaction with the quantum constituents of the apparatus), such interaction being generally supposed to provoke the reduction of the quantum system onto one component of its superposition. Theories of such interactions were tentatively formulated. The interesting point in this last position is that a proper consistent quantum level is conceived and that, contrary to the observationalist conception, the macroscopic, classical, level of physical systems arises from the organization of the lower, quantum one, as is generally considered from a realist point of view.

Another position, in an as well realist direction, is that there is no problem of "reduction", various considerations being advanced for it, for instance those of David Bohm's early theory of non local hidden variables, or Hugh Everett's "relative state" theory and interpretation, and its commentators or followers (such as Bryce de Witt's "many worlds")[25].

A consistent view of Quantum Physics as it seems to be favoured nowadays in the lines sketched here would go in the direction of the absence of reduction of the state vector. The reasons for it would meet some of Everett's pioneer considerations, without the necessity to invoke, as he did, a "wave function of the universe" (related, in Everett's view, tributary in this respect of John A. Wheeler's, with the first prospects in Quantum Cosmology and with the programme of unifying Quantum Theory and General Relativity). Such hypothesis exceeds the present situation of Quantum Physics in its well established corpus and does not appear as central in Everett's arguing. Everett tried to enlighten the meaning of an element of the state vector superposition (that one which we are let with after one measurement), by stating that it was only relative to all the other ones. We let for another time a discussion on this point, and emphasize here merely the rational elimination of the measurement problem and the mention by Everett of the entanglement of the quantum systems[26]

25 Bohm [1950, 1951] ; Everett [1957], Wheeler [1957] ; «on Everett's work, see the detailed historical and contextual study : Osnaghi, Freitas & Freire [in press]»; DeWitt, B. [1968] ; DeWitt & Graham [1973].

26 Among the non reduction conceptions are to be counted also in particular David Bohm (Bohm [1951, 1952, 1980]), Mario Bunge [1973], John Bell (Bell [1989]), See Ben Dov [1987, 1990].

(and of their representative state vectors) in their interactions (including those with the quantum components of the measurement apparatus), which would anyhow forbid, after the (irreversible) chain of interactions, to extract one of them that would supposedly by he original one. Some of these ideas have been taken later on in the formulation of the theory of decoherence, which we shall evoke in the end.

We shall restrain now our own considerations with respect to the problem of quantum measurement to the most simple (and nearly obvious) ones.

The very conditions of the measurement (through classical means) and of its "preparation" offer clearly no alternative than to get only one of the components of the system (which is supposed to exist as individual) at the same time. One should not therefore be surprised by the fact that it effectively gets only one at a time. How could it do otherwise ? It was compelled to this by the very definition given in quantum physics to measurement (wich is always performed with a macroscopic, i.e. classical, non quantum, apparatus). This being so, the only problem is : why this one, and not another one among the various possibilities ? On the whole, by measuring a number of identical systems, the measurement device gives all the possibilities, each one with its affected frequency from calculated probability. This resembles to a purely statistical answer. It is reasonable, indeed, to think that if the operation of measurement chooses the events one by one in a statistical way, it is because what is offered to it at the outset is just a statistical distribution of (possible) states. The "preparation" sets the display of the spectrum of the states, this spectrum being implicitly considered as split into independent components (when in the quantum system they are in phase coherence). One should admit logically that what is given as measured is not the quantum system itself, with its phase coherence corresponding to the linear superposition, but that system *after* it has lost its phase coherence property, although the system has entered the device still in its coherent state. The conclusion is that a lost of coherence must have happened for the system at a first stage of its interactions within the measurement device.

7. The "principle of superposition": from a "formal" to a "physical" principle.

This consideration leads us directly to the question of *decoherence* which is one of the most recent topics in the fundamental aspects of Quantum Physics and its relation with experiments. The knowledge of the processes of "decoherence" would permit to go inside the chains of interactions that occur between the considered quantum system and the material environment it travels through, such as the atomic quantum constituents of the macroscopic measurement system. In the description that we have just tried, we were led to consider a two stages process occurring to the quantum system under study when it is being kept by the detection or measurement device. The first stage, where it looses its phase coherence, or quantum superposition state, to become a simple mixture, would actually correspond to a decoherence process of the kind that has been investigated in laboratory for simple configuration events. The second stage is properly that of measurement, which bears no more on the quantum system itself, but on its "remains", of seemingly classical properties, after decoherence has operated.

We shall specify a little bit more when concluding what exactly decoherence is and how it proceeds. It is useful, before coming to it, to go back to one of the most important principles of the so-called "mathematical formalism" of Quantum Theory, namely the "principle of linear superposition of the state function". What we have in mind is the epistemological status of this principle from the physical point of view. When it has been formulated as a rule to get the wave or state function from the values given by observation, it was considered as merely a formal principle. The solutions of the state equation relationing the state function and its dynamical variables (expressed as operators) are the "eigenfunctions" corresponding to the dynamical "eigenvalues".

The superposition principle of Quantum Mechanics states that any linear combination (or superposition) of eigenfunctions (with coefficients as complex numbers) is also a solution of the state equation : this property comes out directly from the mathematical form of the quantities and their equation, and it means formally that the system is invariant under rotations of the state vector in its (abstract) Hilbert space[27]. Such rotations may be obtained by adequate "preparations" of the system. For example, consider two pure states of a physical system (let us consider two energy states, *a* et *b,* of an atom, represented respectively by the state functions $|a\rangle$ and $|b\rangle$), and the system being submitted to an interaction (let us suppose an electromagnetic pulse on the atom), that mixes these states by transforming them into linear superpositions of the two : $|a\rangle$ becomes $(c_a|a\rangle + c_b|b\rangle)$ and $|b\rangle$ is transformed into $(c_a|a\rangle - c_b|b\rangle)$. The fact that a system (or its state function) is, or can be put, in a state of superposition, is what allows to give account of its specifically quantum properties, such as

27 Hilbert space is the space of functions (vectors in this space) with integrable squared modulus.

interference of sub-states (analogously to the linear superposition of wave amplitudes in classical optics : the amplitudes there are complex numbers too, with a modulus and a phase). All specifically quantum phenomena can be traced back to the superposition principle of the states which follows from the superposition principle of the state functions.

The non-factorizability of the state functions of a system of "quantum correlated states" such as those considered in quantum distant correlation experiments, and more generally in "entangled" quantum systems, is directly responsible, from the theoretical point for view, of the specific quantum properties. The superposition principle is of common and universal use in all quantum domains (from atomic to subnuclear levels), and permits to select the significant variables or parameters (for example in Quantum Theories of Gauge Fields).

This "formal" principle of Quantum Mechanics, as it had been initially considered, has been of a fundamental and systematic use since the beginning of this theory, although there were doubts on its epistemological status, as regards its truly physical or purely formal character. It appears, actually, that the superposition principle is not only an effective and useful tool to give account of the quantum phenomena, but a compelling means to think the quantum systems and their correlated phenomena, through the operation of the quantum quantities (state vectors, "observable"-operators) in accordance with the so-called "quantum formalism". Such an adequateness might rightly lead us to the idea that this "formal" or " mathematical" principle has, in Quantum Physics, the same central and universal function as what is directly called a "physical principle" in non-quantum physics.

A *principle* (in the expression "physical principle") is definitely not a statement of property that would need an explanation ; but it stands as a primary conceptual reference toward which the other quantum concepts must be consistently obliged (in a way similar to the principle of special relativity ruling the transformation laws of the concepts related with the motion of bodies). One does not see, in that case, why one would not recognize the "principle of superposition" of states (which would follow from "of state functions") as properly a *physical principle*, standing at the core of the quantum theoretical description, for it contains the kind of relationships between magnitudes (namely, here, state functions) that allows to grasp the genuine characteristics of all quantum phenomena. It would be the most fundamental and overwhelming physical principle in the domain of quanta, for it implies most of the specific properties of quantum systems[28].

In the same direction of thought, it would be logically consistent with that claim to admit that what has been considered for a long time as the "quantum formalism" is actually the *theoretical* framework of Quantum Theory, where the physical meanings are not any more in need for external physical interpretations; one has already found, in the practice of quantum physical thinking, that the physical meanings of the quantum quantities are given to them from the inside, i.e from the system of relations that has already proven to carry all the physical content that one can expect. In brief, there would be no more a "Quantum Formalism" but only a Quantum Theory (with its known formulation), for it contains (nearly) all the elements of its own interpretation in physical terms. By stating it we have only taken in account what the quantum physicists themselves have obtained in their field of knowledge.

8. A simple reflection on "entanglement" and "decoherence"

The above considerations find also a strong support in the phenomenon of "*decoherence*", which has been recently put in evidence in laboratory. "Decoherence" is the transition from a quantum characterized physical system with "intricate" or "entangled" states to a system that does not exhibit any more its quantum properties, having suffered the lost of its phase coherence due to its interactions with the material environment it goes across. The Theory of Quantum Decoherence, and the experiments which have shown in the last decade the corresponding physical phenomenon at the mesoscopic level, enlighten in a decisive way the physical meaning of the superposition principle, and support strongly the claim that it is a truly *physical principle*.

The Theory of Decoherence, developed since 1981[29], supposes explicitly that the quantum level is fully described by Quantum Theory considered as self-consistent at its proper level and that the macroscop environment is constituted by the aggregation of its constituent quantum systems. (This theory supposes, at least implicitly, a realist point of view, and is in all points neatly in contradiction with the strict Copenhagen "interpretation" of Quantum Mechanics). It is inspired by Schrödinger's thought experiment of the "entangled cat and radioactive atom" which evidenced (in 1935, as a contribution to the debate on interpretation) the difficulty to think of the connection between quantum and macroscopic systems.

28 See, for ex., Paty [2003c].

29 Zurek [1982, 1991], Omnès [1994b], ... See Giulini [1995].

Experiments of "decoherence", designed thanks to the outlines of the Theory of Decoherence, and making use of the most recent techniques to produce and analyze individual atoms[30] (f.i. of the Rydberg kind, of large dimensions and few energy states) and quantified electric fields (with very few virtual photons) have permitted to produce "intricate" (or "entangled"), that is, superposed, physical states of atom and photon (the atom with two energy states and the photon with two phase states) and to identify their state by analyzing it with the help of another two energy states atom sent to interact with the system they form. The resulting more entangled system is in its turn analyzed after its crossing the electric cavity, in such a way as to respect all quantum mechanical exigencies, and to keep the propagating quantum entangled state undestroyed. Varying the parameters (number of photons of the electric fields, distance to be crossed), one is able to follow, step by step, the evolution of the entangled state. Decoherence occurs rapidly but after a finite time which is the longer as interactions with the environment are the fewer.

The main conclusion drawn by the authors of decoherence experiments is that quantum decoherence unavoidably occurs within a very short time, due to the interaction of the original quantum system with the material environment, which explains why, although macroscopic bodies are constituted by micro-quantum systems, they do not exhibit quantum properties such as linear superposition, that show only at the quantum level. The experiment has also shown the frontier region between the quantum and the classical domains and the corresponding systems behaviour.

But we would like to emphasize also that the decoherence experiments have given, at the same time, another quite important result, which is the *evidence for a physical quantum entangled state, caught in the course of its propagation in space and time*: they have made, so to speak, *visible* that quantum superposition state systems *exist physically*. Actually, physicists did not doubt of it previously, except when they had to position themselves with respect to the "interpretation problem". But for those (physicists or philosophers) that do make a difference, as for physical reality, between the quantum and the classical domains, they should admit now that in the quantum domain also one can qualify well defined (real) "physical objects" travelling in space.

When we spoke of "practically visualizing" quantum entangled states, we obviously meant it not as an effectively visible (optical) effect, which would

30 Haroche, Brune & Raymond [1997].

be difficult and even impossible to attain in the atomic and subatomic level, but as an analogy with real physical objects as given to our common knowledge through vision or touch. In such experiments one is in the domain of mesoscopic dimensions, intermediary between atomic and macroscopic, for which classical images might possibly be, in some cases, a nearly legitimate approximation. Here, I don't mean literally an image, but the very precise characterization of (conceptual) quantities that are subject to analysis by the testing quantum systems. The description given of it in the accounts of the experiments is so precise that it gives the impression of visualizing : this is because, actually, the corresponding knowledge and the possible understanding of it (as an immediate intuition) is as complete as if it was given (in common knowledge) by visualization.

It has succeeded to be so by the only means of the theoretical concepts (the quantum quantities) and their theory-given meaning. The meaningful (quantum) quantities are directly put in correspondence with the relevant elements of the phenomena generated by the state interacting with the operating apparatus (electric cavity, analyzer, detector). Only in the end of the experiment, at the final detection, do classical quantities and numerical values corresponding to measurement intervene, and yield statistical results, that are then duly theoretically "interpreted". The *reference for the physical meaning*, in such an experiment, is clearly not the classical measurement device at the detection stage (as it would be according to the Copenhagen School), but *the quantum systems* and the (quantum) phenomena they generate when they cross the whole array. It is in terms of these that the physical meaning of the (numerical, statistical) result registered in the ending, at the detection place, in the last moment, is given.

To end with this, let us emphasize that the phenomenon of decoherence is not to be confused with that of "reduction" : there is no physical projection of the superposed state on one of its components – all are preserved -, but a continuous sequence of interactions giving rise to multiple intricate states. The entangled states are still present in the final system, but they have been multiplied by further interactions. The initial components (with their phases) have not disappeared, but they have been diluted in the correlated many-systems, in such a way that coherence has been lost in the flood and we are let with a mixture of states that look as if there were independent, and they are therefore to be dealt with as a mere statistical distribution.

Conclusion

The philosophical reflections I have proposed here must by no means be considered as a supplement of meaning given to physics by philosophy. Philosophy has not the power to substitute science, even partially, nor is it its proper role. It has only the ability, through critical reflection and analysis, to clarify, up to some point, the meanings of the propositions of the scientific knowledges, by taking the point of view of the rational thought and of the cognitive organization, or architectonics, of the mind's representations. In this sense, philosophical reflection, informed with precision of the acquisitions of the various sciences (one or another), which it acknowledges with enthusiasm and modesty, may help in evaluating and situating the local (disciplinary) and more general (conceptual-categorial and cognitive) reorganizations of knowledge. Such reorganizations are constantly needed, for knowledge – even of the real, external, world - is not already given, but elaborated and built by the minds of human beings, that are always in the quest of fully understanding what they came to know. The case of Quantum Physics, which we tried to think of here, is typical of the conceptual-categorial interconnected changes, the evaluation of which has not yet been performed in a wholly satisfactory way. It is my hope to have helped, although in a limited way, in clarifying the lessons that can be got from its developments.

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Marxism and quantum controversy: responding to Max Jammer's question[1]

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According to Jammer, an important research question is to investigate the manner and the extent to which opposition to the Copenhagen interpretation of quantum mechanics "was fomented and supported by social-cultural movements and political factors, such as the growing interest in Marxist ideology in the West." This paper reconstructs the role played by physicists such as Blokhintsev, Fock, Terletsky, Bohm, Vigier, Rosenfeld, and Schönberg in the controversy over the interpretation of quantum mechanics in the 1950s, making comparisons between the national contexts of countries such as France, the USSR, and Brazil. I conclude by answering Jammer's question positively. In addition, my claim is that by reinforcing the growing opposition to the Copenhagen interpretation, Marxist physicists contributed to the creation of the field of research dedicated to the foundations of quantum mechanics. This occurred despite the dogmatic intellectual practices prevalent among Marxists at the time. This paper also suggests that insofar as the ideas of Marxist physicists can be found spread across the whole spectrum of quantum interpretations, in dealing with such a topic it is more useful to speak of Marxisms rather than Marxism.

1 This paper is dedicated to Prof. J. M. F. Bassalo who has been an enduring source of information and encouragement for my own work. Earlier versions of this paper were presented at the conference Intelligentsia: Russian and Soviet Science *on the World Stage, 1860-1960,* University of Georgia, Athens GA, on 29-31 October 2004, and at the "table ronde" *Interactions entre sciences, politiques et institutions au sortir de la 2e guerre mondiale,* Equipe REHSEIS, Université Paris 7, February 2004. I am thankful to Alexei Kojevnikov, Michel Paty, and Patrick Petitjean for the invitations to present this paper, and to Joan Bromberg, Alexei Kojevnikov, and Shawn Mullet for their comments on a first version of this paper. Grants and Fellowships from CNPq (303967/2002-1), Université Paris 7, and the Dibner Institute for the History of Science and Technology allowed me to finish it.

According to Jammer, an important research question is to investigate the manner and the extent to which opposition to the Copenhagen interpretation of quantum mechanics "was fomented and supported by social-cultural movements and political factors, such as the growing interest in Marxist ideology in the West." This paper reconstructs the role played by physicists such as Blokhintsev, Fock, Terletsky, Bohm, Vigier, Rosenfeld, and Schönberg in the controversy over the interpretation of quantum mechanics in the 1950s, making comparisons between the national contexts of countries such as France, the USSR, and Brazil. I conclude by answering Jammer's question positively. In addition, my claim is that by reinforcing the growing opposition to the Copenhagen interpretation, Marxist physicists contributed to the creation of the field of research dedicated to the foundations of quantum mechanics. This occurred despite the dogmatic intellectual practices prevalent among Marxists at the time. This paper also suggests that insofar as the ideas of Marxist physicists can be found spread across the whole spectrum of quantum interpretations, in dealing with such a topic it is more useful to speak of Marxisms rather than Marxism.

O historiador da física Max Jammer sugeriu, em 1974, que se deveria investigar em que medida a oposição à interpretação usual da física quântica, dita interpretação de Copenhague, havia sido estimulada pela crescente influência do marxismo nas sociedades ocidentais. Desde então, a questão tem se tornado mais interessante para a história da ciência, em decorrência do fato de que a crítica a essa interpretação foi uma condição necessária para o florescimento, na década de 1970, do campo de pesquisa em fundamentos da teoria quântica. Este trabalho buscou reconstituir o papel desempenhado na década de 1950 por físicos marxistas, soviéticos e ocidentais, tais como Blokhintsev, Fock, Terletsky, Bohm, Vigier, Rosenfeld, e Schönberg. Comparações foram feitas entre contextos locais de países como a França, a União Soviética, e o Brasil. A conclusão que emerge do trabalho é que há evidência histórica para se responder afirmativamente à questão sugerida por Jammer. Físicos marxistas contribuíram para a crítica à interpretação de Copenhague, e essa contribuição integra o leque de fatores que levou ulteriormente à emergência do campo de fundamentos da teoria quântica. Essa conclusão se sustenta a despeito de práticas dogmáticas vigentes nos meios marxistas. Esse trabalho sugere, ainda, que quase todo o espectro de possíveis posições relativas à interpretação da teoria quântica se reproduziu entre físicos marxistas, e que nesse sentido é mais razoável se falar em marxismos que em marxismo.

1. Introduction

I shall use the term "quantum controversy" as a shorthand for the controversy over the interpretation of the formalism of quantum mechanics. This controversy is over 80 years old and has involved great names such as Niels Bohr and Albert Einstein in argument since its inception between 1925 and 1927. However, it was only from 1970 on that it became a widespread controversy with a meaningful number of physicists and philosophers working on it (Freire, 2004). The controversy concerns scientific matters and philosophical issues in varying degrees of combination.[1] The existence of phenomena predicted by the mathematical formalism, such as the entanglement of photons and electrons spatially separated, became a hot topic of theoretical and experimental scientific research, while debates on the status of determinism and probabilistic descriptions and on realism and objectivity in quantum physics have always been philosophically loaded. Due to its philosophical implications, the resonances between this controversy and ideological trends such as Marxism, in particular during the Cold War times when the ideological tensions contaminated almost all branches of culture, were inevitable.

1 See Bromberg (2006; 2008); Cross (1991), Forstner (2008); Freire (2004; 2005; 2006; 2007); Freitas & Freire (2008); Jacobsen (2007); Osnaghi, Freitas, and Freire (2008); and Pessoa, Freire and De Greiff (2008).

In broad terms, the quantum controversy opposed two camps: the Copenhagen team centering on Bohr defending the complementarity view, and its critics around Einstein with the former being much more influential and even dominant throughout the controversy. The previous statement has been challenged in several recent studies as an oversimplification of the subject (Chevalley, 1997; Freire, 2004, 1743-44; 2005, 24-26), although for the purposes of this paper it satisfies. It is worth mentioning that the Copenhagen team maintained that quantum theory is a probabilistic description of quantum systems and that quantum phenomena include both systems under study and the experimental devices required for such study. In addition, they considered it a complete theory for the quantum domain. Such a statement also implied that there were no relevant questions to further investigate in the foundations of quantum physics.

The physics historian Max Jammer wrote that "in the early 1950s the almost unchallenged monocracy of the Copenhagen school in the philosophy of quantum mechanics began to be disputed in the West," and suggested that the manner in which opposition to the Copenhagen interpretation of quantum mechanics "was fomented and supported by social-cultural movements and political factors, such as the growing interest in Marxist ideology in the West, deserves to be investigated just as diligently as the influence of the 'Weimar culture' on early quantum theory has recently been studied."[2] My main point in this short paper is that there is now historical evidence to positively answer Jammer's question.[3]

Interest in such a question has been growing because breaking the "monocracy of the Copenhagen school" has been an indispensable condition to afford the quantum debate the status of a genuine scientific controversy and to the rising of a field of research dedicated to the foundations of quantum mechanics. Or, stating it in a weaker manner, that to create such a field some physicists "fought against the dominant attitude among the physicists according to which foundational issues in quantum mechanics were already solved

2 Jammer (1974, 250-1). Again, the content of such "monocracy" may be disputed, but the whole statement is useful for the purposes of this paper. The referred study on "the influence of the 'Weimar culture' on early quantum theory" is Forman (1971). One should note that Forman opens his paper citing a question from a former book by Jammer: "In perhaps the most original and suggestive section of his book on The Conceptual Development of Quantum Mechanics Max Jammer contended 'that certain philosophical ideas of the late nineteenth century not only prepared the intellectual climate for, but contributed decisively to, the formation of the new conceptions of the modern quantum theory.'" Forman considered that it was a far-reaching proposition, and that "properly construed" it is "essentially correct," but added that Jammer did not go very far toward demonstrating [it]."

3 In a previous work (Freire, 1997, on p. 150) I conjectured that "there is historical support to answer the question speculated by Jammer positively," but I did not articulate such an answer.

by the founding fathers of the discipline (Freire, 2006, 611)." Indeed, some of the researchers who have played a major role in the creation of this field of research have stated that it was necessary to break the monocracy in order to create this new field of research. Alain Aspect, who performed some of the more telling experiments on Bell's inequalities, wrote that "questioning the 'orthodox' views, including the famous Copenhagen interpretation, might lead to an improved understanding of the quantum mechanics formalism, even though that formalism remained impeccably accurate."[4] Thus, I will argue that while criticizing the Copenhagen interpretation Soviet and Western Marxist physicists played a positive role in the birth of this contemporary scientific field. This conclusion stands irrespective of the way in which the Marxist camp in the USSR and in the West made their criticisms in the 1950s.

Jammer's question can interest both historians of Marxism or the USSR as well as historians of science. The former tend to see in this Soviet case a minor example of post Second World War "Zhdanovshchina" which was the main Soviet ideological campaign against what they saw as bourgeois trends in art, literature, philosophy, and science.[5] However, this case deserves closer inspection since it is a case in which the relationship between science and ideology seems to exhibit richer features than at first glance. For historians of science, however, the fact that interest in research into the foundations of quantum physics has steadily increased, to the point of turning philosophical debates into potential technological applications in the 1990s, makes Jammer's question all the more meaningful. In addition, as claimed by historians such as Anja Jacobsen, Andrew Cross, and myself, "the postwar Marxist debate over quantum philosophy cannot be bypassed in the historical development of attitudes toward the interpretation of quantum mechanics" (Jacobsen, 2007, 32).

In the following section of this paper I analyze the influence of Marxism among those who were critics of the complementarity view, then I focus on those who were supporters of complementarity, and I finish the paper with a few conclusions. While discussing Bohm's, Rosenfeld's, France's, and Brazil's cases I support my argument with primary sources many of them used in my previous papers. However, for discussing the Soviet case I based my argument on the work of historians such as Loren Graham and Alexei Kojevnikov.

4 Aspect (2004, xix).

5 Exceptions to this trend are works by historians of Soviet science, such as Graham (1972) and Kojevnikov (2004).

2. Marxist criticisms of complementarity

According to the historian of Soviet science Loren Graham (1987, 322), "before World War II the views of Soviet physicists on quantum mechanics were quite similar to those of advanced scientists elsewhere. Russian physics was in many ways an extension of central and Western European physics." Not all of the physicists interested in the interpretation of quantum mechanics were influenced by Marxism; the case of L. I Mandelstam being the best documented until now (Pechenkin, 2000 & 2002). However, the relative weight of each position in the spectrum of interpretations of quantum mechanics in the West and in the USSR was different even before World War II. According to Alexei Kojevnikov (2004, 225) in his *Stalin's Great Science – The Times and Adventures of Soviet Physicists*, "Bohr's complementarity was often criticized as an idealistic interpretation, and it was much more common for the Soviet authors to exclude it from the body of accepted theory of quantum mechanics." However, after the war that stage would dramatically change following the ideological tensions related to the Cold War. Indeed, according to Graham (1972, 80), the period between the late 1940s and the middle of the 1950s was "the age of the banishment of complementarity" in the USSR.

Kojevnikov (2004, 186-244) reviewed the historiography on Soviet physics and added new contributions to and new perspectives on our knowledge of this theme. He analyzed the particularities of "the campaign of ideological discussions in science," between 1947 and 1952 showing that official decisions resulted from five of these campaigns: philosophy in 1947, biology in 1948, linguistics in 1950, physiology in 1950, and political economy in 1951. In addition, he showed that a similar campaign concerning physics was being prepared and that the roots of this campaign were more related to professional disputes among the Soviet physicists, opposing the Academy's physicists and the University of Moscow's physicists, than to philosophical differences. However, for reasons probably related to bureaucratic intrigues the meeting that would be the stage for establishing official decisions concerning this campaign was postponed and eventually cancelled. Instead of the meeting a number of papers criticizing idealistic tendencies in modern physics were published in the journal *Voprosy Filosofii.* Kojevnikov thus considered that these papers played a minor role in the workings of Soviet physics. As he remarked, "a journal publication did not have the same authority as a resolution of the representative meeting would have had and did not have administrative consequences." If the consequences of the failed campaign were minor in the USSR these papers had a wider resonance in the West, in particular among Marxist physicists, as we will see later.

Two of the most influential Soviet critics of complementarity were the physicists Dmitry I. Blokhintsev and Iakov P. Terletsky. For Blokhintsev quantum mechanics was a statistical theory in the sense that it deals with a large number of quantum systems equally prepared or, in an inverse way, it is not an appropriate theory for singular quantum systems. Blokhintsev shared with other critics his repulsion for the emphasis put by complementarity supporters on the role of observation in quantum mechanics. For the Soviet critics the inclusion of observation conditions into the quantum phenomena, as maintained by Niels Bohr, was a kind of idealism and an inclination towards what they considered to be bourgeois trends in the philosophy of science. According to Blokhintsev,[6]

> The Copenhagen school does not emphasizes the fact that quantum mechanics is only appropriate to statistical ensembles; instead, it focuses the analysis on the mutual relationship between the individual phenomenon and the measurement device. [...] The statistical character of the quantum phenomena results from the mutual link between microscopic and macroscopic phenomena. [...] The wave function is not a feature of the particle "in itself," it is rather *the feature of its inclusion in this kind of ensemble*. [...] Thus, *quantum mechanics studies the properties of an individual microscopic phenomenon by way of the study of the statistical laws of a group of such phenomena*.

The Soviet physicist Iakov P. Terletsky expressed similar views, but emphasized that these positions were the result of a debate suggesting a kind of semi-official position to be followed by Marxist physicists. According to Terletsky,[7]

> The results of the 1947–1948 debates established that quantum theory is not a theory of individual micro-objects, as maintained by complementarity principle. It is an adequate theory for statistical ensembles of micro-objects. Quantum mechanics cannot completely represent the movement of an individual micro-object (electron, photon, etc) but only that of an ensemble of identical micro-objects that are simultaneous events or events in a series of successive experiments.

A comprehensive presentation of the complementarity interpretation is beyond the scope of this paper. However, a reader who is not familiar with these

6 Blokhintsev (1952). Emphasis is in the original.

7 Terletsky (1952).

issues might like to know the view being criticized by Blokhintsev and Terletsky. For this reason, I cite a few fragments of Bohr's writings, at least allowing the reader to make the contrast between the complementarity and its critics. According to Bohr,[8]

> In a lecture on that occasion [Congress in commemoration of Volta, Como, 1927], I advocated a point of view conveniently termed "complementarity," suited to embrace the characteristic features of individuality of quantum phenomena, and at the same time to clarify the peculiar aspects of the observational problem in this field of experience. For this purpose, it is decisive to recognize that, *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.* The argument is simply that by the word "experiment" we refer to a situation where we can tell others what we have done and what we have learned, [...].

The target of the criticisms raised by the Soviet physicists comes in the continuation of the same text,

> The crucial point, which was to become a main theme of the discussions reported in the following, implies the *impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.*

Except for its ideological flavor, Blokhintsev and Terletsky's criticisms were similar to those held in the West by physicists and philosophers such as Einstein, de Broglie, and Popper. However, due to their ideological loads, the influence of such Soviet critics was not limited to Soviet boundaries. The web of cultural and political organizations linked to the Western Communist parties took upon themselves the task of spreading criticism everywhere, presenting them, at least among the French Marxist physicists of the time, as "the" Marxist stance on the interpretation of quantum mechanics. Soviet texts on quantum mechanics and other issues laden with ideology were translated and published in several countries on the initiative of cultural associations and journals and magazines related to Marxism. As far as I am aware, the first Western publi-

8 Bohr (1958, 39-40). Emphasis is in the original. Complementarity is presented in several papers by the Danish physicist; Bohr, however, considered that in his review of his discussions with Einstein he well expressed the major ideas of the complementarity interpretation of quantum mechanics.

cation of these papers appeared in France in 1952. They appeared as a volume under the title *Questions Scientifiques – Physique* by Les éditions de la nouvelle critique, a publisher related to the French Communist Party. It included papers by G. Souvorov, Kouznetsov and N. F. Ovtchinikov, D. I. Blokhintsev, and Ia. P. Terletsky, on the interpretation of quantum mechanics and philosophy of modern physics, and by V.A. Fock, on philosophical problems of general relativity. However, before the French publication the papers on quantum mechanics were already known to Marxist physicists in the West. David Bohm, who would become a critic of the complementarity interpretation from 1951 on, stated that two influences were meaningful in his becoming a critic of the complementarity: discussions with Einstein at Princeton and readings of papers by Soviet physicists, maybe those by Blokhintsev or Terletsky (Jammer, 1974, 279). Soviet criticisms were also noted early on by Western Marxist physicists who were critics of Soviet Marxism or supporters of complementarity, such as Léon Rosenfeld, a Belgian physicist and former assistant of Niels Bohr. Rosenfeld wrote to Bohr on 31 May 1949 warning him about "the different misunderstandings which appear when trying to blend complementarity and all kinds of mysticism (whether idealism a la Eddington and others or the Russian pseudo-Marxism)."[9] In other cases cultural and political associations took the initiative of circulating the Soviet criticisms among physicists and trying to publish them in specialized journals. This was the case of Pauline Yates, Secretary of the Science Section of the British organization called "The society for cultural relations between the peoples of the British Commonwealth and the U.S.S.R." She translated a paper by the Soviet physicist Ya. I. Frenkel into English, which "foreshadows the development of a non-statistical theory of microphenomena and final elimination of indeterminacy," and tried to publish it in *Nature,* but she gave up because of opposition from Rosenfeld to be commented on later. She just reproduced it as a stenciled translation for the members of the society.[10]

However, the main criticism of complementarity in the 1950s did not come from the USSR but from a young American physicist who had close ideological links with Marxism, David Bohm. I have discussed elsewhere (Freire, 2005) the reception given to Bohm's proposal in the 1950s, here I just summarize the relevant information for discussing Jammer's question. Bohm's causal interpretation was a direct challenge to complementarity since it replicated the standard results

9 Bohr Scientific Correspondence in the Archives for the History of Quantum Physics.

10 See letters from P. Yates to L. Rosenfeld, 02.07.1952 & 02.19.1952. Rosenfeld Papers. Niels Bohr Archive, Copenhagen.

of non relativist quantum mechanics but it obtained them with a model embedded in a quite different philosophical framework. In this framework quantum systems, like electrons, are treated as microbodies evolving continuously in space-time. [11] Even if Bohm's philosophical views, i.e. strong causality and realism, were central values shared by physicists with different ideological trends, such as A. Einstein and D. I. Blokhintsev, Bohm's causal interpretation did not get full support from them. In fact, the causal interpretation was given a poor reception among physicists at the time. However, it gathered a small but active group of physicists, mainly in France under the leadership of Louis de Broglie, the Nobel Prize, and Jean-Pierre Vigier, a young Marxist. In spite of the ideological neutrality of de Broglie, not a Marxist, adhesion to causal interpretation in France was favored by the strong presence of the Communist Party among the French intellectuals.

In France there was a certain resonance between the defense of the causal interpretation and the criticisms of the idealistic taste of complementarity.[12] The French case is the best locus to see how Soviet criticisms of complementarity reinforced the acceptance of the causal interpretation because there was an overlapping of the strong influence of Marxism among young physicists and the leadership of Vigier and de Broglie supporting the causal interpretation. The influence of Marxism is a well-known feature of French intellectual history as one can deduce from the two following examples. The French physicist Evry Schatzman, who also supported the causal interpretation, suggested that in the early fifties about a quarter of the students at the Ecole Normale were Communists. Didier Eribon, the biographer of Michel Foucault, while analyzing the adhesion of Althusser and Foucault to the Communist Party in the same decade, commented that "Althusser swung toward Marxism and communism at a time when almost the entire Ecole Normale and a large proportion of French intellectuals were doing so."[13] The criticism against the complementarity interpretation and sympathies towards the causal interpretation can be found in journals related to the communists, such as *La nouvelle critique* and *La pensée.* An illustration of this resonance can be found in the speech of Eugène Cotton reporting the conclusions from the "Journées nationales d'études des intellectuels communistes,"[14]

11 Bohm (1952).

12 For a detailed analysis of the French case in the early 1950s, see Freire (1993).

13 Evry Schatzman, interviewed by Spencer Weart, on August 21, 1979, to the "Sources for History of Modern Astrophysics", Center for History of Physics – American Institute of Physics. Eribon (1991, 33).

14 Cotton (1953), 170.

> Since the important discussion of 1947, the true character of quantum mechanics was demonstrated: The Heisenberg principle is a statistical theory of an ensemble of micro-objects. [...] Papers seriously criticizing the old complementarity view appeared in the scientific journals of several countries. Janossy, a collaborator of Einstein, returned to Hungary, Vigier, Régnier, and Schatzman in France, Bohm, a radical American physicist working in Brazil, all have published theoretical attempts to go beyond current quantum mechanics.

David Bohm, who at the time was living in exile in Brazil due to the McCarthyist persecution, received this news from France with excitement. "I have been in communication with Regner + Schatzman. They tell me about all sorts of wonderful discoveries using these new ideas, but as yet no details. I have sent them letters recently urgently asking for details," wrote Bohm to an American correspondent, Miriam Yevick; and later, "I have heard from someone that in a debate on causality given in Paris, when our friend Vigier got up to defend causality, he was strongly cheered by the audience, (which contained a great many students) I would guess that many of the younger people in Europe recognize that the question of causality has important implications in politics, economy, sociology, etc." The same resonance does not seem to have existed in the Soviet Union or, at least, it was less than that wished by David Bohm. "I ask myself the question 'Why in 25 years didn't someone in USSR find a materialistic interpretation of quantum theory?' […] But bad as conditions are in US etc, the only people who have thus far had the idea are myself in US, and Vigier in France," wrote Bohm in 1952, and three years later he would reproduce the same complaints this time to Melba Phillips: "Their orientation is determined strongly by the older men, such as Fock and Landau, who in addition to their training, are influenced by the fear of a sort of 'Lysenko affair' in Physics. […] It is disappointing that a society that is oriented in a new direction is still unable to have any great influence on the way in which people work and think."[15]

As a consequence of the joint effect of the seduction of the causal interpretation and the ideological appeal of Soviet criticisms, one can identify plenty of critics of complementarity influenced by the context. Besides Bohm, Blokhintsev, and Terletsky, of note are Jean-Pierre Vigier, Mario Bunge, Hans Freistadt, and Janossy among the quantum dissenters connected, to a certain degree, to

15 David Bohm to Miriam Yevick, 24 Dec [1952]; David Bohm to Miriam Yevick, 5 Nov 1954; David Bohm to Miriam Yevick, 7 Jan 1952; David Bohm to Melba Phillips, 18 Mar1955. Bohm Papers, Birkbeck College, London.

Marxism. These criticisms were influential beyond the boundaries of the Marxist camp, because sometimes physicists who were critics of the complementarity interpretation took into account these criticisms, stripping them of their Marxist clothes. An example of this is the physicist H. S. Green, writing from the University of Adelaide to the British engineer and philosopher Lancelot Whyte: "You are doubtless aware that some eminent men, notably Einstein and Schrödinger, have questioned the generally accepted interpretation of quantum theory, though they have not been able to suggest an alternative." In a footnote to this comment he added: "If you disregard interpolated nonsense about bourgeois society and the omniscience of certain Russian physicists, a very good general discussion of the question had been given by Janossy in a recent number of the Hungarian Acta Physica." In the received letter Whyte underlined the name "Janossy," and the name of the journal.[16] The influence of that ideological context on the Western Marxist physicists was studied by Andrew Cross who analyzed the idea of the existence of a crisis in physics at the time put forward by these physicists in particular. Cross has the merit of not restricting his study to national boundaries revealing how widespread the influence of such an ideological trend was. Nevertheless, he did not analyze the scientific content of these criticisms and for this reason he could not grasp the relationship between these criticisms and the lasting controversy over the foundations of quantum physics. In particular, he did not see that some of these criticisms absorbed the interest of physicists even after the ideological context he studied.[17]

After Stalin's death the ideological tension in the Soviet cultural sphere began to fade. Marxist philosophical engagements continued to play a role in the quantum controversy but they were less influential than they had been in the 1950s.[18] However, the influence of positions such as that of Bohm and Blokintsev increased with time. Two brief remarks give us an idea of this kind of influence. Firstly, the reach of the causal interpretation goes beyond the de-

16 H. S. Green to Lancelot Whyte, 29 Jan 1953, Lancelot L. Whyte Papers, Boston University, on Box 4, folder 3(12). The referred paper was Jánossy (1952).

17 Cross (1991 & 1992), Freire (1992).

18 For later and weaker incidences of this ideological bias and on the quarrel between Eugene Wigner and Léon Rosenfeld, see Freire (2007). For the survival of this dispute, see the following letter from Otto Frisch to Hugo Tausk, on September 16, 1967, cited in Pessoa, Freire and De Greiff (2008): "The questions which he [Klaus Tausk] addresses (essentially the question of the reality of the external world) seems very interesting. The orthodox Copenhagen interpretation says that physics does not deal with things but with measurements. That sounds like idealism and is therefore rejected by the communists. The opposite also applies, since anyone here in the West who doubts the orthodox interpretation–even for objective reasons–is suspected of being a communist. All this with the complexities and meaninglessness of a religious war, complete with converts: the greatest defender of the orthodoxy is a communist [Rosenfeld], and many in the opposition are fully bourgeoise."

bates of the 1950s and its poor reception among the physicists of the time. Historically it opened the doors to John Bell's inequalities formulated in the middle of the 1960s which gave great impetus to research into the foundations of quantum theory. Furthermore, physicists such as Bohm and Vigier actively worked on foundations of quantum mechanics until their last days, even with further disagreements about the content of their positions. Indeed David Bohm in the 1990s was largely recognized for his contributions to this field of research.[19] Secondly, it became common practice among physicists when speaking about the so-called "statistical interpretation" of the quantum theory, which has remained as one of the alternative interpretations to the complementarity one, to include among its ancestors Einstein, Slater, Popper and the Soviet physicist Blokhintsev.[20]

So far there is already evidence to say that "factors, such as the growing interest in Marxist ideology in the West," fomented and supported opposition to the Copenhagen interpretation of quantum mechanics. There was, however, an interesting by-product of the Soviet and Marxist criticisms. They stirred up disputes among Marxists on the one hand, and moved some of the founder fathers of quantum mechanics to enter in the debate; and all these events amplified the controversy on the interpretation of quantum mechanics.

3. Marxist support for complementarity

The 1950s not only saw the criticisms of complementarity set forth by Marxists but also a dispute among Marxists as a number of them defended complementarity and its compatibility with dialectical materialism. Among the defenders we should include Léon Rosenfeld in the West, and V. A. Fock in the USSR. They were both orthodox in quantum mechanics as they supported the orthodox interpretation of quantum mechanics and heterodox in Marxism insofar as they opposed to what was considered by many the point of view of Soviet Marxism.

The role played by Fock in the USSR was studied in detail by Graham (1972, 69-110; 1987, 320-353; 1988; and 1993, 112-117) and for this reason I will only comment on his possible influence on Niels Bohr himself. Graham was able to identify the two parties in dispute in the USSR, although the dominant one was that of the criticisms of complementarity for its idealistic flavor. In the second one, that of the supporters of complementarity, he recognized the singu-

19 Freire (2005).

20 See Home & Whitaker (1992).

larity of V. A. Fock's ideas, an attempt to combine complementarity with dialectical materialism. He emphasized that it is an open question to the philosophy and history of physics to assess the real influence of Fock on some of Bohr's last texts. Graham analyzed this question because Fock visited Niels Bohr in Copenhagen in 1957 with the goal of discussing and arriving at an agreement on the interpretation of quantum mechanics. The Soviet physicist agreed in general with Bohr's views on this issue but thought that some texts of the Danish physicist lacked terminological clarity.[21] According to Graham, "more work on this issue is in process, but it already seems rather clear that some transition did occur in Bohr's thought, a transition away from emphasis on the interaction of the measuring instrument and the micro-object as the key to quantum mechanics, away from a renunciation of causality, and toward a greater recognition of the physical reality of the microbodies of quantum mechanics. And it is entirely possible that the conversations with Fock were an important cause for Bohr's shifts, although proving this causation would be quite difficult."[22] Graham asked Aage Bohr, Niels Bohr's son and also a Physics Nobel Prize winner, for the records concerning these conversations between Fock and Niels Bohr. Aage Bohr replied that those papers could not yet be released.[23]

In contrast to Fock's case, Rosenfeld's activities have only recently been comprehensively scrutinized from a historical point of view yet.[24] Rosenfeld had been Bohr's former assistant since the 1930s, and was a physicist who was very sensitive to epistemological matters. He had been engaged in Marxist philosophy since the thirties, but Rosenfeld's Marxism was closer to Western Marxism than it was to Soviet Marxism, to use the terms introduced by Perry Anderson in order to make sense of Marxist trends in the 20th century.[25] So as to preserve what seemed to him to be a dialectical feature of complementarity, Rosenfeld criticized the Soviet and Marxist physicists who were themselves critics of complementarity.[26] In the 1950s, Rosenfeld used both political and scientific personal links to fight against the Soviet critics of complementarity and the

21 Fock's visit took place at the time of the dissolution of the "Zhdanovshchina" campaign after Stalin's death. Therefore, his visit had political implications beyond scientific and philosophical ones. He invited Bohr to visit the USSR and organized what was the first visit of the Danish physicist to USSR after the war. Fock's visit to Copenhagen and its political implications are discussed in Freire (1994).

22 Graham (1993), 116. See also Graham (1988).

23 Loren Graham to the author, Athens, Ga, 30 Oct 2004.

24 See Jacobsen (2007) and, for Rosenfeld's reaction to Bohm's causal interpretation, Freire (2005).

25 Anderson (1979).

26 Rosenfeld (1953).

causal interpretation supporters. Among the people with whom he exchanged letters defending complementarity against its Marxist critics we find scientists, philosophers, historians, and social activists, such as Frédéric Joliot-Curie, David Bohm, Evry Schatzmann, E. Burhop, V. A. Fock, Hans Freistadt, Adolf Grunbaum, Lancelot Law Whyte, Robert Cohen, J.L. Destouches, John D. Bernal, Benjamin Farrington, Abraham Pais, Koefed, Guido Beck, Gerard Vassails, Pauline Yates, and Nature's editors.[27]

A comprehensive assessment of Marxism's role in Rosenfeld's defense of complementarity was carried out by the Danish historian of science Anja Jacobsen. In addition to the previous remarks on Rosenfeld and by showing the "philosophical, political, and socio-cultural factors that induced him to take on the role as defender of Bohr's ideas," Jacobsen (2007, 4) was able to reveal "the complexity of his thought, contrary to the general perception that he was simply a vicious or 'vitriolic' attacker of unorthodox interpretations (p.4)." Indeed, she showed that Rosenfeld's view of complementarity as an expression of a dialectical relation derived both from his attachment to Bohr and complementarity and from his view of Marxism. As regards the latter, he was critical of Lenin's *Materialism and Empirio-criticism* and Stalin's *Dialectical and Historical Materialism* views which he saw as mechanistic materialism and preached that Marxist philosophy of science could only be truly found in some texts by Friedrich Engels.

Disputes among Marxists about the philosophical status of complementarity were not restricted to the discussions with Fock and Rosenfeld as protagonists on the side of complementarity. In Brazil, for example, during David Bohm's exile in this country while persecuted by McCarthysm, Bohm met Mario Schönberg as a colleague at the same Physics Department of the University of São Paulo. Being both Jews and Communists their mutual solidarity was reinforced, however, they failed to agree on one issue, the philosophy of quantum mechanics. Schönberg's views were close to Rosenfeld's views and he suggested to Bohm the study of Hegel in order to get a broader understanding of causality, remarking that Lenin had recommended that Communists should read the German philosopher.[28] Bohm took the suggestion seriously and some of that influence can be found on Bohm's later reflections on causality first ex-

27 All of these letters are at ***Rosenfeld Papers***, Niels Bohr Archive, Copenhagen. I am indebted to Felicity Pors for her assistance while consulting them. For the discussions between Rosenfeld and Joliot-Curie, see Pinault (2000), 508.

28 Peat (1996).

pressed in his "Causality and Chance in Modern Physics."[29] This influence of the Bohm – Schönberg conversation in Brazil on Bohm's reflections did not mean that Bohm enjoyed such discussions as Schönberg's stance was against his causal interpretation. According to Bohm's perception at the time,[30]

> Schönberg is 100% against the causal interpretation, especially against the idea of trying to form a conceptual image of what is happening. He believes that the true dialectical method is to seek a new form of mathematics, the more 'subtle' the better, and try to solve the crisis in physics in this way. As for explaining chance in terms of causality, he believes this to be 'reactionary' and 'undialectical.' He believes instead that the dialectical approach is to assume 'pure chance' which may propagate from level to level, but which is never explained in any way, except in terms of itself.

A fair debate among Marxists, such as that between Bohm and Schenberg, was not common at the time. On the one hand, Rosenfeld publicly and unfairly criticized Bohm as a tourist in physics, and privately refereed against the publication of Bohm's papers in *Nature* (Freire, 2005, on pp, 11, 20-22; Jacobsen, 2007, on pp. 23). On the other hand, many of those who were aligned with the criticism of complementarity, the dominant position in the Marxist camp, reproduced the dogmatic practice in cultural and philosophical issues so common in the Western Communist parties and in the USSR. Consequently, in the many publications and debates organized at that time papers by Marxist defenders of complementarity were not published. The French Marxist milieu went as far as to make up the intellectual heritage of one of its predecessors – physicist Paul Langevin – in order to present Langevin's ideas as compatible with the dominant criticism of complementarity.[31]

Returning to Rosenfeld, there is another by-product of this whole affair that contributed to heated discussions, this time among some of the founding fathers of quantum mechanics. In fact, Rosenfeld faced opposition not only from the Marxists aligned with the criticism of complementarity but also, for a different reason, from physicists and supporters of complementarity, such as Werner Heisenberg, Max Born, and Wolfgang Pauli. They did not accept

29 Bohm (1957).

30 David Bohm to Miriam Yevick, 24 Oct 1953. Bohm Papers.

31 Freire Jr. (1993 and 2002).

Rosenfeld's mixture of Marxism with complementarity. With Heisenberg, Rosenfeld kept a public and lasting debate until 1970 criticizing his leanings towards idealism.[32] He kept a private struggle with Born and Pauli. As part of the debate, Max Born wrote and sent to him a 10-page typed text in which he argued that dialectical materialism could not include achievements of contemporary science among its corroborations.[33] Eventually Born abandoned the idea of publishing the text once he saw the beginning of a détente between West and East in the late 1950s. Wolfgang Pauli used his famously ironic and bitter correspondence style to hit Rosenfeld. When editing a volume in honor of Bohr he wrote to Heisenberg, saying he would prevent Rosenfeld from embellishing his paper with banalities on Materialism and labeled Rosenfeld "$\sqrt{Bohr x Trotzky}$."[34]

4. Conclusion

Let me finish with three conclusions. The first is that we have enough historical evidence to positively answer Jammer's question. Marxists, in the West, and Soviet critics fomented and supported opposition to the Copenhagen interpretation. They were not the only reason why the opposition grew, and not even the main ones, but they contributed to the increasing opposition. This included not only their direct criticisms but also the indirect and unintentional effect, i.e., the disputes among Marxists and the reaction of some of the founding fathers, which heated the quantum controversy. The same can be said for the support to Bohm's alternative interpretation, which had a lasting influence on the research on the foundations of quantum mechanics and motivated the production of the most influential scientific result from this field of research, Bell's theorem (Freire, 2005 and 2006). While focusing on the Marxist component of the quantum controversy it should not lead us to overestimate its role and, by the same token, it should not lead us to underestimate its role.[35] As because we consider that in fact it was necessary to criticize the complementarity view as a condition to create a field of research dedicated to the foundations of quantum mechanics, in which complementarity is considered one, even the main, but not the only pos-

32 Rosenfeld (1960 and 1970).

33 Freire (2001).

34 Letter from Pauli to Heisenberg, 13 May 1954. The same expression can be found in a letter to Rosenfeld, 28 Sep 1954. The letters are in Pauli (1999).

35 For an evaluation of the several factors leading to the emergence of "foundations of quantum theory" as a field of research, see Freire (2004).

sible interpretation of quantum mechanics, the role played by Marxist criticisms needs revaluation. Therefore, Marxism can be said to have played a favorable role in the development of this field of scientific research.

The quantum controversy is not the only case found by historians in which Marxism played a fruitful role in science in the 20th century. Stéphane Tirard argues that the historical and materialist approach used by John D. Bernal in his study on the basis of life contributed to the interesting scientific results that Bernal achieved. Kent Staley applies Gerald Holton's *thematic hypothesis* to explain the heuristic influence of Marxism on the Japanese Nagoya School of particle physics. The Physics 2008 Nobel Prize for two physicists from that school, Makoto Kobayashi e Toshihide Maskawa, who shared it with Yoichiro Nambu, updated Staley's remarks. To frame these kinds of influences as well as the different cases in which Marxism became a hindrance for scientific development, Loren Graham uses the distinction between the "authentic phase," in which he includes the works of V. A. Fock, A. I. Oparin, and L. S. Vygotsky, and the "Stalinist ideology," whose most telling example was the Lysenko affair and the suppression of research in genetics in the USSR.[36]

My second conclusion derives from the fact that quantum controversy, which has opposed on philosophical grounds such giants of the 20th century physics as Niels Bohr and Albert Einstein melded science, philosophy and ideology to varying degrees, according to the people, time and places involved.[37] Therefore, Marxism's influence on this story is as legitimate as other philosophical views. As early as 1966 Paul Feyerabend arrived at a similar conclusion. His argument, however, was grounded on a philosophical analysis of Bohr's thought while mine is based on a historical analysis of the quantum case.[38] This is why my first conclusion is that, unlike Joravsky's conclusion when studying the Lysenko affair,[39] one cannot establish a sharp distinction between science and ideology in the quantum case.

However, and this is my last conclusion, the relationship between Marxism and the spectrum of stances in the quantum controversy was not one-to-one. Instead, Marxism had a fruitful influence on critics of complementarity, such as Blokhintsev, Terlestskii, Bohm, and Vigier, as well as on complementarity supporters such as Fock and Rosenfeld. This multisided relationship between

36 Tirard (1997 and 2004); Graham (1993, 99-134); and Staley (2001)

37 Jammer (1974), Freire (2004).

38 Feyerabend (1966).

39 Joravsky (1970, 1-17).

Marxism and quantum controversy should be no surprise for those who think that while speaking of Marxism in the 20th century it is better to use the plural Marxisms rather than the singular Marxism.

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Independent Discoveries following Different Paths: The Case of the Law of Spectral Reversal (1848-59)[1]

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1. Introduction

History of science could have been different. For example, if Sadi Carnot had published his 1826 calculation of the mechanical equivalent of heat before dying, then the principle of conservation of energy could have been anticipated perhaps by 20 years.

When studying a field of the history of science, the historian sometimes gives an opinion about "what could have taken place, if something had not happened". Such assertions are called "counterfactual" statements. The present work is part of a project (initiated in Pessoa, 2001) that tries to explore the possible paths (factual and counterfactual) available for the evolution of science.

The historian is usually suspicious of speculations about histories that did not happen. However, it is not difficult to recognize that the notion of *cause* can usually be translated into a set of counterfactual statements, and vice-versa. One may even define "cause" in the following way: "*A* is cause of *B*, if *A* and *B* occurred, and the absence of *A* would affect the probability of occurrence of *B*". In experimentally reproducible systems, such as those set up in laboratories of natural science, one may study in a controlled way the effect of the absence or presence of a cause or condition, and in this way stipulate the causal structures involved.

But in the case of historical disciplines, such as biological evolution or the history of science, one cannot control the individual causes in order to have a secure notion of the possible histories. In the case of history of science, however, a certain familiarity that we have with the way the mind of a scientist works, together with the advantage of hindsight that the present stage of science offers of past science, allows one to have an *intuition* of simple counterfactual scenarios with a modest degree of confidence, in the same way that we one can tentatively discern the causes involved in an historical episode.

The proposal of the project, of which this work is part, is to develop an approach, within the philosophy of science, that may assist the intuitive task of postulating counterfactual histories. This is done by expressing the evolution of science by means of *probabilistic causal models*, which involve units of scientific knowledge that may be called "advances", "contributions" or "achievements" (Pessoa, 2001, 2005). Examples of an *advance* are the calculation of the mechanical equivalent of heat, the construction of the electrochemical pile, and an empirical law such as the law of spectral reversal, subject of the present study. One assumption of the present approach is that, for small variations in the history of science (such as the publication of the aforementioned calculation of Carnot), the advances maintain their identity (i.e., they don't change in an essential way).

The general aim of the method is to acquire an idea of what are the historical possibilities for the evolution of science, so that one may try to explain *why* science evolved in a certain way and not in another.

As an example of how the explanation of an episode in the history of science may be expressed by a causal model, we examined previously (Pessoa, 2006) the case of the beginning of the science of magnetism that occurred in China and in Europe, in which the *same* path was taken independently in two different contexts, until a bottleneck was not crossed in Europe, which resulted in the stagnation of the field. In contrast to this, in this paper we consider the situation in which two *different* paths lead to the same advance, which acted as a bottleneck to the beginning of the old quantum theory.

2. The Law of Spectral Reversion

Independent discoveries are quite common in the history of science, although we tend to attribute all the merit to the scientist that arrived at it first or to the one that is culturally closer to us. Quite common are also those independent paths that are abandoned after the announcement of the sought for discovery is made by other scientists.

Let us consider the independent discovery of the law that a medium absorbs the same wavelengths of visible light or of thermal radiation that it emits. This fact is sometimes called the "law of spectral reversion" (Cornu, 1872), because when the experimental conditions of a gas are varied, an emission line may revert into an absorption line (Fig. 2). The law states that, for each wavelength, absorption and emission are proportional, i.e., that if a body absorbs well (or poorly) in a certain wavelength, it will radiate well (or poorly) in this *same* wavelength (assuming the same temperatures).

Nowadays this property is better known as "Kirchhoff's radiation law", since Gustav Robert Kirchhoff announced it in October, 1859, providing a mathematical proof in December of that year and, in a more general form, in January of 1860 (see Siegel, 1976, pp. 577-83). Stating it more precisely, this law says that, considering electromagnetic radiation of a single wavelength λ, emitted at a same temperature T, for any two bodies 1 and 2 (made of any material), the ratio between the rates of emission E and absorption A are the same:

$$\frac{E_1(\lambda,T)}{A_1(\lambda,T)} = \frac{E_2(\lambda,T)}{A_2(\lambda,T)} . \tag{1}$$

Another way of announcing this, due to Kirchhoff, is to establish a "univeral function" $e\,(\lambda,T)$, which has the same value for any body in the Universe, where e may be interpreted as the emissivity E of a "blackbody", defined as that which absorbs all incident radiation, that is, for which $A = 1$. Kirchhoff's work led to many experimental and theoretical developments which culminated with the beginning of quantum theory, or more specifically, with the announcement of the law of blackbody spectral radiation by Max Planck in October, 1900, followed by the explanation presented by him in December of that year, which postulated the existence of "quanta" of energy at the molecular level of the radiating bodies.

It is interesting, however, that the law announced by Kirchhoff in October, 1859, had already been published, in a less general form, by the Scottish experimental physicist Balfour Stewart, in March, 1858. His statement was less general, since it only referred to thermal (infrared) radiation, and only in March, 1860, would he extend his law to visible light. Furthermore, his mathematical derivation of the law was inferior to that presented by Kirchhoff 1½ years later (see Siegel, 1976, pp. 583-7). One can therefore understand that Kirchhoff's work would have a greater impact in the following generations, but this does not remove the merits and priority of the Scot, justifying that this advance may be referred to as the "Stewart-Kirchhoff law". A study of the ensuing controversy over priority, involving British and German scientists, is presented by Daniel Siegel (1976, pp. 587-600).

Stewart and Kirchhoff followed quite *different* paths: Stewart did experiments in the field of thermal radiation, and Kirchhoff concentrated on visible light spectroscopy. In fact, a very similar path to that taken by Kirchhoff was previously followed by Léon Foucault (1848), who stated qualitatively the law of spectral reversion but did not go any further. In Sweden, the same spectroscopic law was discovered in 1853 by Anders Ångström, who explained it in terms of Euler's resonance theory. Our interest, in the present paper, is to describe these different paths in terms of causal models.

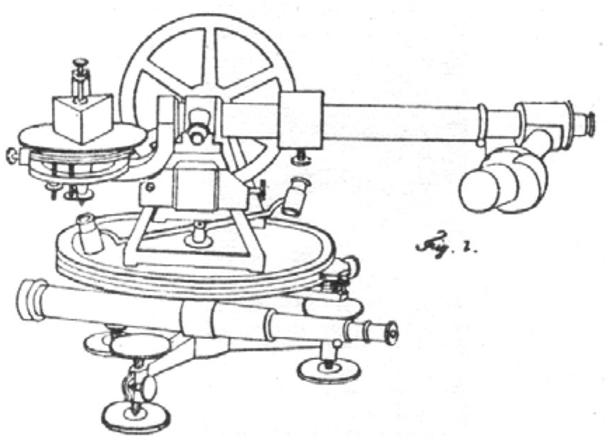
3. Visible Light Spectrosopy

In 1666, Isaac Newton used a prism to study the colored spectrum of sunlight. It would take almost a century before light from other sources was also investigated with a prism. This was done around 1752 by the Scottish Thomas Melvill, who would die a year later, only 27 years old. Melvill discovered that when different substances become incandescent, they emit light with a characteristic color which, when passed through a prism, forms a spectrum of discrete lines. The most intense line was a yellow one, present in every substance he examined (Woolf, 1964, pp. 628-30).

In 1802, William Wollaston studied the properties of refraction and dispersion of many different transparent substances, passing sunlight through a slit 1/20 of an inch wide, which was narrower than the round hole used by Newton. Examining the resulting spectrum, by placing the flint glass prism in front of his eye, he noticed a few dark lines in the otherwise continuous sunlight spectrum (Pearson & Ihde, 1951, p. 267; Woolf, 1964, pp. 621-2).

This study was conducted in a more refined way in 1814 by Joseph Fraunhofer, in Munich. As a young man, Fraunhofer developed excellent technical skills for manufacturing lenses, receiving from the Swiss Pierre Guinand his secrets for making striae-free glass, thus producing prisms and lenses of superior quality. In order to study the optical properties of different glasses used as lenses, he analyzed the refraction of the yellow line emitted by flames by means of an equipment of high sensitivity that combined a prism (made of the glass to be studied), a theodolite for measuring angles precisely, and a 25 mm telescope (Fig. 1). When he analyzed sunlight, he rediscovered the dark lines and mapped hundreds of them. He also discovered that the yellow line from incandescent sources was in fact two close lines, and that they were located exactly in the positions of two dark lines in the solar spectrum, which he had called the "D lines" (Jenkins, 1970, pp. 142-3). What was the origin of these dark lines in the solar spectrum?

Figure 1:
Spectroscopic apparatus of Fraunhofer (Woolf, 1964, p. 631).



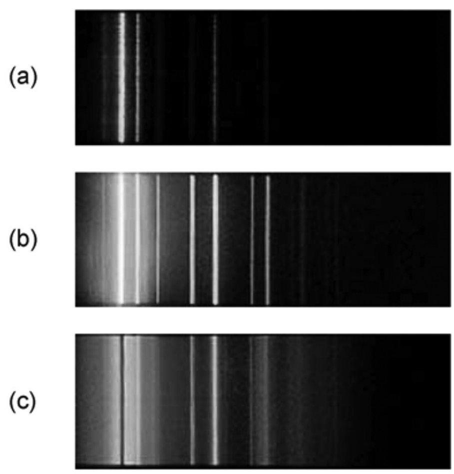
Spectral analysis of flames was further developed, after 1822, by John Herschel and Henry Fox Talbot. Another technique, introduced by Charles Wheatstone in 1834, consisted of the analysis of the spectrum of metals present in the electrodes of a carbon Voltaic arc (the carbon arc was developed by Humphry Davy, in 1821).

Taking into account the similarities between the positions of various dark lines (of the solar spectrum) and bright lines (of the spectrum of flames and voltaic arc), such as the D lines, many scientists, such as David Brewster, in the 1840's, proposed that this coincidence was a general phenomenon. Others, however, like William Swan, denied that this coincidence was general, although they admitted a few coincidences, such as the case of the D lines, which Swan himself was able to show, in 1856, as being due to the element sodium (Siegel, 1976, pp. 568-70).

This identification was delayed because of the difficulty of purifying chemical substances: minute contamination by sodium made almost all materials emit light at the D lines. A decisive step towards this confirmation was the introduction of the so-called "Bunsen burner" by the German Robert Bunsen and his English student Henry Roscoe, around 1855 (Faraday has already prepared a similar equipment in 1827, see Partington, vol. 4, 1964, p. 288). It consisted of a flame of high temperature and low luminosity that introduced a smaller background spectrum than other flames, due to the correct mixture of coal gas and air, which led to the almost complete combustion of the carbon present in the gas.

In 1848, using a voltaic arc, Léon Foucault compared the D lines luminous emissions from the arc and the dark lines in the solar spectrum. Having sunlight pass through the light emitting medium, he observed a curious phenomenon: the dark lines from the sun became even darker after passing through the emitting medium! Foucault thus concluded that the same medium that selectively *emitted* the D lines had the property of selectively *absorbing* the same lines, and this constituted the first statement of the law of spectral reversal (see Fig. 2).

Figure 2:
Discharge of sodium gas lamps. (a) Low pressure gas, with the emission only of discrete lines, the stronger being the pair of D lines (not separated in the photograph).(b) Medium pressure gas, with initial broadening of lines and strong emission of the D lines. (c) High pressure gas, with the occurrence of self-reversion (darkening) of the D lines, due to the absorption, by the sodium gas surrounding the voltaic arc, of the D lines emitted in the continuous spectrum originating at the center (source of the spectra: http://ioannis.virtualcomposer2000.com/ spectroscope/amici.html).



This same phenomenon of darkening of the solar D lines was observed by William Thomson (future Lord Kelvin) in 1854, but he considered that the darkening of the D lines was a psychological effect, like an optical illusion, as described in a letter to George Stokes (Siegel, 1976, pp. 571-2).

In Stockholm, in February, 1853, Ångström clearly stated the law of spectral reversion, after comparing the solar spectrum with that of metals in a voltaic arc. He based his conclusion on Euler's theory of resonance (Maier, 1970, p. 166). Later, Stokes would also justify this law as being a special case of the phenomenon of resonance, explaining that any mechanical system, which has a natural frequency of oscillation, when perturbed will emit mechanical oscillations at this same frequency, and if submitted to incoming vibrations, will mainly absorb at this same frequency (Dampier, 1929, p. 241). This is the origin of the conception that an atom contains mechanical oscillators which emit light of same frequency, an idea that would only be abandoned after Bohr's atomic theory.

Kirchhoff repeated the experiment in 1859, without knowing the details of Foucault's publication, and was also surprised by the darkening of the solar lines. However, contrary to the Frenchman, he took a further step. He used the continuous spectrum of lime light (developed by Goldsworthy Gurney, in 1823, and Thomas Drummond, in 1826; see Partington, vol. 3, 1962, p. 725), and tried to observe dark lines by passing the lamp light through a sodium flame: he noticed that this only worked with a cooler flame, obtained by mixing alcohol and water. This suggested to him that temperature had an important role in this tkind of phenomenon. Inspired by known results (see the following section) for the total emission of radiation (that is, for all wavelengths), but incorporating the first law of thermodynamics (conservation of energy), he derived the law that a medium absorbs the same wavelengths of light or thermal radiation that it emits, i.e., he obtained his universal function $e\,(\lambda, T)$, as mentioned in the previous section.

Besides Foucault, Ångström and Kirchhoff, other scientists that came close to the law of spectral reversion, besides Thomson and Stokes, were William Hallows Miller and John Tyndall (Pearson & Ihde, 1951, p. 270).

4. Researches on Thermal Radiation

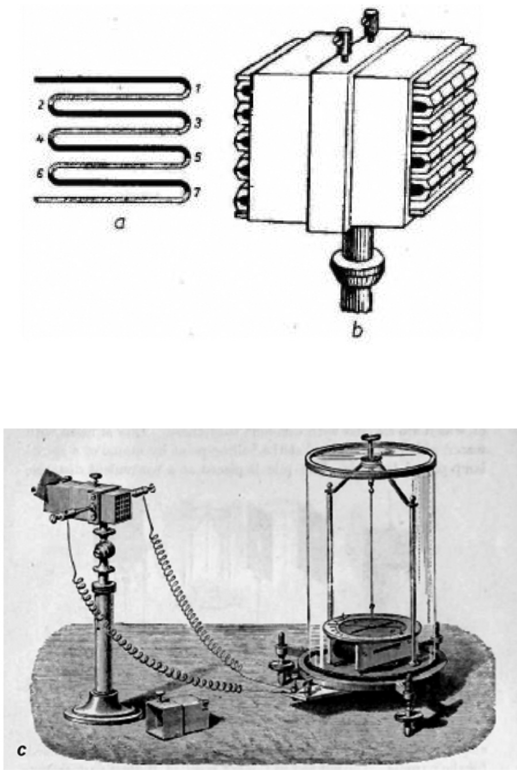
In 1791, the Swiss Pierre Prévost used the notion of caloric (the imponderable fluid of heat, postulated by Lavoisier and others, around 1777) to formulate his "exchange theory". In a situation of equilibrium, all the bodies of an isolated system have the same temperature (satisfying the 2nd law of thermodynamics, which at the time had not yet been formulated); on the other hand, there is a conservation of caloric (expressing the 1st law of thermodynamics, which would only be formulated after 1842). Prévost therefore concluded that a body which absorbs a lot of caloric must emit a lot of caloric. This is in fact what was observed by John Leslie, in 1804, verifying that a good absorber, such as lampblack, is also a good emitter of radiant heat, while a bad absorber, such as polished metal, is also a bad emitter.

Measurements of the thermal radiation emitted and absorbed were made with thermometers, and that was how the great astronomer William Herschel (father of John) discovered in 1800 the existence of infrared radiation. He was studying the Sun, using filters of different colors to darken the solar image. He noticed that when he used some darker filters, he could still experience a sensation of heat, while with other filters that transmitted more light, the sensation of heat was smaller. Passing sunlight through a prism, he measured with his thermometers the rise in temperature as a function of the distance in relation to the visible spectrum (Woolf, 1964, pp. 622-5).

In 1833, with a differential thermometer, William Ritchie verified the equality of total emission and absorption (and not only their proportionality, as Leslie had done), for the same material (Kangro, 1976, p. 7).

The study of infrared radiation increased in the 1830's, after the development of the thermopile by the Italians Leopoldo Nobili and Macedonio Melloni. The principle used in this detector is the "thermocouple", which consists of two metal strips joined at one of their ends, while their loose ends lie at different temperatures, resulting in an electric current proportional to the difference in temperature. Such phenomenon of thermoelectricity was discovered in Berlin by the Estonian Thomas Seebeck in 1821, while measuring, with a rudimentary galvanometer, a current in a ring composed of two metallic semicircles, made of bismuth and copper, with one of junctions being heated. Experimental setups in which the themoelectric couples were mounted in series, alternating the warm and cold junctions, were introduced by Ørsted and Fourier in 1823. These setups were improved by Nobili, in 1829, connecting the system to an astatic galvanometer, which he had developed in 1825, and which achieved higher sensibility by canceling the effect of the Earth's magnetic field by means of a second magnetized needle. The resulting "thermomultiplier" was extremely sensitive to temperature differences between objects in touch with the thermocouples. In the following year, Melloni suggested to Nobili that the detector be adapted to measure radiation, resulting in an instrument which avoided two problems of the thermometer: its slow and small response, and the absorption of radiation by the glass bulb. The resulting thermopile detector (Fig. 3) of Nobili & Melloni (1931) was sensitive to the presence of a person located 10 meters away (Barr, 1960, pp. 45-6).

Figure 3:
(a, b) Detail of the thermoelectric junctions of a thermopile made of the metals antimony and bismuth, as drawn by Nobili, in 1835. (c) General view of the thermopile connected to an astatic galvanometer (Deschanel, 1886, p. 441).



With the thermopile detector, Melloni (1834) showed that thermal radiation had different "descriptions", analogous to the colors of visible light, with different wavelengths, and James Forbes (1836), of the University of Edinburgh, showed that thermal radiation also has polarization states. In 1835, André-Marie Ampère defended the view that thermal radiation and visible light have the same undulatory nature, differing only with respect to their wavelengths. This conclusion only became consensual after the observation of interference fringes in infrared radiation by Armand Hippolyte Fizeau & Léon Foucault, in 1847 (Barr, 1960, p.48).

It was in Forbes' laboratory that Balfour Stewart carried out his studies on radiation, published in March, 1858. He prepared samples of different materials, such as rock salt, glass, mica, and also lampblack. This latter material, being the best absorber of thermal radiation, should, according to Leslie's observations, also be the best emitter, and thus was used as a reference standard. The samples, when heated to 100°C, served as emitters of radiation, and the same samples could be used as filters, when placed before the thermopile radiation detector. Stewart noticed that a rock salt filter absorbed 25% of the radiation emitted by lampblack, but absorbed 68% when the emitter was also rock salt. In other words, rock salt emits radiation of a peculiar "quality", and is quite opaque for that same quality of radiation. This also happened for other materials. From this observation, he concluded that "every body [...] is more opaque with regard to heat radiated by a thin slice of its own substance, than it is with regard to ordinary heat". The novelty of Stewart's statement was that he restricted Ritchie's conclusion, that "absorption is equal to emission", to each wavelength of radiation. Although he wasn't able to observe this directly, he inferred it from the concept of "quality" of radiation (Siegel, 1976, pp. 575-7).

Using the exchange theory, Stewart then derived this result in a theoretical way. When Kirchhoff published the derivation of the same law, in December, 1859, and learned of Stewart's similar conclusions, he criticized him (with good reasons) for lack of rigor in his derivation and for not having done a thorough experiment. Stewart would only extend his results to visible light in a paper published in February, 1860.

5. Counterfactual Questions

We have seen that the law of spectral reversal appeared in two independent paths, both with a strong empirical component, but involving different domains of physical reality: spectroscopy of visible light and infrared radiation.

Why did the two results appear more or less at the same time? Was it a coincidence, or did both fields mature together due to the same set of causes? In other words, if we imagine a set of possible histories developing, say since 1750, in what fraction of these worlds would a similar coincidence occur, in what fraction would the path involving optical spectroscopy arrive first at the law of spectral reversion, and in what fraction would the path involving thermal radiation arrive first?

Since both paths are strongly empirical, dependent on experiments, an answer to the above questions could only be given after a study of the rate in which experimental techniques and instruments were developed in each path. Both fields were involved with the question of the nature of solar radiation, but each was based on different sets of instruments. The optical techniques included the improvement of the manufacture of prisms and lenses, while the infrared techniques included thermal detectors, such as the thermopile, and the preparation of materials that acted as emitters and filters, such as rock salt. One striking difference between these two sets is the use of voltaic currents in the detectors of thermal radiation, and their absence in optical spectroscopy. This indicates that the discovery of the electrochemical pile by Volta has an essential role in one of the paths, but not in the other. Let us investigate these issues in more detail.

6. Rate of Advances in Optical Spectroscopy

Contrary to the case of thermal radiation, which we will examine in the next section, the rate of advances in the field of optical spectroscopy was not dictated by the electrical techniques (except for the use of the Voltaic arc), since the basic apparatus used in this field were prisms, lenses, diffraction gratings and flames, together with precise determinations of angles and positions. The difference in relation to the field of thermal radiation was that the basic detectors were the human eye and (later) photographic plates, and not thermoelectric detectors requiring electrical amplification.

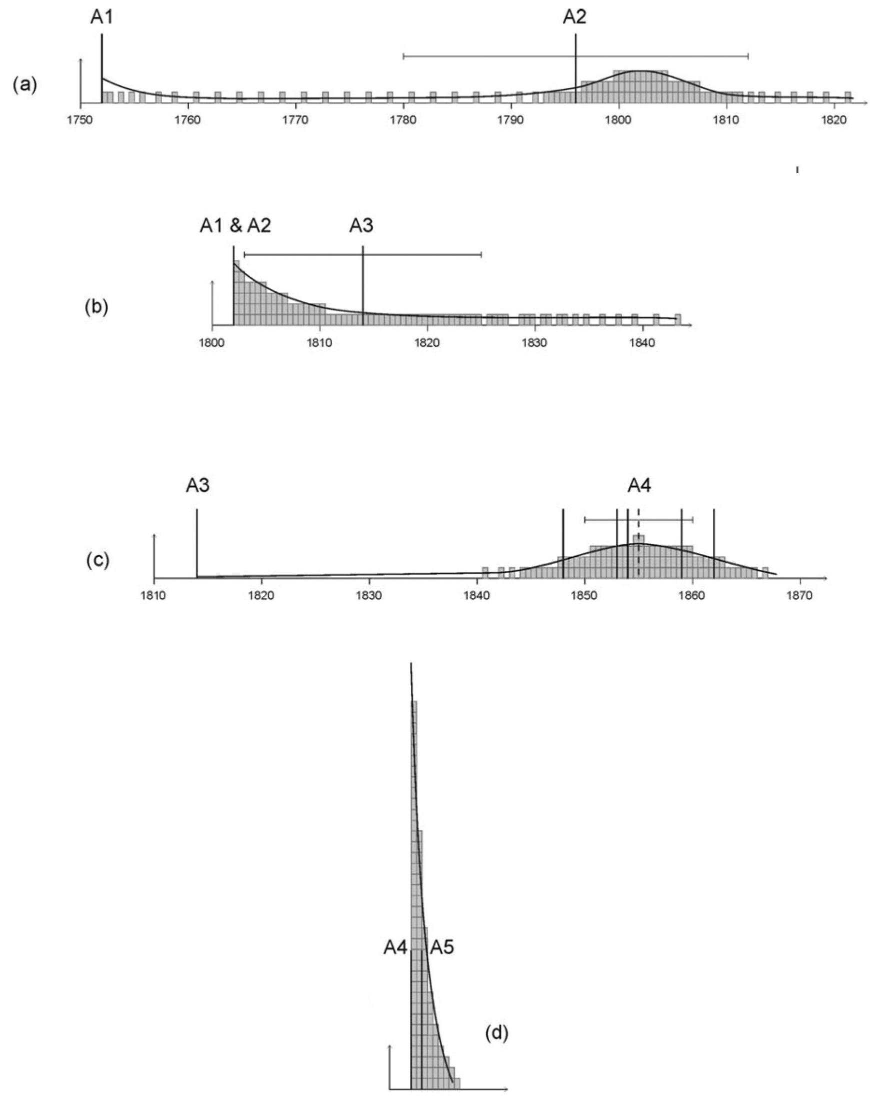
Because of this, one may say that the main advances, in the path to the discovery of the law of spectral reversion in optics, could have occurred before the date that they in fact took place, since the technology required was in general available. The exception to this were the technical innovations introduced by Fraunhofer, especially more homogeneous prisms, which improved significantly the resolution of optical spectroscopy.

The work of Thomas Melvill on the chemical analysis of flames (advance A1), in 1752, did not involve any technique that was not previously available at the time of Newton. These discoveries could have been made by a scientist as bright as Melvill as early as 1700, while the fact that it was not repeated before the 19th century suggests that it could have been made for the first time only around 1800. In this year, William Herschel, while studying the Sun, discovered infrared radiation, which stimulated many other researchers, such as Wollaston, who identified for the first time the dark lines in the solar spectrum. It is not an exaggeration to speculate that Melvill could have made this discovery, if it weren't for his untimely death at the age of 27, when, according to the historian Harry Woolf (1964, p. 628), he "was clearly on the road to major discovery in science".

In Fig. 4a, we chose to begin the causal model in 1750, so that this initial dispersion (standard deviation) of ± 50 years, estimated above for the beginning of chemical analysis, was not included in our analysis. In this year of 1750, we imagine that one hundred slightly different copies of the Universe were generated (see Pessoa, 2008, for more details on this fantasy), and we assume that in each of them Melvill made his discovery of 1752. We suppose, however, that in 1750 the microorganism that led to the death of Melvill had still not encountered him. What would be the probability that a promising scientist would die at an early age? Maybe not more than 20%, at that time. Adopting this estimate, then in 80% of the worlds Melvill would have continued his experimental investigations, and maybe in a fourth of them (to adopt a conservative estimate) he would have discovered the dark lines of the solar spectrum (advance A2). Therefore, in Fig. 4a, in one fifth of the worlds this discovery would have occurred before 1785. In these worlds, A2 would follow causally from A1, but since this would consist of an exploratory process (the discovery would take place when a scientist, fortuitously, decided to examine sunlight), we use an initially exponential distribution (for the criteria of use of this distribution, see Pessoa, 2006). What in fact happened, in our world, was that Wollaston arrived at A2 not influenced by A1, but inspired by the discovery of Herschel. We therefore assume that in most (80%) of the possible worlds the discovery of A2 took place around the year of 1802, with a dispersion of ± 5 years. The resulting distribution has a mean value of 1796, with a standard deviation of ± 16 years.

Once the discoveries A1 and A2 were made, a comparison of the two phenomena could have led some scientist to notice that many of the dark lines of sunlight were in the same position as the bright lines arising in flames of chemical substances, especially the D lines (advance A3). This, however, would require a precise spectrometer, such as the one Fraunhofer built in 1814. We don't know whether Fraunhofer's experimental work could have been done by someone else in the previous decades: that would depend on technical and economical details concerning the manufacture of striae-free lenses. Thus, we will assume that the manufacture of this equipment was "ripe" at that time, and ignore any influence it could have in our estimates of probability. We therefore suppose that A3 would follow from the conjunction of A1 and A2 according to an exponential distribution (since someone familiar with the optical techniques could simply compare A1 and A2, and notice A3), with a mean value of 1814 (see Figura 4b).

Figure 4:
Estimates of the probability distribution of an advance, given a previous one, in the field of optical spectroscopy. Each one of the hundred small rectangles under each distribution curve (of area 1 under each curve) represents the appearance of the advance in a possible world.
(a) Distribution, given A1 (chemical analysis of flames), of A2 (the discovery of dark lines in the solar spectrum). (b) Exponential distribution, given A1 and A2, of A3 (the discovery that the solar D lines are emitted by flames). (c) Distribution, given A3, of A4 (the observation that the solar lines darken as they pass through sodium vapor), represented by five dates, the mean value of which is indicated by the dashed line. (d) Exponential distribution, given A4, of A5 (the law of spectral reversion).



From the knowledge that there is a coincidence between the D lines in the solar spectrum and in chemical substances (advance A3), it took around 34 years for scientists to superimpose both spectra and notice that the flame or voltaic arc containing sodium could darken even more the dark lines from the Sun (advance A4). This step involved many intermediary advances, such as the researches on chemical analysis of flames, on the analysis with a voltaic arc, the recognition that the D lines arise from sodium, besides the improvement of the spectrometer and the Bunsen burner, which were not used in the observations of Foucault (1848), Ångström (1853), and Kelvin (1854), but which helped Kirchhoff (1859). Strictly speaking, Ångström's path to advance A5 did not involve A4, so the spike for 1853 in Fig. 4c should be omitted (see causal model of the situation in Fig. 7).

Given this chain of advances, one should not use an exponential distribution, but one that is similar to a gamma distribution (see Pessoa, 2006), such as the one in Fig. 4c. In this case, in order to estimate the mean value and the standard deviation of the curve, one may make use of the fact that this discovery was made independently by the four scientists. We have also introduced a fifth possible world, that in which Tyndall would have arrived at the discovery some time later (as has been suggested in the literature), say in 1862. With these five data, the mean value is 1855 and the standard deviation ± 5 years. The gamma distribution was drawn on the basis of these two values.

With the observation of A4, Foucault and Kirchhoff immediately noticed that this implied the law of spectral reversal, or, in other words, that if a medium emits well in a certain wavelength, it will also absorb well in that wavelength (the same applying to thermal radiation) (advance A5). However, Kelvin did not take this leap, for he judged that darkening effect was psychological. In Fig. 4d, this situation is represented by an exponential distribution of narrow dispersion.

Calculating the composition (or convolution) of these four probabilistic causal processes, the resulting distribution is indicated in Fig. 6a. It represents an estimate of the year in which the law of spectral reversal would have been discovered, through the path of optical spectroscopy, starting from the factual situation in 1750. The mean value for the year is around 1850, with a dispersion of ± 20 years.

We notice that this calculated value is below the mean value in which A5 actually occurred, although it is above the year of Foucault's discovery. This lower mean value is due to the curve in Fig. 4a, the mean value of which is below the year in which Wollaston actually made his discovery.

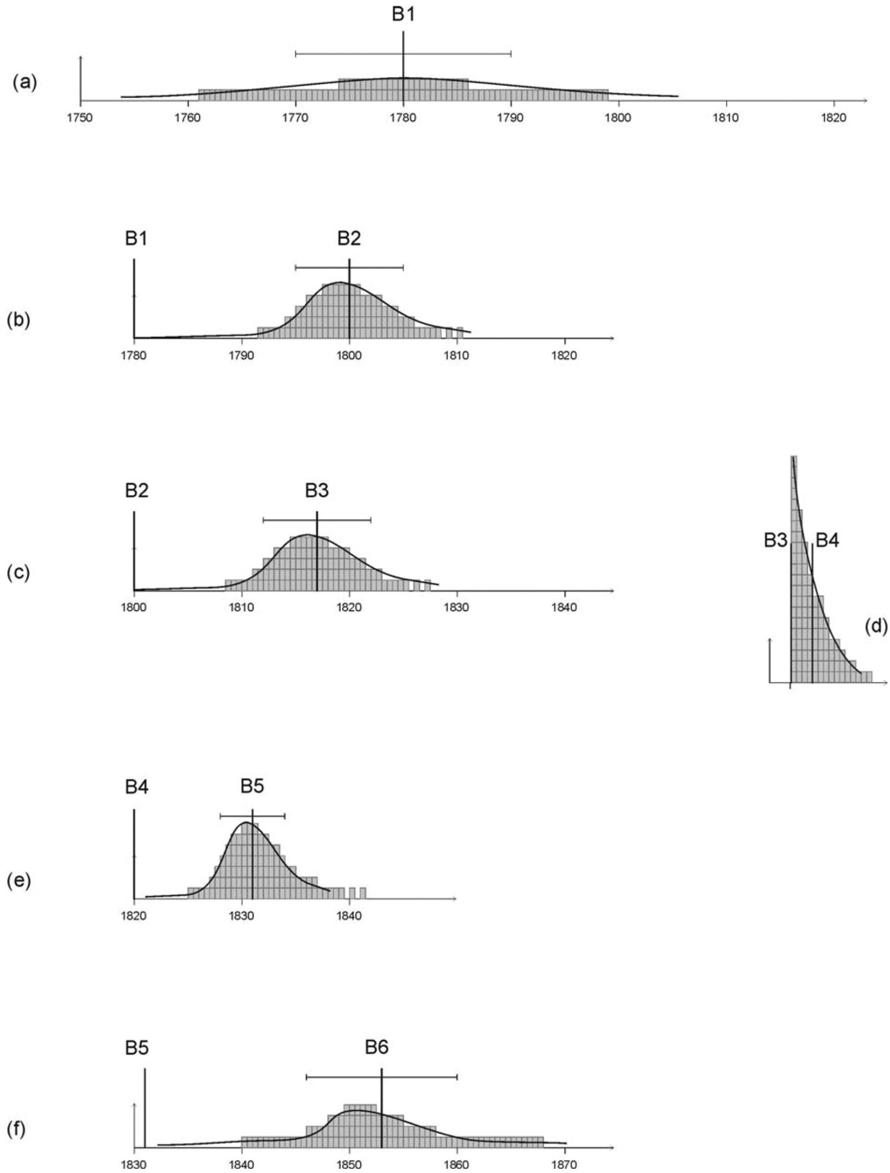
7. Rate of the Techniques in Thermal Radiation

We saw that the field of optical spectroscopy, between 1750 and 1860, did not have its rate determined by technical innovations, but by other factors we were unable to identify clearly, maybe connected to the institutionalization of scientific practice or to random causes. On the other hand, the field of infrared radiation depended heavily on the thermopile, the construction of which was based on the advances in electromagnetic instrumentation. Therefore, the rate of development of this path was dependent on the date in which the electrochemical pile was first built by Alessandro Volta and on the discovery of the conversion process between electricity and magnetism, by Hans Christian Ørsted.

In order to compare the path of thermal radiation with that of optical spectroscopy, we use the same date of 1750 as starting point. At this time, research in electrostatics was well on its way: the machine for generating electricity by friction had already been invented, as well as the Leyden jar, which stored electricity. In 1752, Johann Sulzer made the first observations that the contact of two different metals with the tongue led to a peculiar flavor (Whittaker, 1951, p. 67), but such observation was not followed through to the point of obtaining a pile which generated electricity (a possibility of low probability). The starting point of Volta's advance was the discovery of Luigi Galvani, in 1780, that the spinal marrow or other nerves of a frog, when pressed by a brass hook against an iron lattice, would lead to muscle contractions. Twenty years later, Volta arrived at his electrochemical pile.

In Fig. 5a, the possible histories of Galvani's discovery (advance B1) are represented, in lack of further information, by a Gaussian curve centered around 1780 and with a standard deviation of ± 10 years, since there was no technical obstacle for this discovery being made 10 years before. The path from this discovery to the invention of the electrochemical pile (B2) involved many intermediary steps, taken especially in Italy, by Volta and Giovanni Fabroni. Assuming that each of these steps may be represented by an exponential distribution, then the composition of many such steps would lead to a distribution that is similar to the gamma distribution of Fig. 5b (see Pessoa, 2006). The dispersion around the mean value of 1800 (the year of Volta's invention) was estimated as ± 5 years. If the French Revolution hadn't interrupted scientific research in France, it is plausible to assume that the French might have preceded Volta.

Figure 5:
Estimate of the probability distribution of an advance, given a previous one, in the field of thermal radiation. (a) Initial Gaussian distribution, corresponding to advance B1 (the discovery of galvanism). (b) Gamma distribution which represents the appearance of B2 (Voltaic pile), given B1. (c) Gamma distribution of the appearance of B3 (Ørsted's experiment), given B2. (d) Exponential curve representing the discovery of thermoelectricity (B4), given B3. (e) Gamma distribution for the discovery of the thermopile (B5). (f) Gamma distribution for the appearance of B6 (law of spectral reversal in the field of thermal radiation), given B5.



The next important discovery involving electrical phenomena was made by Ørsted, in 1820, showing that an electric current may deflect a nearby magnetic compass (B3). This discovery did not require a very intense electric current, and it is rather surprising that it didn't happen before. Ørsted himself, in 1812, had planned to investigate the effects of galvanism on magnets. We may thus speculate that the probability distribution between B2 and B3 has a mean value different from 1820, maybe 1817. The gamma distribution with such a mean value and standard deviation of ± 5 years is sketched in Fig. 5c.

Ørsted's discovery had immediate influence over all of Europe, especially France. The exploration of new territories is typically represented by an exponential distribution, and the swiftness with which the new discoveries took place is consistent with this distribution. One of the most important developments of the Dane's discovery was the galvanometer, an instrument for measuring electric currents. In 1821, a rudimentary galvanometer allowed Seebeck to discover the phenomenon of thermoelectricity (B4). This causal influence may be represented by the exponential distribution of Fig. 5d, for which a mean value of 1822 was chosen, since Seebeck's discovery seems to have taken place in a particularly fast way.

The next advance to be considered was the construction of the radiation detector based on the thermopile (advance B5), obtained in 1831 by Nobili & Melloni, after many intermediary steps. This process is represented once more by a gamma distribution, shown in Fig. 5e, with mean value in 1831 and estimated dispersion of ± 3 years.

The thermopile (advance B5) was one of the necessary causes for Stewart's experiments, which resulted in the law of spectral reversal for thermal radiation (advance B6). The path from B5 to B6 had some intermediary stages, mentioned in section 4, but these had already been attained in 1840, after the work of Melloni and Forbes. This suggests that we may move back the mean value of the distribution between B5 and B6 by at least five years, so that the mean value would be 1853, with a standard deviation of ± 7 years, as sketched in Fig. 5f.

The composition of these six processes results in a distribution for B6, starting from 1750, with a mean value of 1853 and standard deviation of ± 13 years, illustrated in Fig. 6b.

It should be stressed that in the present analysis we have neglected the theoretical and experimental advances, related to thermal radiation, which took place before the development of the thermopile.

8. The Computation of Compositions of Causal Processes

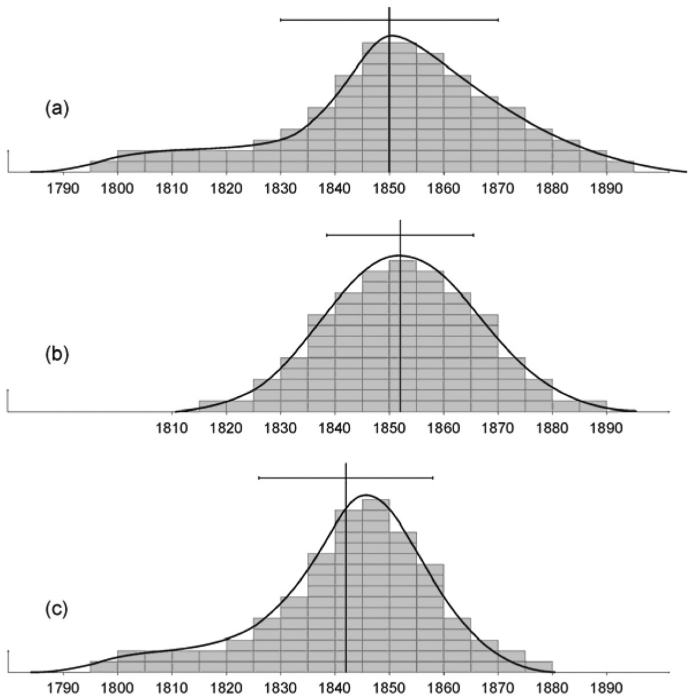
In Figs. 4 and 5, different probability distribution curves are used to describe the appearance of an advance after another. Each small rectangle represents one among a hundred possible worlds, and the actual computations of the composition of causal processes were made from each discrete sample (of one hundred worlds), which approximates the continuous curve in each figure. The shape of the distribution functions is important for attributing a probability for the occurence of a certain advance within a certain time interval (Pessoa, 2006). However, the exact form of the distribution functions involved in a *composition* is not important for computing the overall mean value and standard deviation. The mean value of the convoluted function (obtained by composition) is the *sum* of the mean values of the component distributions. And the *square* of the standard deviation of the convoluted function is the *sum of the squares* of the standard deviations of the component distributions. These properties should allow a significant simplification in the calculation of probabilities of large networks of advances.

Fig. 6 (a and b) presents a comparison between the estimates for the appearance of the law of spectral reversal by the paths of optical spectroscopy and thermal radiation, taking 1750 as the initial date. The advances A5 and B6 are considered the same (strictly speaking, B6 refers only to thermal radiation, but we suppose that the extension to visible light is immediate). In the first path, of optical spectroscopy, the appearance of the law was found to have as mean value 1850 ± 20 years, while in the path of thermal radiation it was found to have the mean value of 1852 ± 13 years.

If we imagine one hundred worlds created in 1750, our estimates would be that in around 51% of them the discovery would occur first in the field of optical spectroscopy, and in 48% of them first in the field of thermal radiation (a little less than 1% would occur in the same year). In factual history, Foucault's 1848 observation would put our world in the first class.

Fig. 6c presents a distribution curve of the year that the discovery of the law of spectral reversal would first take place, if we consider *either path*. In this case of disjunction, the resultant distribution was computed from the distributions of Figs. 6a and 6b. The mean value of the resultant distribution is 1842 ± 16 years.

Figure 6:
Distribution curves for the discovery of the law of spectral reversal, relative to the initial date of 1750. (a) Path of optical spectroscopy, with mean value 1850 ± 12 years. (b) Path of thermal radiation, with mean value 1852 ± 13 years. (c) Disjunction of the previous curves, that is, distribution of the discovery by any of the two paths, which gives the mean value 1842 ± 16 years.



A general view of the main advances involved in these two paths that led to the Stewart-Kirchhoff law is given in the causal diagram of Fig. 7.

9. Conclusion

We examined the two historical paths that independently led to the discovery of the law of spectral reversal. One difference between these two paths is that the one involving research in thermal radiation is strongly dependent on electrodynamical techniques (thermoelectric effect, astatic galvanometer), contrary to that involving optical spectroscopy. Therefore, one can say that the rate of advances in thermal radiation was dictated, until 1840, mainly by the techniques associated with scientific instruments, especially detectors.

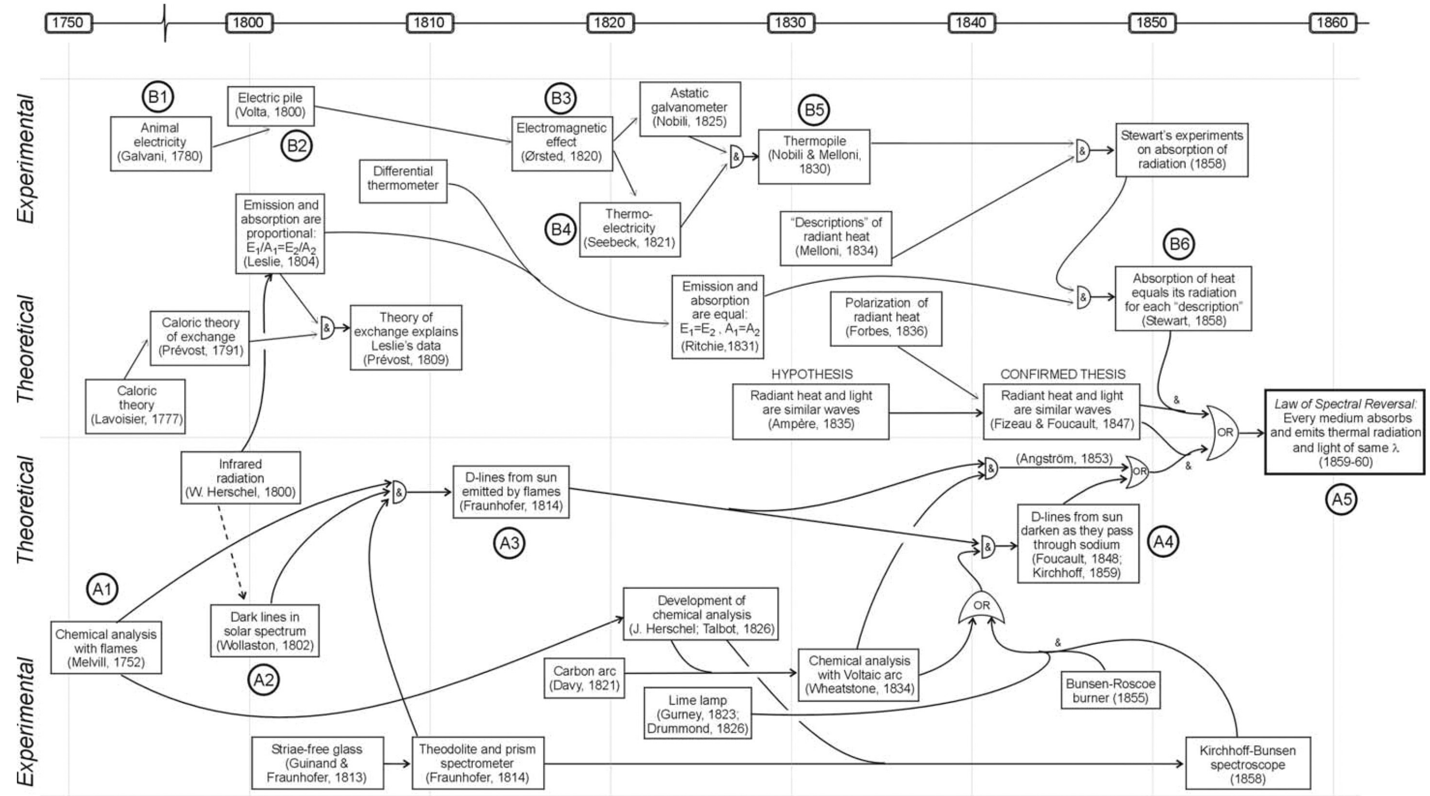


Figure 7:
Causal diagram of the main advances leading to the Stewart-Kirchhoff law, following the path of thermal radiation (upper part) and of optical spectroscopy (lower part), with indication of the advances used for analysis.

We considered that, in most possible worlds, the path involving thermal radiation would arrive at the law of spectral radiation at a previous date than it actually did, due to an anticipation of Ørsted's discovery by an average of 3 years, and of Stewart's results by 5 years.

The rate of development of the field of optical spectroscopy was not initially influenced by technical issues, so we estimated (conservatively) that, averaging over the possible worlds, Wollaston's discovery would have occurred 6 years before the time that it actually did. After Fraunhofer's development of optical techniques, the pace of the field did not suffer further delays (in relation to the estimated mean of the possible worlds).

So we ended up estimating similar delays for both fields, which resulted in curves with similar mean values in Figs. 6a and 6b.

The methodology of causal models and counterfactual histories is still being explored, and the present study is far from being the final word on the subject. The method is based on the historian's intuitions for each individual advance, and it will probably become more useful for large networks, for which human intuitive ability is more limited. One result of the present paper, that will be useful for further work, is that one need not work with distribution functions to compute the composition of causes, since sufficient information is given by the mean value and standard deviation of each individual causal process. However, as far as we were able to investigate, the evaluation of disjunctions of causes (Fig. 6c) does not have a similar simplification.

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Challenges of 21st century physics

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It is a great pleasure to dedicate this paper to my friend Bassalo, collaborating with his festschrift. Our friendship started when Bassalo was a young physicist and worked with me at the University of Brasília in its beginnings. From the first contacts I could recognise some affinities between us on fundamental things in life and on the interest for science; our friendship became stronger and stronger with time. I have always admired his permanent disposition to learning and teaching, which it at the origin of his contribution to the development of physics, to the teaching of physics and mathematics and to the history of science in our country.
Bassalo's interests are wide. I have chosen for this paper a subject in accordance with his mind that looks permanently for challenges trying to understand what has not been understood. We shall see questions for which there are no answers at the present stage of physics.
In spite of the progress in physics we often work with concepts and facts whose origins are unknown. We should pay attention to such questions and have them in mind, otherwise we run the risk of following the negative Aristotle's philosophy, by saying that the world is like it is because it is like it is, not going further. Questions without answers exist in different branches of physics, we shall deal with some of them in elementary particle physics - in the microscopic scale - and in cosmology and astrophysics - in the macroscopic scale.
The paper is divided into two parts. In the first part we shall consider particle physics and in the second cosmology and astrophysics.

1 Extended translation of the paper: "Descobertas Independentes por Caminhos Diferentes: O Caso da Lei da Reversão Espectral (1848-59)", in Martins, R.A.; Silva, C.C.; Ferreira, J.M.H. & Martins, L.A.P. (eds.), *Filosofia e História da Ciência no Cone Sul. Seleção de trabalhos do 5º Encontro.* Campinas: AFHIC, 2008, pp. 347-55.

Part 1 - Questions without answers in particle physics

1.1 The origin of masses and of electric charges

Let us consider an elementary reaction, like

$$p + p \rightarrow p + p + K^+ + K^- + \pi^+ + \pi^- \quad (1)$$

Pions and kaons were created with their masses and electric charges. This simple reaction has four questions for which we have no solutions.

Question 1 – Dynamics of creation of masses – from theory of relativity we know the equivalence between mass and energy, the masses of the pions and of the kaons come from energy of the initial protons. However we do not know the dynamics of the transformation of mass into energy, i.e. we do not know the physical process.

Question 2 - Creation of well definite values of masses – there is creation of masses of definite values in the collision of particles; in reaction (1) the masses of pions and kaons were created. Why are these masses created and no others, and why there is not a continuous spectrum of masses created?

Question 3 – We know more than 350 particles that are created in the Universe in reactions like (1). Why are there so many particles?

Question 4 – When particle masses are created, the electric charges of the particles are also created, positive and negative in equal numbers because there is conservation of charge. However we do not have the slightest idea of how an electric charge is created, how it appears from nothing.

Some of the facts that will be reminded bellow are elementary. We shall recall them in spite of being elementary just to keep some order in the paper. They are the four interactions between particles, the structure of elementary particles, the colour field, the intermediate bosons and the three families of particles.

1.2 The four interactions

There are four types of interaction or force between particles. *Strong interaction* – is the interaction between protons and neutrons inside the atomic nucleus, which maintains the stability of matter. *Weak interaction* – which produces particles decay, as in radioactivity. *Electromagnetic interaction* – due to the electric charges. *Gravitational interaction* - due to the universal attraction between masses.

If we take as reference the intensity of the strong force and represent it by 1, the intensities of the others will be 10^{-14} for the weak force, 10^{-2} for the electromagnetic and 10^{-38} for the gravitational. The range of the strong force is 1 fermi = 10^{-13} cm, of the weak force is 10^{-2} fermi, of the electromagnetic and gravitational is infinite.

1.3 The structure of elementary particles

There are two large groups of particles, *hadrons* and *leptons*. Hadrons have the four interactions. Leptons do not have strong interactions.

Leptons - There are six leptons, three with negative electric charge equal to the electron charge and three neutral neutrinos:

lepton	mass MeV
elétron e^-	0,51
múon μ^-	107
tau τ^-	1777
neutrino elétron ν_e	≈ 0
neutrino múon ν_μ	≈ 0
neutrino tau ν_τ	≈ 0

There are six *anti-leptons* – with masses equal to those of the leptons, three with positive charges and three neutral anti-neutrinos.

Hadrons and quarks - Hadrons are composed of six particles called *quarks*, whose charges and masses are given below, where e = charge of proton and electron. The names u, d, s, c, b, t are called *flavours* of quarks.

There are six *anti-quarks*, with masses equal to those of the quarks and charges of opposite signs. We shall represent them by antid, antiu, antis, antic, antib and antit.

quark		electric charge	mass MeV
u	up	+2/3 e	4
d	down	−1/3 e	8
s	strange	−1/3 e	150
c	charm	+2/3 e	1200
b	bottom	−1/3 e	4700
t	top	+2/3 e	175 000

There are two types of hadrons, *baryons* and *mesons*.

Baryons are composed of three quarks. Example, the proton has two quarks u and one quark d, the neutron has two quarks d and one quark u:

p = u + u + d charge = + (2/3)e + (2/3)e – (1/3)e = +e
n = d + d + u charge = – (1/3)e – (1/3)e + (2/3)e = 0

Mesons are composed of one quark and one anti-quark. Example, the positive pion has one quark u and one anti-quark d, the negative kaon has one quark s and one anti-quark u:

π+ = u + antid charge = + (2/3)e + (1/3)e = +e
K^- = s + antiu charge = – (1/3)e – (2/3)e = –e

All experimental results can be interpreted with point-like quarks and leptons, without structure.

Question 5 – Are quarks and leptons really particles with no structure, or do they look like point-like because our experiments are not accurate enough to detect a structure?

There are theoreticians working on the subject, but there are no results.

Question 6 – Why are there six quarks and six leptons? There are no fundamental principles that lead to the prediction of six quarks and six leptons.

1.4 Mass without mass

Masses of all bodies come essentially from the masses of protons and neutrons in the atomic nucleus. Protons have two quarks u whose mass is 4 MeV and one quark d whose mass is 8 MeV; neutrons have two quarks d and one quark u. The quark contribution to the proton mass is therefore 16 MeV and to the neutron mass is 20 MeV. However, proton and the neutron masses are much larger, 940 MeV. Therefore quark masses contribute to only 2% of the proton and neutron masses; there are 98% of mass missing.

Question 7 – Where do the 98% of the proton and neutron masses come from? They come from energy, but we do not know the physical process.

This applies to all bodies. For example a person whose mass is 70 kg has only 1.4 kg of quarks; 68.6 kg come from energy.

1.5 The colour field of force

There is experimental evidence that there are three types of each quark, i.e. there are three quarks u, three quarks d, three quarks s, and so on. This means that there are three fields of force for the quarks. These fields are called *colours*. We can give them the names of any three colours; we shall call red, blue and green.

When quarks stick together to form particles the colours neutralise each other and the particle is without colour, it is *white*. Examples:

proton = green quark + red quark + blue quark form a white proton
π+ = green u + green antid, or red u + red antid, etc form a white π+

Question 8 – Why are there the three colour fields of quarks?

1.6 The intermediate bosons in the interactions

The four interactions occur via the exchange of spin 1 particles called *intermediate bosons* of the interactions. They are:

interaction	intermediate boson
strong	gluons
weak	W^+ W^- Z^0
electromagnetic	foton
gravitational	graviton

The name *gluon* comes from *glue*. There are 8 gluons, and each gluon has two colours. The masses of the intermediate bosons are (we give also the proton mass for comparison):

$$m_{foton} = m_{gluon} = m_{graviton} = 0$$
$$m_W = 80{,}4 \text{ GeV} \quad m_Z = 91{,}2 \text{ GeV} \quad m_{proton} = 0{,}94 \text{ GeV}$$

When two quarks interact one of them emits a gluon that is captured by the other. When two electric charges interact one of them emits a foton that is captured by the other. In the same way in the weak interactions there are exchanges of W^+, W^- or Z^0.

1.7 The three families of quarks and leptons

Quarks and leptons appear in three families, given in table 1. Quarks and leptons interact only with members of the same family. For example, e^- and ν_e interact only with quarks u and d and do not interact with other quarks; μ^- and ν_μ interact only with quarks s and c; τ^- and ν_τ interact only with quarks b and t.

Table 1 represents a summary of the components of matter existing in the Universe and their interactions: six quarks and six anti-quarks with three colours, six leptons and six anti-leptons, the four interactions and the intermediate bosons.

Table 1 -
Summary of particles and interactions in the Universe

leptons	quarks (3 colours)			family
	red	blue	green	
ν_e	u	u	u	1
e^-	d	d	d	
ν_μ	s	s	s	2
μ^-	c	c	c	
ν_τ	b	b	b	3
τ^-	t	t	t	

there are also: 6 anti-quarks, 6 anti-leptons

intermediate bosons: foton, gluon, graviton, W^+ W^- Z^0

four interactions: strong, weak, electromagnetic, gravitational

Question 9 – Why are there three selective families of quarks and leptons?

1.8 Unification of the theories of weak and electromagnetic interactions

Dirac made a quantum electromagnetic interactions theory and Fermi made the weak interactions theory, both in the 1930s. Many physicists, among the greatest of the 20th century, have thought of unifying the theories of the four interactions in one single theory. There was no possibility of making any unification before we knew the existence of quarks, which have been introduced by Gell-Mann and Zweig in 1962. After about twenty years of efforts the unification of the theories of weak and of electromagnetic interactions was made, mainly due to the works of Glashow, Illiopoulos, Maiani, Salam and Weinberg. It is called *electro-weak theory*.

In the electro-weak theory, an electromagnetic interaction with the exchange of a foton and a weak interaction with the exchange of a Z^0 are treated with the same mathematical formalism, as if they were a single phenomenon.

1.9 The Standard Model

The Standard Model is a theoretical model for three interactions, strong, weak and electromagnetic, the last two being unified into the electro-weak theory. It is based on the existence of quarks, anti-quarks, the three colour fields, leptons, anti-leptons, the three families of particles, the intermediate bosons. The underlying theories are quantum mechanics and relativity. However it does not come from some fundamental principles; it was developed gradually, with new parts being added as new experimental results were obtained.

The Standard Model cannot deal with gravitational interactions. In fact there is no theory that can include gravitational interactions with the other three. This is a great challenge of physics. A great difficulty arises because we do not know how to introduce quantum mechanics into gravity.

Question 10 – Is it possible to make a unified theory of the four interactions?

The Standard Model is mathematical coherent, is extremely useful because it has a great predictive power. Its predictions are always confirmed by experiments. Examples: the existence of the charm quark was predicted before it was found experimentally; the model predicted for the top quark mass 174 GeV, the experimental value is 175 GeV; the model predicted the ratio $M_z / M_w = 1.13$, which agrees with experiment.

The Standard Model has, however, a great problem: it has to assume total symmetry for particles masses and that all masses are zero. This is not our real world in which particles have no zero mass and have different masses.

Question 11 – How to explain that particles masses are not zero?

1.10 Higgs field and particle

A proposal to explain no zero values of particle masses is that perhaps there is a field that fills matter and interacts with particles when they are created and the interaction produces the masses. It is called *Higgs field*. This assumption has not been proved.

A field has always a particle associated to it. If the Higgs field exists there must exist a particle associated to it, called *Higgs particle*. Much theoretical work has been done about possible properties of this particle. All experiments that will be performed with the future Large Hadron Collider will look for Higgs particle.

1.11 Are some fundamental principles missing?

The questions without answers show that the physics we know is not enough to explain the phenomena that we know. We can ask if in order to explain nature there are not some fundamental principles unknown to us. It is probable. However if such principles exist we do not know where to look for them.

If fundamental principles are missing they might be discovered in some contradictions found in the study of some process. For example, the experiments to be done with the Large Hadron Collider have been carefully planned to detect processes that exist or should exist; they would give an important contribution to physics if they find some surprise, some unexpected type of events whose detection has not been predicted.

Part 2 – Questions without answers in cosmology and astrophysics

The absence of anti-matter in the Universe, dark matter, gravitational waves and extremely high energy cosmic rays are amongst the most important unsolved questions in cosmology and astrophysics.

2.1 Absence of anti-matter in the Universe

To each particle corresponds an *anti-particle* of equal mass and electric charge of opposite sign. The existence of anti-particles, i.e. anti-matter, was theoretically predicted by Dirac in 1928. The experimental discovery was made in 1932 by Anderson, who detected the *anti-electron*, or *positron*, in a cosmic ray experiment. Particles and anti-particles had been detected in the production of mesons in the 1950s, but a great sensation was produced by the discovery of the anti-proton in 1954 by Chamberlain, Segré, Wiegand and Ypsilantis. The first atom of anti-matter produced in laboratory was the anti-hydrogen, composed of an anti-proton and a positron, detected at CERN in 2000.

When matter and anti-matter collide they annihilate each other and their masses are converted into energy.

A great difficulty to make assumptions about the origin of the Universe is that we do not know if the laws of physics at that time were the same laws we know today. We have to assume that they were the same, because in the absence of any signs of how they were each one could make his own assumptions about the laws and make his own interpretation of the origin of the Universe.

According to the Big-Bang model in the beginning of the Universe matter and radiation existed at extremely high temperatures and densities. The Universe expanded and matter cooled down. We believe that the four forces were unified, i.e. the strong, weak, electromagnetic and gravitational forces had the same intensity. In the first second quarks, protons and neutrons were formed. After three minutes deuterium, helium and other light nuclei were formed. After about one million years protons, neutrons and electrons were at temperatures low enough to stick together and form atoms. After some million years stars and galaxies were formed.

Whatever assumption we adopt about the origin of the Universe we must admit that equal amounts of matter and of anti-matter have been formed. They should have annihilated each other. However, even if some part of matter was annihilated there was a predominance of matter at the end because in the Universe there is matter and no anti-matter. Here we have another question.

Question 12 – What happened to the initial anti-matter of the Universe?

Dirac made the assumption that perhaps there is a Universe of anti-matter. A proof of this assumption would be, for example, the collision of a galaxy of matter with a galaxy of anti-matter, which would give an extraordinary explosion.

Experiments are being made to try detecting anti-matter in the Universe. They are made with counters sent at high altitudes in balloons with electronics controlled by radio. A different and audacious experiment is being made with a complete spectrometer to detect particles, with the dimensions of the spectrometers used in experiments with accelerators, including a magnet, sent in a satellite. It is called Alpha Magnetic Spectrometer – AMS. We are waiting for results.

An explanation for the absence of anti-matter could be a possible breaking of symmetry in laws of physics. We can apply to a state of a particle the parity operation P that changes the signs of the space-time coordinates, and the charge conjugation operation C that transforms a particle into its anti-particle. The two operations applied successively, CP, change a particle into its anti-particle as well as the signs of the coordinates. A great violation of P and C when applied separately was discovered in 1960s but the product CP was supposed to be a good symmetry, not violated. Cronin and Fitch detected in an experiment in 1964 a small violation of CP in the decay of neutral K mesons. A violation of CP is also expected in the decay of mesons containing the b quark.

If there is violation of CP in many events perhaps in the beginning of the Universe matter and anti-matter have not been produced in equal amounts. In this case our assumption of equal production of matter and anti-matter would not be correct and the problem would be solved. However a large violation of CP would be needed.

Experiments to be performed with the accelerator LHC at CERN will look for CP violation in some processes.

2.2 Dark matter

Galaxies are visible because they emit light. Galaxies have huge masses and move in space attracted by the gravitational forces that exist between them. In 1930 the Swiss astronomer Fritz Zwicky showed that the gravitational forces between galaxies are too small to account for their observed velocities. He concluded that there must be some invisible matter attracting the galaxies: it is the *dark matter*, which does not emit light. The young American astronomer Vera Rubin showed around 1960 that most of the stars of spiral galaxies move with nearly the same velocity; she concluded that this could be explained if there is some invisible matter to accelerate the stars. Her work was criticised, she was even ridiculed, until it was shown that she was right. There is no doubt about the existence of dark matter.

This is one of the great problems of cosmology because dark matter contains 90% of matter in the Universe: the visible galaxies contain only 10%.

There are two types of assumption about the composition of dark matter. One type of assumption is that dark matter would be some kind of *normal* matter; the other is that it would be some *special* matter not yet detected. Normal matter could be neutrinos, dwarf stars, matter from planets, like rocks or dust, or some super-symmetrical particle (which was never detected in accelerators experiments). As special matter have been proposed other types of neutrinos with mass and particles called WIMP and MACHO.

WIMPs – Weak Interacting Massive Particles – would be large particles with dimensions of atoms produced in large numbers. They have not been found in experiments.

MACHOs – Massive Compact Halo Objects – would be of large dimensions, for example 10% of the mass of the Sun. There is only one way to detect them: by their gravitational effect on the light of stars whe*n we are observing*. They should attract light and bend light rays, as it occurred at the Sun eclipse observed in 1919 in Sobral, in Ceará, which proved the prediction of the theory of relativity that large masses like the Sun mass attract light. Their detection is very difficult; a MACHO with mass equal to 10% of the Sun mass would be produced only once in 50000 years. They have not been detected.

Experiments to be performed at the LHC accelerator at CERN will look for WIMPs and MACHOs.

A surprising result was recently obtained with an unexpected method, the detection of electromagnetic waves in the radio wave lengths. A group of Australian, British, French and Italian astronomers have discovered in 2005 with the Radio Telescope of the Manchester University electromagnetic waves with large energy emitted by a galaxy near our galaxy. Such large energy waves that are not visible could be a type of dark matter. If this result is confirmed it could be the first evidence for dark matter in the Universe.

2.3 Gravitational waves

In Newton's theory of gravitation the forces between masses would occur at distance. Einstein's theory of relativity predicts without ambiguity the existence of gravitational waves that would transmit the forces. Gravitational waves should be produced by large masses moving at high velocities; for instance binary neutron stars or black holes. Waves produced by the system Earth-Moon would not been detectable because the masses and the velocities are not large enough.

One of the consequences of gravitational waves is that they would produce variation of distances in all places in Earth. This variation would be extremely small, of the order of one thousandths of the proton diameter in a distance of 4 km, making their detection extremely difficult.

Several experiments to try detecting gravitational waves are working or beginning to work:

In INPE in São José dos Campos, Brazil, with large spheres

LIGO	United States
GEO 600	Germany
VIRGO	France-Italy (installed in Italy)
TAMA	Japan
LISA	Europe-United States with satellite

The last five are made with Michelson interferometers with two perpendicular arms with several kilometres each to detect small variations in distances.

2.4 Extremely high energy cosmic rays

Cosmic rays with extremely high energies, 10^{21} to 10^{22} eV have been detected. For comparison, the largest particle accelerator, LHC of CERN, will produce two proton beams with 7×10^{12} eV each. The collision of two beams will have 14×10^{12} eV, one billion times smaller than the energy of those cosmic rays. The existence of such cosmic rays arises few questions:

Questions 14

- what is the nature of these cosmic rays, i.e. which type of particles are they?
- which parts of the Universe do they come from?
- what is the physical process that accelerates them to such high energies?

There are several experiments being made in different parts of the world. They are very difficult because the flux of those rays is extremely small, on the average one particle per square kilometre per year. The Auger experiment being made in Argentine by a large international collaboration has detectors covering an area of 3600 square kilometres.

A Brazilian group works in the Auger Experiment with members from Unicamp, Centro Brasileiro de Pesquisas Físicas, Federal Fluminense University, Federal University of Rio de Janeiro, Pontifical Catholic University of Rio de

Janeiro, University of São Paulo, State Universities of Feira de Santana and Sudoeste of Bahia, Federal Universities of Bahia and ABC, under the coordination of Carlos Escobar from Unicamp. The Brazilian contribution to the experiment is on parts of the detectors made by Brazilian industry and on data analyses.

An outstanding result of the experiment was obtained on the days this article was being written. It detected as origin of extremely high energy cosmic rays the active nucleus of a galaxy near our galaxy. If confirmed it will be one of the great discoveries of our time.

At present we have no ways to solve some of the 14 questions mentioned in the paper.

Superfluidos atômicos e formação de vórtices por oscilações: observações preliminares

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Apresentamos a produção e estudo de excitações topológicas em um condensado de Bose-Einstein em átomos de Rubídio-87. O condensado é produzido através de resfriamento evaporativo forçado por rádio-freqüência em uma armadilha puramente magnética do tipo QUIC. Essa armadilha é carregada por um sistema de duplo-MOT. A temperatura de transição é de cerca de 150 nK. Observamos condensados puros de ^{87}Rb com 1-2×10^5 átomos. Realizamos uma caracterização da amostra em relação às suas características fundamentais.

Com o objetivo de observar excitações coerentes do condensado entre os estados da armadilha, adicionamos um campo magnético do tipo quadrupolo esférico oscilante no tempo. Observamos, no entanto, a transferência de momento angular para a amostra com a formação de vórtices e arranjos de vórtices. Definimos regiões de amplitude que geram números de vórtices crescentes. Também observamos a formação de estruturas de três vórtices não convencionais donde supusemos a possibilidade de excitação conjunta de vórtices e anti-vórtices. Observamos evidência de turbulência quântica, um estado onde os arranjos dos vórtices não são regulares nem as linhas de vórtices têm um eixo de rotação comum.

1. Introdução

Condensação de Bose-Einstein, em inglês Bose-Einstein Condensation ou BEC, é uma história que começa em 1925 com os trabalhos seminais de Bose[1] para fótons e a generalização feita por Einstein[2] para partículas massivas. Uma história que passa pela justificativa da superfluidez do Hélio líquido[3], mesmo que a teoria original não consiga ser aplicada completamente ao Hélio devido à forte correlação do sistema. Uma história que tem contribuições pontuais e predominantemente teóricas ao longo dos 70 anos seguintes, costumeiramente voltadas ao próprio Hélio líquido ou a sistemas de física nuclear. A condensação de Bose-Einstein como fenômeno físico antes de 1995 era quase apenas um objeto mais imaginário e menos controlado como um laboratório de experimentação. A condensação de Bose-Einstein como objeto de estudo científico torna-se realidade em 1995, e uma realidade impressionante. Os trabalhos de E. Cornell[4], W. Ketterle[5] e R. Hulet[6], que independentemente e quase simultaneamente observaram condensação em amostras gasosas de átomos alcalinos aprisionados, são o marco da transição de uma fase para a outra. Com a possibilidade de realização experimental deste fenômeno, ocorre um desdobramento de trabalhos, atingindo praticamente todos os campos da física. Dentre estes campos esta o da superfluidez. Originalmente observado em Hélio abaixo da transição Lambida, a superfluidez sempre foi associada com sistemas quânticos de muitos corpos, e, portanto seu entendimento ficou atrelado a várias áreas como teoria de campos, física nuclear, dentre outras. A produção dos condensados atômicos, teve uma rota interessante e cheia de desenvolvimentos. Esses desenvolvimentos incluem técnicas de resfriamento de átomos a laser e do subseqüente confinamento desses átomos em armadilhas feitas essencialmente de luz e/ou campos magnéticos[7,8,9]. Desenvolvimentos que permitiram um melhor entendimento das interações desses átomos entre si e com a luz[10]. Finalmente, o desenvolvimento da técnica de resfriamento evaporativo[11] somada às outras já mencionadas e a lapidação de todas juntas tornou possível à obtenção de amostras gasosas em regimes nunca antes observados. Amostram tão próximas ao zero absoluto que todas as partículas acumulam-se no estado de mais baixa energia do sistema, onde todas as partículas são descritas pela mesma função de onda, onde todas juntas comportam-se como uma única partícula e uma partícula não pode mais ser descrita independentemente das outras e nem mesmo como partícula. Amostras em escala humana que se comportam segundo as leis microscópicas da física quântica. Amostras de condensados de Bose-Einstein. Incontáveis fenômenos foram explorados à exaustão.

A equação de Gross-Pitaevskii, que corresponde ao modelo básico que descreve esse tipo de sistema, foi testada em muitos dos seus aspectos e sempre se mostrou bem sucedida. Em paralelo, novos sistemas surgiam e com eles, novas descobertas e áreas de interesse. Até hoje, diferentes espécies atômicas bosônicas foram levadas à degenerescência quântica e cada uma delas representa de certa forma, um avanço diferente na área de átomos frios e, sem dúvidas, o estado da arte até aquele momento nas técnicas utilizadas para sua observação. Cronologicamente, $^{87}\mathbf{Rb}$[4], $^{23}\mathbf{Na}$[5] e $^{7}\mathbf{Li}$[6] em 1995, Hidrogênio[12] em 98, $^{85}\mathbf{Rb}$[13] no ano 2000, $^{4}\mathbf{He}$[14,15] e $^{41}\mathbf{K}$[16] em 2001 e $^{131}\mathbf{Cs}$[17] em 2003. Ainda em 2003, $^{174}\mathbf{Yb}$[18] inaugura as espécies não-alcalinas. Em 2005 $^{52}\mathbf{Cr}$[19], abrindo um novo campo de gases degenerados com interações dipolares significativamente grandes. Finalmente, em 2007, $^{170}\mathbf{Yb}$[20] e $^{39}\mathbf{K}$[21] fecham a lista. Mas os sistemas quanticamente degenerados não se restringem a bósons. Férmions também foram resfriados até a degenerescência quântica[22] e combinações de férmions em "moléculas" bosônicas[23] tiveram o mesmo destino.

Em São Carlos, no IFSC/USP, a tradição de estudos com átomos frios, evoluiu para os estudos de BEC, e um condensado de Na foi produzido em 2004[24,25], mas obtivemos um baixo número de átomos e uma reprodutibilidade afetada por problemas técnicos. Modificamos nosso sistema e obtivemos um condensado de

Rubídio em 2007[26], desta vez com maior potencialidade de estudos. Dentre os estudos que iniciamos esta a investigação da formação de vórtices no superfluido atômico, e a possível rota para um regime de turbulência na amostra quântica, que, portanto pode ser chamada de Turbulência Quântica.

No que segue, na seção II teremos uma descrição do aparato experimental para a obtenção do condensado de Rb e nas seções III e IV, iremos descrever observações com relação à formação dos primeiros vórtices com indicativos da evolução para o regime de turbulência quântica.

2. Breve descrição da obtenção experimental do condensado brasileiro de Rubidio

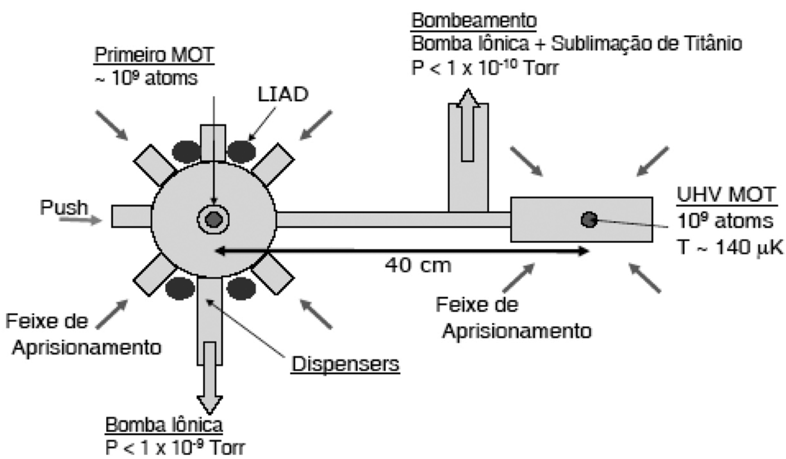
Até o momento, a única técnica experimental reconhecidamente bem sucedida na produção de um gás de bósons quanticamente degenerado chama-se resfriamento evaporativo[11]. Em linhas gerais, essa técnica consiste em retirar seletivamente os átomos mais quentes de uma amostra aprisionada, favorecendo a retermalização em uma temperatura mais baixa e assim continuamente até a observação da condensação.

Há algumas condições essenciais para que esta técnica seja bem sucedida. É necessário que a amostra atômica esteja bem fria, pois esse processo tem temperaturas finais abaixo de 1μK. No processo de resfriamento evaporativo há também uma perda inerente e significativa de átomos ($N_{final} \approx 0.01 - 0.001 N_{inicial}$). Assim, o número absoluto de átomos deve ser grande devido a essa perda. Finalmente, a amostra deve ser significantemente densa para que o processo de retermalização, totalmente devido às colisões elásticas, seja eficiente e rápido, mas não tão densa de forma a promover uma taxa de colisão de três corpos elevada e que invariavelmente induz perda de átomos da armadilha. Quanto ao “ambiente” externo à armadilha, esse deve ser suficientemente limpo, ou seja, livre de vapor de fundo. Dessa forma, as colisões entre o vapor de fundo e os átomos aprisionados se dão a uma taxa baixa o suficiente para que os átomos aprisionados tenham tempo de retermalizar continuamente à medida que o resfriamento evaporativo avança.

Assim, para a realização de resfriamento evaporativo em uma armadilha puramente magnética como a do sistema experimental descrito aqui, devemos ser capazes de acumular cerca de 10^9 átomos a uma densidade de $10^{10}\,cm^{-3}$ a $10^{11}\,cm^{-3}$ a temperaturas da ordem de 200 μK em uma câmara mantida a uma pressão de cerca de 10^{-11} Torr. Em um sistema de vácuo simples, com uma câmara preenchida por vapor de Rubídio, essas condições são completamente incompatíveis entre si. Se tivermos, por exemplo, uma pressão da ordem de 10^{-11} Torr, então o número de átomos com temperatura abaixo de $1mK$ em $1cm^3$ é aproximadamente de 10^3. Se tivermos, ao contrário, cerca de 10^8 átomos/cm^3 (equivalente a uma pressão parcial de Rubídio de $\approx 10^{-6}$ Torr) nessas mesmas condições ($T < 1mK$), a taxa de colisão dos átomos aprisionados com os não aprisionados será tão grande que o tempo médio de permanência na armadilha será menor que 1 s.

Diversas estratégias são usadas para unir, numa mesma câmara de vácuo, um grande número de átomos aprisionáveis com uma pressão de fundo baixa. Uma das principais técnicas consiste em unir duas câmaras de vácuo por um fino tubo que permite uma diferença de pressão considerável entre ambas (algumas ordens de grandeza).

Figura 1:
Esquema simplificado do sistema de vácuo mostrando as duas câmaras de aprisionamento, tubo de transferência e saídas de bombeamento, bem como alguns números típicos.



Uma das câmaras é preenchida por vapor de Rubídio e apenas átomos previamente resfriados são enviados à outra câmara, possibilitando, assim, o acúmulo de um grande número de átomos na armadilha da segunda câmara sem a presença de vapor de fundo. A variante desta técnica usada neste trabalho chama-se duplo-MOT, e consiste em aprisionar átomos na primeira câmara em uma armadilha magneto-óptica (MOT- em inglês magneto optical trap) e empurrá-los continuamente com um feixe laser através do tubo de transferência para a outra câmara onde acumulam-se em um outro MOT, agora em um ambiente de alto-vácuo. Uma vez acumulados no MOT em ambiente de alto-vácuo, os átomos são transferidos para uma armadilha puramente magnética onde se dá o resfriamento evaporativo.

Nas subseções a seguir, descrevemos brevemente o sistema de vácuo, as armadilhas magneto-ópticas e o processo de transferência. Descrevemos ainda o sistema de lasers usado para produzir as armadilhas e todas as outras freqüências usadas no experimento. Em seguida, descrevemos as técnicas usadas para transferir os átomos para a armadilha magnética, bem como o processo de resfriamento evaporativo e seus resultados que nos levam a observar a degenerescência quântica.

Uma descrição mais detalhada de nosso sistema pode ser encontrada na Tese de dotorado de Emanuel Henn[27] e no artigo da Brazilian Journal of Physics[26].

2.1- Sistema de Vácuo

A Figura1 mostra uma visão geral do sistema de vácuo utilizado. A primeira câmara é em forma de balão, feita de vidro e possui oito janelas e duas saídas com flanges metálicas. Uma dessas saídas conecta-se ao tubo de transferência e a outra ao sistema de bombeamento e à fonte de átomos. Com o tubo de transferência ligando as duas células de vidro, um bombeamento diferencial possibilita uma pressão no lado de alto-vácuo da ordem de 5×10^{-11} Torr.

A célula de alto vácuo mede 6 *cm* de comprimento e tem seção quadrada de 30 $\times$ 30 mm^2, e possui alta qualidade óptica em toda a sua extensão. Ela se liga à parte flexível do tubo de transferência por uma conexão tipo-T. Da porta livre ramificam-se uma série de conexões onde são ligados um medidor de vácuo, uma bomba iônica, uma bomba de sublimação de Titânio e uma válvula de selo metálico.

Como indicado na Figura 1, temos na primeira célula de vidro um conjunto de *dispensers* (SAES Getters), os quais são pequenos reservatórios que contém um cromato de Rubídio e uma liga metálica como agente redutor. Quando uma corrente elétrica atravessa o *dispenser*, Rubídio gasoso é liberado, restando no reservatório apenas os outros elementos. Este tipo de fonte de átomos é extremamente limpo, gerando apenas Rubídio gasoso a um fluxo bastante reprodutível quando operado sob a mesma corrente elétrica. Algumas semanas de operação são necessárias para se observar diminuição desse fluxo, que pode ser corrigido aumentando-se ligeiramente a corrente de operação. Tipicamente, a emissão de Rubídio ocorre para correntes acima de 3A e sua operação contínua pode ser feita até correntes pouco acima de 6A. Além dessas correntes, só é possível o uso pulsado como forma de preservar o vácuo existente.

Nas nossas primeiras experiências com *dispensers*, estes permaneciam ligados durante todo o tempo de operação do experimento. Dessa forma, a primeira câmara de vapor permanecia recebendo vapor de Rubídio durante os processos de transferência de átomos entre as câmaras, mas também durante o período de armadilhamento magnético e resfriamento evaporativo. Isto permitia uma transferência de átomos eficiente entre as câmaras, mas, devido ao grande volume de vapor não aprisionado na primeira câmara e da posição das bombas iônicas na segunda câmara, não tão favorável, sempre havia algum vapor "quente" presente na segunda câmara e isso prejudicava sensivelmente o tempo de vida dos átomos na armadilha magnética, cerca de $1/e = 6s$, inviabilizando o resfriamento evaporativo, que, em nosso sistema, demanda cerca de 30 s.

Para resolver esse problema, usamos a técnica chamada LIAD, acrônimo inglês para "dessorção atômica induzida por luz" (*Light-Induced Atom Desorption*).

LIAD consiste em usar luz para induzir a desorção de átomos adsorvidos às paredes de uma câmara de vácuo e aumentar significativamente a pressão parcial desse átomo dentro da câmara. A vantagem dessa técnica reside no fato de que a pressão parcial decai muito rápido assim que a luz de desorção cessa, permitindo conjugar uma grande quantidade de vapor de fundo e um bom vácuo em instantes subsequentes de tempo. Juntamente à técnica de aprisionamento de átomos na superfície de *chips*[28], LIAD foi usada para a produção de condensação de Bose-Einstein em um sistema de vácuo portátil[29], composto por uma única câmara de vácuo. Em nosso sistema, posicionamos seis LEDs que geram luz UV ($\lambda \approx 400nm \pm 5nm$) ao redor da primeira câmara de vácuo.

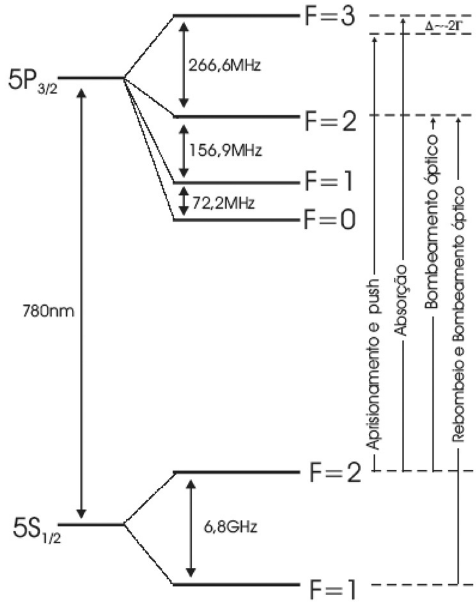
O vapor de fundo gerado na primeira célula é muito maior que o obtido com o uso de *dispensers*, permitindo que a carga do primeiro MOT e, por conseguinte, do segundo MOT, seja muito maior. Adicionalmente, findo o processo de carga do segundo MOT e desligada a luz UV, o vapor de fundo desaparece quase que instantaneamente, permitindo tempos de vida dos átomos na armadilha magnética muito superiores, cerca de 33 s.

Nesse contexto, os *dispensers* são usados apenas para refazer, periodicamente, o filme de Rubídio que recobre as paredes da primeira célula.

2.2- Sistema de Lasers

O sistema de lasers deste experimento é composto por três lasers de diodo de alta potência - Toptica; DLX110 (1) e DLX110L (2). Os lasers são travados individual e ativamente em uma transição hiperfina apropriada do átomo de ^{87}Rb, observada por espectroscopia de absorção saturada em células de vapor de Rubídio. A largura de linha dos lasers é da ordem de 1 *MHz*.

Figura 2:
Estrutura de níveis do átomo de ^{87}Rb relevantes ao experimento e freqüências laser utilizadas.



As diversas freqüências utilizadas, juntamente com a estrutura de níveis relevantes do átomo de Rubídio-87 são mostradas na Figura 2.

O laser de rebombeio (Toptica - DLX-110) gera luz que conecta o estado $5S_{1/2}$ (F = 1) ao estado $5P3/2$ (F' = 2) e tem papel fundamental no aprisionamento dos átomos, tanto no MOT quanto na armadilha magnética. Esse laser é sobreposto a um outro feixe ressonante com $5S_{1/2}$ (F = 2) $\rightarrow$ $5P_{3/2}$ (F' = 2) e inserido numa fibra mantenedora de polarização. As duas freqüências sobrepostas fazem parte do processo de bombeamento óptico dos átomos, processo utilizado para preparar os átomos para a armadilha magnética.

Esse feixe é então dividido em dois. Um feixe é sobreposto ao feixe de aprisionamento do primeiro MOT e segue para um estágio de expansão, de onde então é dirigido à região da armadilha. O segundo feixe faz um caminho semelhante. É sobreposto à luz de aprisionamento do segundo MOT, passando por um *pinhole* de 50μm para limpeza de seu modo espacial. Os feixes são então expandidos e seguem para a região de aprisionamento do segundo MOT.

O laser que gera a luz de aprisionamento do segundo MOT (TOPTICA - DLX110L) é também o que gera o maior número de freqüências usadas no experimento. A luz de bombeamento óptico, sintonizada em $5S_{1/2}$ (F = 2) $\rightarrow$

$5P_{3/2}$ (F' = 2), a luz de prova, em $5S_{1/2}$ (F = 2) $\rightarrow$ $5P_{3/2}$ (F' = 3) e as freqüências do feixe de *push*, que empurra os átomos de uma armadilha à outra, e do próprio aprisionamento, sintonizadas para o vermelho ($\approx -20MHz$) da transição $5S_{1/2}$ (F = 2) $\rightarrow$ $5P_{3/2}$ (F' = 3).

O laser de aprisionamento do primeiro MOT (Toptica - DLX110L) é dos três o de configuração mais simples. A luz é enviada ao ponto onde é sobreposta à luz de rebombeio e em seguida sofre expansão, sendo ambos enviados à região do primeiro MOT.

2. 3- Transferência de átomos entre as duas armadilhas magneto-ópticas.

A idéia geral por trás do funcionamento de armadilhas magneto-ópticas (MOT, do inglês *Magneto Optical Trap*) é simples e baseia-se na idéia de que a luz faz força pela transferência de momentum em um ciclo de absorção-emissão de um fóton por um átomo, a chamada força de pressão de radiação[30,31]. Desde sua demonstração experimental[32], MOTs têm sido o principal instrumento de trabalho de amostras ultra frias de átomos, não só como estágio de pré-resfriamento em experimentos de condensação, mas também no estudo de colisões atômicas, interação luz-matéria, etc.

Por causa do espalhamento não-ressonante de fótons, e da existência de outros estados que não apenas o fundamental, sempre há a possibilidade de o átomo cair para um outro estado, saindo de ressonância o feixe laser. Para evitar esse problema, adiciona-se uma outra freqüência, chamada de rebombeio, ressonante com o estado de "fuga" e um outro estado superior qualquer que se conecte com o estado original da freqüência de aprisionamento. Como já mostrado anteriormente, para o átomo de ^{87}Rb, o laser de aprisionamento é sintonizado a cerca de -20MHz (-3Γ, onde Γ é a largura de linha da transição) da transição $5S_{1/2}$ (F = 2) $\rightarrow$ $5P_{3/2}$ (F'= 3) e o laser de rebombeio é sintonizado ressonante com a transição $5S_{1/2}$ (F = 1) $\rightarrow$ $5P_{3/2}$ (F = 2).

Em nosso sistema, temos dois MOTs. O primeiro é formado por três feixes independentes retrorefletidos, formando assim os três pares necessários para a armadilha. O gradiente de campo magnético é cerca de 20 Gauss/cm. Três pares de bobinas Helmholtz, nas três direções do espaço, contribuem para o posicionamento do MOT com relação ao feixe que empurra os átomos desta câmara para a outra. De fato, todos os parâmetros desta armadilha, tal como o gradiente de campo, o alinhamento dos feixes e sua potência relativa são ajustados de forma a maximizar a transferência dos átomos de uma armadilha para a outra.

O segundo MOT é mais delicado e por isso os seis feixes são independentes. Seu alinhamento é finamente ajustado, sua potência é cuidadosamente balanceada e sua polarização também recebe muito mais atenção. As bobinas em configuração Helmholtz, nesse caso, são usadas para zerar ou diminuir campos espúrios na região de aprisionamento.

A avaliação desses parâmetros pode ser feita tanto se observando a transferência dos átomos para a armadilha magnética, como também se desligando o campo magnético do MOT e observando-se a expansão dos átomos. Em situação ideal, a nuvem expande-se isotropicamente e um melaço óptico grande e razoavelmente denso toma o lugar do MOT, apesar de nesse caso os átomos não estarem aprisionados.

O MOT da primeira câmara de vácuo é mais bem definido como um "MOT de sete feixes", onde o sétimo feixe é o feixe de *push*. A definição se justifica pelas características típicas do feixe que empurra os átomos de uma câmara para a outra: sua intensidade, polarização e freqüência são compatíveis com as dos feixes de aprisionamento.

Especificamente, o feixe de *push* é inserido na câmara de vácuo com 800 μW de potência, circularmente polarizado, e levemente focalizado cerca de 1 cm antes da primeira armadilha. A focalização permite que o feixe atinja o primeiro MOT com uma intensidade grande, a fim de empurrar os átomos para a outra armadilha, mas chegue à região do segundo MOT (40 cm à frente) com um diâmetro maior e, por conseguinte, menor intensidade, não sendo capaz de perturbá-lo. O alinhamento do primeiro MOT e do feixe de *push* é feito em conjunto de forma a maximizar a transferência de átomos para a segunda armadilha.

Cerca de 5×10^8 átomos são aprisionados em cerca de 30 s, o que permite estimar uma taxa aproximada de $1\text{–}2\times10^7$ átomos/s entre as duas câmaras.

Quando o feixe de *push* é desligado, observamos o tempo com que o número de átomos diminui, observando a fluorescência da amostra. Quanto mais longo esse tempo, melhor é o vácuo na região da armadilha, pois as colisões dos átomos aprisionados com os átomos não aprisionados é um dos fatores que força a descarga da armadilha. De fato, o tempo de vida na armadilha magnética será de 2 a 3 vezes o tempo de vida do MOT.

2. 4- Transferência para Armadilha Magnética

O aprisionamento magnético de átomos neutros só é possível em átomos com momento de dipolo magnético permanente em seu estado fundamental, como é o caso dos átomos alcalinos. A energia de interação entre um dipolo magnético μ e um campo magnético B é dada por $V = -\mu \cdot B$. O momento de dipolo magnético em um átomo alcalino é dado por $\mu = \mu_B g_F F$, onde $\mu_B = 9.27x10^{-24} J/T$ é o magnetón de Bohr, g_F é o fator giromagnético de Landé e F é o momento angular atômico total, composto pelos momentos angulares eletrônico $J = L + S$ e nuclear I.

No caso do átomo de ^{87}Rb, $I = 3/2$ e, no estado fundamental $5S_{1/2}$, $J = 1/2$, o que dá origem aos estados hiperfinos F=1 e F=2. Assim, a energia de um estado hiperfino do estado fundamental do átomo de Rubídio é dada por

$$V_{F,m_F} = -\mu_B g_F m_F \left|\vec{B}\right|$$

onde m_F é a projeção do momento angular atômico na direção do campo magnético. Para o estado F=1, $g_F = -1/2$ e $m_F = -1, 0,1$ e para o estado F=2, $gF = 1/2$ e $mF = -2, -1, 0, 1, 2$.

Chamando esses estados de $\left|F, m_F\right\rangle$, os estados $\left|2{,}2\right\rangle$, $\left|2{,}1\right\rangle$ e $\left|1{,}-1\right\rangle$ são chamados de *low-field seekers*, pois sua energia cresce com o campo magnético. Esses estados também são os chamados "estados aprisionáveis", haja vista, num espaço livre de correntes, somente ser possível obter configurações com mínimos locais de campo magnético.

Esta é uma das limitações intrínsecas às armadilhas puramente magnéticas: nem todo estado é aprisionável. Assim, para transferir uma fração considerável de átomos do MOT para a armadilha magnética é necessário um processo que deixe a amostra spin-polarizada. Em nosso experimento, aplicamos o chamado bombeamento óptico antes de transferir os átomos para a armadilha magnética.

Veja ainda que, para campos máximos típicos produzidos em laboratório ($\approx$ 100 *Gauss*), a barreira de potencial ainda é baixa para o aprisionamento dos átomos, permitindo apenas o confinamento de átomos com temperaturas menores que 1 *mK*.

Além das restrições do processo de evaporação a altas temperaturas iniciais, a armadilha magnética também não permite o aprisionamento de átomos sem o devido pré-resfriamento realizado em um MOT. Esse pré-resfriamento consiste basicamente do resfriamento sub-Doppler.

Utilizamos em nosso sistema, a configuração de bobinas chamada QUIC, em inglês Quadrupole Ioffe Configuration. Essa Configuração de bobinas para armadilhamento magnético foi primeiramente proposta por Esslinger *et al.*[33]. É uma das mais simples que permitem suprimir o canal de perdas da armadilha de quadrupolo. Para isso, adiciona-se às duas bobinas de quadrupolo apenas uma terceira bobina com eixo perpendicular ao eixo de simetria do quadrupolo.

Em linhas gerais, uma vez que os átomos são transferidos para a configuração final da armadilha de quadrupolo, faz-se passar corrente na bobina de Ioffe, aumentando-a continuamente ao longo de 800 ms. Inicialmente, o mínimo de campo onde os átomos estão confinados move-se na direção da bobina e um segundo mínimo de campo surge próximo a esta. Aumentando-se mais a corrente os mínimos se aproximam ainda mais até que a barreira entre eles torna-se baixa o suficiente para que os átomos comecem a popular ambos os poços de potencial. Finalmente, próximo ao seu nível de corrente final, os dois mínimos de potencial fundem-se e o mínimo de campo torna-se não-nulo.

Em nosso sistema isso acontece para uma corrente na bobina de Ioffe equivalente a 80% da corrente das bobinas de quadrupolo. Nesse estágio os átomos terão se deslocado cerca de 7 mm em direção à bobina de Ioffe e o potencial confinante próximo ao mínimo será harmônico onde há um campo de bias com aproximadamente 1Gauss e $\omega_x = 2\pi \times 24\ Hz$ e $\omega_y = \omega_z = 2\pi \times 210\ Hz$, onde adotamos a direção x como o eixo de simetria da bobina de Ioffe e a direção-z paralela ao eixo de simetria das bobinas de quadrupolo.

Neste momento, as condições para iniciar o resfriamento evaporativo estão satisfeitas: cerca de 2×10^8 átomos frios ($T < 100\ \mu K$) aprisionados em um potencial magnético harmônico em um ambiente de alto-vácuo. Na subseção seguinte falaremos do processo de resfriamento evaporativo, o qual é próximo para obter o condensado.

2. 5- Resfriamento Evaporativo

O resfriamento evaporativo de uma amostra atômica aprisionada consiste na retirada seletiva de átomos mais quentes da distribuição atômica de velocidades a uma dada temperatura T e na subseqüente retermalização dos átomos remanescentes a uma temperatura mais baixa $T' < T$. A repetição contínua desse processo permite produzir amostras a temperaturas muito reduzidas[34,35]. A limitação intrínseca ao resfriamento evaporativo é que os átomos permaneçam tempo suficiente na armadilha para que a retermalização ocorra. Definem esse tempo de evaporação/retermalização: (i) a taxa de colisões elásticas dentro da amostra, pois são essas colisões que redistribuem a energia e (ii) a taxa de colisão com átomos térmicos (não-aprisionados), pois estas retiram átomos da amostra indevidamente. Na verdade, a taxa de colisões elásticas deve se manter mesmo com a diminuição do número de átomos aprisionados. Isso só é possível se a densidade da amostra também crescer à medida que a evaporação avança.

De fato, o que define o ponto inicial do resfriamento evaporativo e sua taxa de evolução são a temperatura e densidade iniciais da amostra atômica e como essas grandezas evoluem à medida que o resfriamento avança.

A retirada seletiva dos átomos mais quentes em armadilhas magnéticas baseia-se no comportamento da energia dos seus subníveis Zeeman com o campo magnético. Átomos mais quentes alcançam valores de campo mais altos no potencial magnético e, por conseguinte, a separação energética entre dois estados vizinhos m_F e $m_{F\pm1}$ é maior. A separação é tipicamente da ordem de 10 a 20MHz para os átomos mais energéticos de amostras típicas de átomos frios antes da evaporação e diminui com o campo magnético. Assim, se aplicarmos rádio-freqüência (RF) com essa energia aos átomos aprisionados, induziremos transições de spin do tipo $m_F \rightarrow m_{F-1}$, levando os átomos de um estado aprisionável para outro não-aprisionável. Diminuindo continuamente a freqüência da radiação aplicada acessaremos átomos cada vez mais frios e, se a retermalização acompanhar a taxa de varredura da RF, então amostras cada vez mais frias e densas serão produzidas.

No sistema experimental que utilizamos, a RF é aplicada por meio de pequena bobina de apenas duas voltas, feita em fio de cobre (ϕ = 0.7mm), a qual é posicionada entre uma das bobinas de quadrupolo da armadilha magnética e a célula de vácuo. A RF é produzida por um gerador de onda (Stanford DS345) acoplado diretamente a essa bobina com apenas uma resistência de 50Ω em série com o circuito. A eficiência no acoplamento da RF gerada com a bobina é avaliada adicionando-se um acoplador direcional ao circuito e medindo-se a reflexão de RF da bobina. Temos uma baixa reflexão da bobina (e conseqüente bom acoplamento RF-bobina) em toda a faixa de interesse, que vai de alguns 1,6 a 30MHz.

A avaliação da eficiência da evaporação é feita pela observação da evolução das imagens de absorção dos átomos frios em expansão livre. Com o objetivo de otimizar o processo ao máximo, a varredura de RF é dividida em segmentos de varredura linear, dos quais podemos controlar o valor inicial, final, taxa de varredura e potência de RF aplicada durante aquele segmento. Uma vez otimizado um segmento, outro é adicionado à varredura, iniciando-se na freqüência final do anterior. Usualmente, a taxa de varredura é diminuída a cada segmento adicionado, permitindo aos átomos cada vez mais tempo de retermalização. São adicionados segmentos até atingirmos freqüências da ordem da separação energética dos estados de átomos no fundo do potencial ($\approx 1.6\ MHz$ para Bo $\approx$ $1 Gauss$). Finalmente, um pequeno deslocamento é adicionado (ou subtraído) à curva para o ajuste fino das freqüências finais. Esse deslocamento (±10 KHz) não altera a eficiência dos segmentos iniciais, mas faz toda a diferença na sintonia fina da freqüência final, haja vista a diferença energética entre uma amostra logo acima da temperatura de transição e o condensado puro e subseqüente evaporação de todos os átomos para fora da armadilha ser de apenas ≈ 30 *KHz*.

A varredura otimizada para as condições do nosso sistema experimental consiste de 5 segmentos de reta, todos com potência máxima de RF fornecida pelo gerador, com o primeiro iniciando-se em 16 MHz e terminando em 7.5 MHz após 6s. Para os primeiros segmentos, quando a nuvem ainda é muito quente para avaliarmos a temperatura T em tempos de expansão suficientemente longos, apenas nos guiamos pela absorção central da nuvem. Avaliamos N e T para os segmentos finais da evaporação nas imagens de absorção após 10 ms de expansão livre.

A densidade no espaço de fases aumenta mais de 5 ordens de grandeza apenas nesse estágio final da evaporação e supera 2.612, indicando a presença de um condensado de Bose-Einstein na amostra, resultado da obtenção de degenerescência quântica. Nosso conjunto de medidas indica que a transição ocorre a cerca de 160 *nK* com 7×10^5 átomos na amostra. Evaporando um pouco mais, obtemos condensados puros com cerca de 2×10^5 átomos no estado fundamental do potencial.

2. 6- Diagnóstico por Imagem de Absorção

A observação e estudo de átomos frios é realizada através de imagens. Três técnicas básicas são utilizadas: imagens por contraste de fase[36], por fluorescência e por absorção.

No primeiro caso, luz não-ressonante é utilizada e os átomos são observados não-destrutivamente. A deformação da frente de onda do feixe de imagem é o parâmetro observado e esta deformação é devida à nuvem aprisionada de átomos frios. Por ser uma técnica que não se utiliza da interação da luz com estados internos dos átomos, ela pode ser utilizada para observação *in-situ* da nuvem atômica, ou seja, com os átomos ainda confinados no potencial.

Nas técnicas de fluorescência e absorção, ao contrário, utiliza-se feixes ressonantes a uma transição atômica. Entretanto, devido ao deslocamento espacialmente dependente dos níveis energéticos dos átomos aprisionados (deslocamento Zeeman para armadilha magnética e Stark para armadilha óptica), imagens *in-situ* não são de fácil interpretação, a menos que o potencial confinante seja exatamente conhecido e seu efeito seja considerado[24].

Para evitar esse tipo de efeito os átomos são liberados da armadilha e observados em expansão livre. O tempo entre o desligamento da armadilha e a observação dos átomos, que varia tipicamente de alguns milissegundos até dezenas de milissegundos, chama-se "tempo de vôo" e dá nome à técnica, chamada TOF (*Time-of-Flight*).

A imagem por fluorescência consiste em iluminar os átomos com um pulso de luz e coletar em uma câmera CCD a luz espalhada. Usualmente, esta técnica é bem sucedida para amostras com grande número de átomos $N > 10^7$, restringindo seu uso a nuvens não condensadas ou a nuvens condensadas muito grandes[37]8]. Para nuvens condensadas típicas $N \approx 10^5$, o sinal é baixo. Nesse caso, imagem por absorção é a técnica mais indicada.

A imagem de absorção é complementar à de fluorescência. Ao invés de observar a luz espalhada pelos átomos, observamos a "sombra" dos átomos em um feixe de prova. A "ausência de fótons" no feixe proporciona as informações relevantes sobre a nuvem atômica.

A imagem de absorção na prática não corresponde a uma única imagem, mas a três imagens combinadas. A primeira consiste na imagem do feixe de prova que atravessa a nuvem atômica. A sombra dos átomos usualmente pode ser vista impressa sobre o feixe. Em seguida, uma imagem apenas do feixe de prova é tomada. Finalmente, a câmera CCD coleta a luz de fundo ("dark image") presente. As duas primeiras imagens são subtraídas da terceira e a imagem resultante que contém a sombra dos átomos é dividida pela que não contém. O resultado é uma imagem normalizada, com intensidade unitária onde não há átomos, e menor que a unidade onde houve absorção de luz pela presença dos átomos.

Nosso sistema de imagens possui uma câmera CCD (Cooke corp. - PixelFly QE) a qual é uma câmera de baixo-custo, com sensor de 1392 x 1024 pixels ($6.45 \times 6.45\ \mu m^2$) e baixo ruído. O feixe de imagem sai de uma fibra óptica com cerca de $300 \mu W$ de potência total e é colimado com cerca de 8 mm de diâmetro. O feixe é superposto ao feixe de MOT em um dos braços do plano em um cubo polarizador e alinhado na posição da armadilha magnética. Após passar pelos átomos o feixe é desviado do caminho do MOT por um outro cubo polarizador exclusivamente colocado com esse objetivo.

Um sistema de lentes dá um aumento de 1.4X na imagem. Esse sistema de imagens é semelhante ao esquema *4f*, onde o foco da primeira lente é equivalente à sua distância aos átomos e o foco da segunda equivale à sua distância à câmera, mas com aumento maior que a unidade. Nesse esquema, apenas a imagem dos átomos é focalizada na CCD. Qualquer luz espúria presente estará fora de foco, contribuindo muito pouco no ruído da imagem.

2. 7- Caracterização do Condensado de Bose-Einstein

A observação da densidade no espaço de fase maior que a densidade crítica

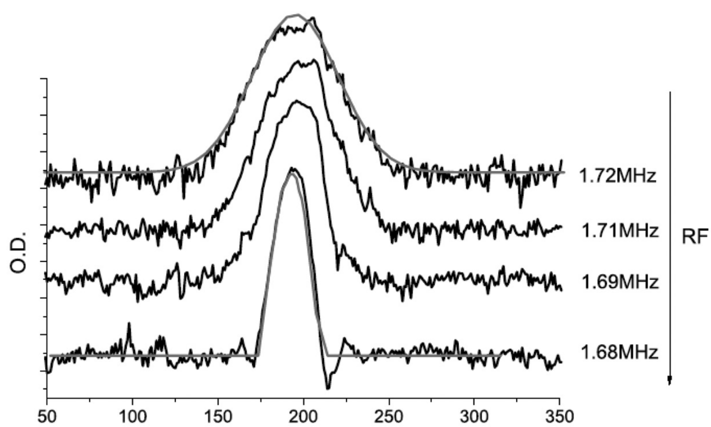
2.612 é prova irrefutável da obtenção da degenerescência quântica na amostra atômica. No entanto, há algumas outras evidências da presença de BEC na amostra. Discutiremos aqui algumas dessas evidências bem como apresentaremos a caracterização da nossa amostra.

A primeira e, com certeza, a mais conhecida evidência de BEC é a mudança na distribuição atômica de densidade, de uma distribuição gaussiana característica da nuvem clássica para uma distribuição bimodal, fruto da mistura das nuvens clássica e quântica.

A nuvem parcialmente condensada apresenta duas distribuições de densidade espacialmente sobrepostas. Uma é gaussiana, mais alargada e correspondente aos átomos não-condensados. A segunda, mais estreita, corresponde ao perfil de Thomas Fermi e a função de onda do estado fundamental do potencial confinante.

Na Fig.3, mostramos a evolução do perfil de densidade como função da freqüência final de evaporação. Enquanto o primeiro perfil é bem ajustado por uma gaussiana, o último, onde há um condensado quase puro, é ajustado por uma parábola invertida, uma clara assinatura do perfil de Thomas-Fermi, o inverso do potencial confinante. Nos perfis intermediários há uma mistura das fases condensada e não-condensada, de forma que o ajuste é feito por uma distribuição bimodal produto da mistura de uma gaussiana com um parabolóide invertido.

Figura 3:
Evolução do perfil de densidade da nuvem atômica como função da rádio freqüência final da seqüência de evaporação. O perfil muda claramente de uma distribuição gaussiana para uma distribuição com perfil de Thomas-Fermi, uma parábola invertida, passando pela distribuição bimodal característica da obtenção de degenerescência quântica na nuvem. O. D. é a densidade óptica que é relacionada à densidade atômica.



A possibilidade de separar espacialmente as nuvens clássica e quântica permite avaliar o parâmetro de ordem da transição, ou seja, a fração de átomos no estado fundamental da armadilha. A Fig.4 mostra a evolução da fração condensada com a temperatura da amostra e um ajuste de acordo com expressões existentes, que fornece a temperatura crítica de 142 *nK*.

Uma das assinaturas mais marcantes da condensação, além da distribuição bimodal, é a expansão anisotrópica da nuvem condensada. Uma nuvem clássica típica, mesmo em regimes ultra-frios, quando liberada de uma armadilha, primeiramente possui o formato do potencial confinante. No entanto, à medida que se expande livremente acaba por atingir uma distribuição isotrópica de densidade. A nuvem quântica comporta-se de forma diferente. Inicialmente, a nuvem possui o formato da armadilha, por exemplo, alongada na direção perpendicular à gravidade, como é o caso em nosso sistema. À medida que se expande livremente, a direção mais confinada expande-se mais rapidamente e a nuvem torna-se isotrópica para, em seguida, alongar-se na direção perpendicular à original. Esse comportamento é completamente devido ao domínio das interações na energia total da nuvem, uma vez que, durante a expansão, o potencial não mais existe e a energia cinética é muito reduzida.

Figura 4:
Fração de átomos no estado fundamental do potencial como função da temperatura da nuvem, mostrando bom acordo com a previsão teórica Eq.2.12 e fornecendo a temperatura crítica de transição de ≈ 142*nK*.

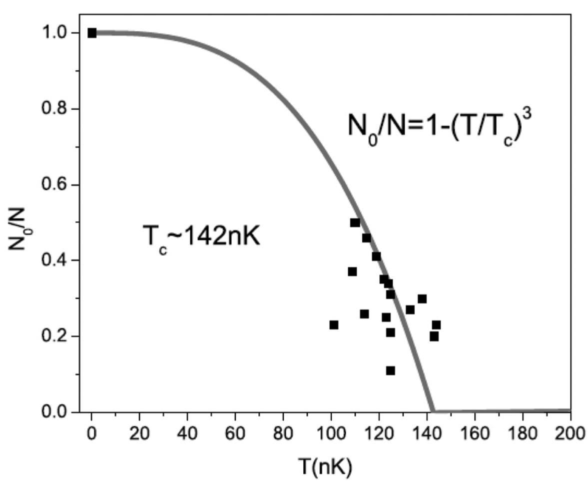
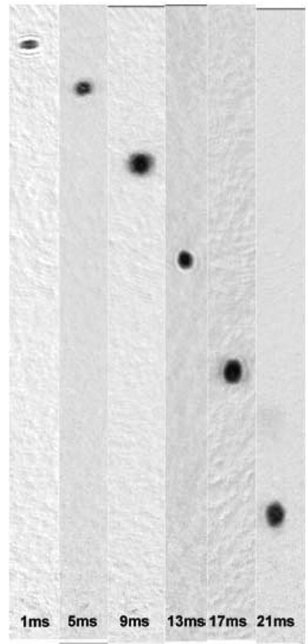
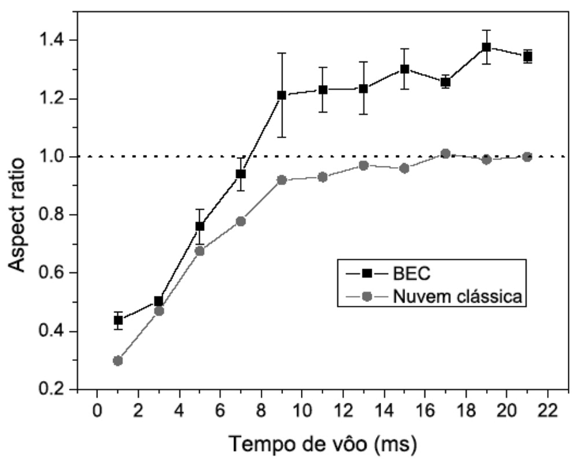


Figura 5:
Expansão de uma nuvem quântica como função do tempo mostrando a queda devido à gravidade e a inversão do *aspect ratio* da nuvem, assinatura típica de nuvens condensadas.



A razão entre a duas dimensões da nuvem é chamada de *aspect ratio*. A Fig.5 mostra a expansão de uma nuvem quântica, evidenciando a clara inversão do *aspect ratio* devido à expansão anisotrópica. Na Fig.6 mostramos o *aspect ratio* de ambas as situações. Veja que enquanto a nuvem clássica evolui para a unidade, a nuvem quântica vai de um valor de cerca de 0.4 para um valor de ≈ 1.4 em 21ms de expansão livre. A inversão ocorre em torno de 8 ms, valor que concorda com o teórico 7.8 ms para as freqüências da armadilha.

Figura 6:
Aspect ratio das nuvens clássica e quântica mostrando a evolução para a isotropia no primeiro caso e a inversão no segundo.



3. Observações dos vórtices excitados por excitações oscilatoriais

Fluidos quânticos caracterizam-se por trazer, para uma escala macroscópica, fenômenos típicos do domínio microscópico. Em especial, a fase da função de onda, que se estende por toda a amostra, dá origem ao fenômeno da superfluidez[38]. Tal qual na supercondutividade, onde também a extensão macroscópica da fase é a origem do fenômeno, uma corrente, de matéria neutra em um caso e de partículas carregadas no outro, pode fluir por toda a amostra sem dissipação.

Adicionalmente, se momento angular for imposto a essas amostras, haverá o aparecimento de vórtices, tal qual em fluidos usuais. A grande diferença é que vórtices em fluidos quânticos carregam quantidades discretas de momento angular. Na verdade, a observação de vórtices com circulação quantizada é prova irrefutável da superfluidez de determinadas amostras, tanto que, em 2005, o grupo do MIT usou a nucleação de vórtices em um gás de férmions quanticamente degenerado para atestar definitivamente sua natureza superfluida[39].

O estudo de vórtices em fluidos quânticos remonta aos tempos de estudos em Hélio líquido superfluido[40]. No caso específico do Hélio, os vórtices eram formados localmente por instabilidades geradas pela presença de impurezas ou rugosidades nas paredes do recipiente que girava. No caso de fluidos quânticos gasosos não há impurezas ou rugosidades no "recipiente" magnético. Assim, o momento angular fornecido à amostra é devido, completamente, à rotação do potencial e/ou a qualquer outro processo externo que seja capaz disso[41,42].

A primeira observação de vórtices em um condensado de Bose-Einstein foi feita com a utilização conjunta de laser e microondas para imprimir, na amostra, a fase adequada[43]. Isso originou um condensado com átomos em um estado hiperfino rodando em torno de outro estático, em um outro estado hiperfino. Em seguida, técnicas mais simples foram implementadas e, de fato, muito semelhantes aos experimentos com recipiente rodando dos estudos com Hélio líquido[44]. Nessas técnicas, um laser é focalizado na amostra e rotacionado. Além de uma determinada freqüência crítica, momento angular é transferido para a amostra de forma a nuclear um vórtice[45].

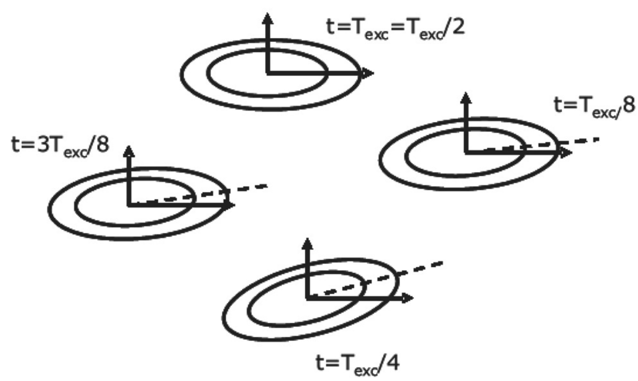
Vórtices em condensados são facilmente observados devido às baixas densidades. Isso permite que sejam grandes o suficiente para observação direta no perfil de absorção. Eles caracterizam-se por um ponto de densidade nula ou diminuída na distribuição de densidade. O aumento da freqüência de rotação favorece a nucleação de um número crescente de vórtices na amostra[43]. De fato, apesar de já terem sido observados vórtices com circulação bem maior que a unidade[46], eles não são estáveis e decaem em vórtices com circulação unitária[47,48], ou seja, com um quantum de momento angular. Redes gigantescas de vórtices já foram observadas e, tipicamente, essas redes têm arranjos matematicamente bem descritos[49,50].

O estudo de vórtices, ao contrário de outros tópicos no estudo de condensados, nunca sofreu perda de interesse. Ao longo dos anos, tanto estudos envolvendo novos aspectos da formação e dinâmica de vórtices foram realizados, como novas técnicas de formação destes foram desenvolvidas. Nesse tópico, vórtices foram formados não apenas pela impressão de uma fase específica na amostra ou rotação de um feixe focalizado sobre a mesma, mas também pela indução de assimetria em um potencial magnético girante[51], a primeira formação de vórtices puramente magnética.

Recentemente, vórtices foram formados pela recombinação de condensados independentes[52,53] e pela transferência de momento angular orbital de um feixe de luz com perfil espacial de Laguerre-Gauss[54,55].

Durante a fase final deste trabalho, descobrimos que Möttönen e colaboradores[56] propuseram, recentemente, um método muito similar ao aplicado em nosso experimento para adicionar momento angular em uma amostra Bose-condensada. No entanto, nessa proposta teórica, vórtices com grande circulação são produzidos. Não é isso exatamente que observamos, como mostramos nas seções a seguir.

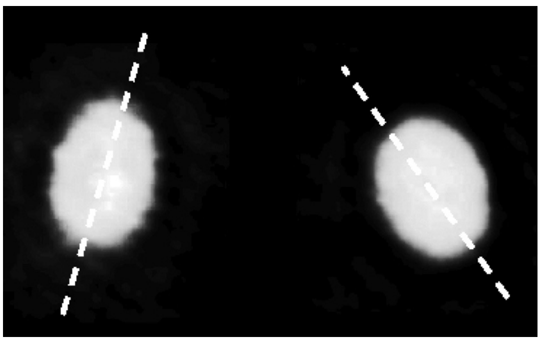
Figura 7:
Representação esquemática do movimento do potencial confinante como função do tempo, dado em função do período (*Texc*) de oscilação da excitação externa.



Nossa explicação para a geração de vórtices reside no fato de que nosso campo de excitação não está centrado com a nuvem condensada. Dessa forma, geramos algum tipo de torção no potencial de aprisionamento, transferindo momento angular para a nuvem aprisionada. Pictoricamente, podemos imaginar o movimento de torção do potencial confinante como o mostrado na fig. 7, lembrando sempre que esse movimento deve ocorrer em todas as direções do espaço e não apenas em um plano específico. Essa interpretação é reforçada pela observação da inclinação do eixo de simetria da nuvem expandida quando as amplitudes de oscilação do campo são pequenas (<40 mGauss/cm), como é mostrado na fig.8. Todas as imagens são de nuvens após 15ms de expansão livre.

Veja que o eixo de simetria da nuvem inclina-se para ambos os lados e tanto o valor como o sinal do ângulo de inclinação variam a cada imagem do experimento. Tanto nesse regime como nos regimes de amplitude descritos a seguir, onde observamos vórtices na amostra, a variação do eixo de simetria e/ou do número de vórtices formados varia de imagem para imagem. Há duas possíveis explicações que se completam para a observação desse comportamento.

Figura 8:
Observação da inclinação do eixo da nuvem condensada para baixas (< 40 mGauss/cm) amplitudes de excitação.

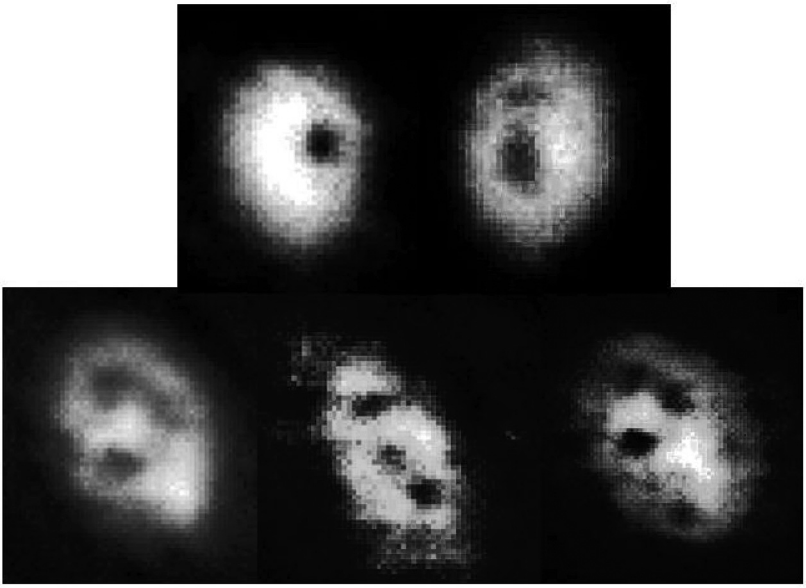


Primeiramente, a torção no potencial confinante, da forma que é feita, pode sempre ser pensada como a combinação de duas torções independentes, uma para cada sentido, em cada plano de simetria da nuvem. Assim, é sempre possível esperar que a torção, ou a transferência de momento angular para a nuvem ocorra em uma das duas direções ou mesmo em ambas simultaneamente, mas sem uma direção preferencial e mesmo sem amplitudes iguais. Dessa forma, tanto a inclinação da nuvem para um dos lados quanto à geração de vórtices, com circulação também para qualquer um dos lados, deve ocorrer de uma maneira aleatória e naturalmente variar de imagem para imagem.

A segunda remete à dinâmica de formação dos vórtices e da transferência de momento angular para a nuvem condensada. Como já mencionado, temos uma variação temporal lenta do fundo do potencial de forma que, para as mesmas condições experimentais, observamos amostras com diferentes números de átomos finais e diferentes frações condensadas ao longo do tempo. Além disso, a rotação aplicada aos átomos pode ser tomada como rotações nas duas direções, como já discutido. Assim, podemos transferir momento angular nas duas direções e mais, de forma diferente a cada vez, dado que tanto o número total de átomos quanto a nuvem não condensada devem desempenhar algum papel nesse processo. A função e a contribuição específicas de cada uma dessas partes ainda não é clara, mas com certeza existe. Além disso, a possibilidade de rotação em duas direções opostas é algo novo, no sentido de que todas as técnicas até então empregadas para geração de vórtices, aplicavam rotação apenas em uma direção[42]. Em nosso caso, é possível que estejamos gerando vórtices com circulação em ambas as direções ao mesmo tempo. Há pouca teoria sobre o assunto, mas é sabido que essas estruturas não são estáveis, de forma que, um par vórtice/antivórtice seria, em princípio, aniquilado[51,57]. De qualquer forma, se isso realmente ocorre, por sua fragilidade, o processo deve apresentar uma dinâmica de formação bastante delicada, dando origem às variações observadas. Discutiremos brevemente esse assunto adiante.

Isto posto, o que fizemos para entender os resultados de formação de vórtices foi agrupar as imagens por faixas de amplitude, contando o número de vórtices observados e fazendo uma média dessas observações. Assim, acabamos por definir, para este trabalho, três zonas distintas. A primeira, onde a amplitude de oscilação encontra-se na faixa de 40 a 90 mGauss/cm, chamamos de zona de um vórtice. Para essa faixa de amplitudes, tipicamente observamos entre zero e dois vórtices formados, com uma média de 0.6 ± 0.3 vórtices formados. A faixa superior, chamada de zona de múltiplos vórtices, corresponde a amplitudes entre 90 e 190 mGauss/cm. Nessa faixa observamos uma média de 2.6 ± 1.2 vórtices, apesar de algumas imagens não apresentarem nenhuma estrutura e em algumas ser possível observar 5 ou mais vórtices. A fig. 9 mostra imagens típicas com um, dois, três e vários vórtices na nuvem condensada. Veja que a torção do eixo de simetria se mantém, mesmo quando vórtices são gerados. No regime de amplitudes acima de 190 mGauss/cm, um fenômeno interessante ocorre: uma série de estruturas aparece e apesar de ser possível identificar estruturas circulares tipo-vórtice, outras mais ocorrem, e todas com pouco contraste e bordas pouco definidas. Nos primeiros trabalhos que estudam vórtices há referência a um regime dito turbulento acima de freqüências de excitação muito grandes. No entanto, usualmente, existe apenas a menção a esses regimes e, até onde sabemos, nenhum trabalho reporta sua observação ou mesmo estuda esse regime. Na seção a seguir, exploramos brevemente esse tema.

Figura 9:
Resultados típicos de observação de um, dois, três e múltiplos vórtices na nuvem condensada para as diversas faixas de amplitude de excitação.



A fig.10 sumariza o comportamento observado na geração de vórtices como função da faixa de amplitude de oscilação. Uma observação importante recai sobre o grupo de imagens onde três vórtices são observados. Tipicamente, em experimentos usuais de formação de vórtices, como os que usam feixes de laser girantes, formações de três vórtices aparecem como arranjos triangulares regulares[42]. A justificativa usual é que vórtices com a mesma circulação se repelem e a configuração mais estável é a de triângulo eqüilátero. Em nosso sistema, imagens com três vórtices nem sempre aparecem nessa formação (fig.11). De fato, se medirmos o maior ângulo da formação de três vórtices, observamos que esse ângulo cai preferencialmente entre 60 e 100 graus e entre 140 e 180 graus, como indicado na fig. 12. Isso indica que formamos a configuração usual, do triângulo regular, mas, também, uma formação exótica, onde os vórtices estão alinhados, o que pode indicar que um dos vórtices tem circulação contrária aos outros, apesar de ainda não termos fundamentação teórica para afirmar isso categoricamente.

Figura 10:
Contagem de vórtices como função da amplitude de excitação, onde vê-se a clara formação de zonas de um vórtice, múltiplos vórtices e incontáveis vórtices como função da amplitude de excitação do campo externo.

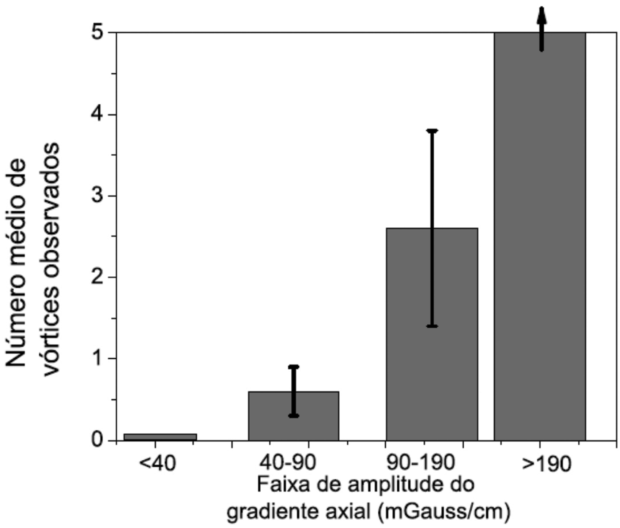
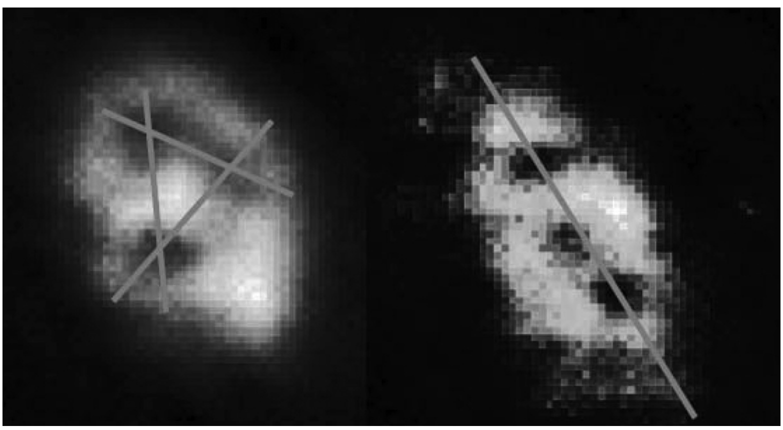
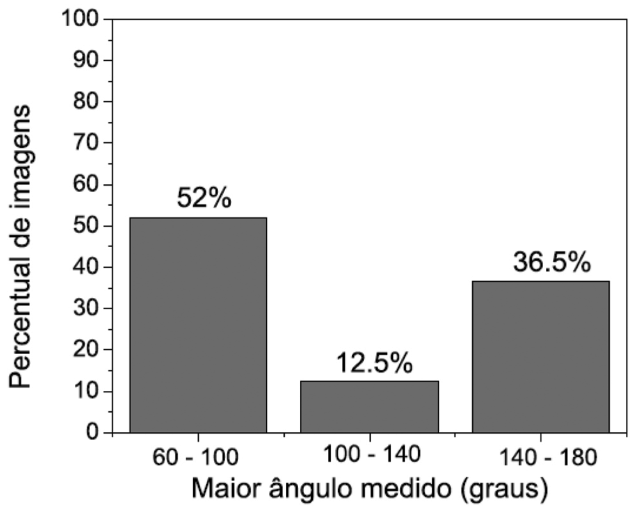


Figura 11:
Formações típicas de três vórtices observadas em nosso sistema experimental. A da direita é a formação do triângulo equilátero típica. A da esquerda é uma estrutura nova, quase linear.



De fato, uma forma de provar a existência de circulação nas amostras e mesmo de observar circulações opostas, se for o caso, é fazer um processo de interferência tal qual é feito em[58]. Outra opção, em princípio, é tentar observar o tempo de vida dessas estruturas como função do tempo de expansão, haja vista pares de vórtices e antivórtices terem tendência de aniquilarem-se. Estes são alguns dos procedimentos que devem ser implementados em breve. Na seção a seguir, discutimos brevemente a observação do regime turbulento.

Figura 12:
Distribuição percentual do ângulo máximo medido nos arranjos de três vórtices, mostrando concentração na formação do triângulo regular e distribuição colinear.



4. Rota para turbulência quântica

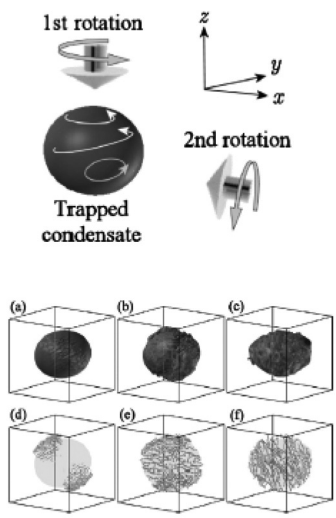
O estudo de sistemas quânticos turbulentos remonta também aos estudos com Hélio líquido[59]. No entanto, a investigação de regimes clássicos turbulentos é muito mais antiga e seu entendimento e eventual controle tem grande interesse tanto por parte da ciência, como no campo tecnológico, desde o projeto de aviões até a carros de corrida. Apesar de antigo, o estudo de turbulência no regime clássico ainda possui várias questões em aberto. No entanto, similaridades entre os regimes clássico e quântico foram observadas experimentalmente e confirmadas por simulações numéricas em sistemas de Hélio líquido[60,61].

Dessa maneira, o estudo de turbulência quântica se tornou interessante como um protótipo para o entendimento tanto da estatística como da dinâmica de nucleação de vórtices nesses regimes e eventual extensão para os regimes clássicos. No entanto, o Hélio ainda não é um bom sistema para observações desse regime, haja vista as dificuldades experimentais no controle dos parâmetros e da própria geração da turbulência.

Ao contrário, amostras de Condensados de Bose-Einstein, onde os parâmetros experimentais são altamente sintonizáveis e a adição de momento angular pode ser feita de uma forma controlada, são as amostras ideais para esse tipo de estudo. No entanto, experimentalmente, não houve estudos nesse regime até hoje. As poucas referências experimentais mencionam, brevemente, imagens com pouco contraste, sem estruturas claras[42]. Mesmo no campo teórico, poucos foram os trabalhos nessa área.

Recentemente, em uma série de trabalhos teóricos, M. Tsubota e colaboradores[62,63,64] estudaram diversos aspectos relacionados à turbulência quântica em BECs.

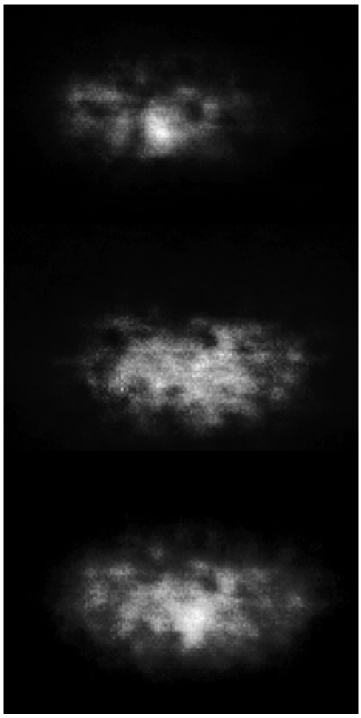
Figura 13:
Acima, esquema de excitação do regime de turbulência quântica proposto por Tsuboto e colaboradores (veja texto para referência), com a aplicação de rotação em duas direções distintas e abaixo, (a), (b) e (c) são os resultados de simulações numéricas da superfície do condensado e (d), (e) e (f) são as linhas de vórtices ao longo da amostras. (a) e (d), (b) e (e) e (c) e (f) correspondem à amplitudes iguais e crescentes da rotação aplicada.



Em um desses trabalhos[62] eles propõem que a aplicação de rotações em direções distintas em um condensado aprisionado seria suficiente para a geração o regime turbulento. Nesse mesmo trabalho, o regime turbulento é caracterizado por um emaranhado (no sentido clássico e não quântico) de vórtices e linhas de vórtice nas mais diversas direções ao longo da amostra. Tanto a idéia da excitação como os resultados obtidos por eles, em simulações numéricas da equação de Gross-Pitaevskii, são mostrados na fig.13.

Compare com as estruturas que observamos, mostradas na fig.14, para amplitudes de excitação além de 190 mGauss/cm. Há uma evidente distribuição confusa de vórtices, de vários tamanhos e formas, além de "caminhos", como se fossem linhas de vórtice atravessando a amostra perpendicularmente à direção de imagem.

Figura 14:
Observação experimental do regime de turbulência quântica, mostrando diversas estruturas não homogêneas e exoticamente distribuídas ao longo da amostra. Note que não houve inversão do *aspect ratio*. Imagens com 15ms de expansão livre.



Há ainda outro fato importante que indica que nossos resultados evidenciam turbulência quântica. No trabalho da ref. [65] mostra-se que um BEC com muitos vórtices não sofre inversão do seu *aspect ratio*. Veja que as imagens da fig. 14 têm seu eixo de simetria na horizontal, ao contrário das imagens mostradas na seção 4.3, onde nuvens condensadas com 15ms de expansão livre tais quais as mostradas aqui, já sofreram a inversão de seu *aspect ratio* e por isso têm seu eixo mais longo na direção vertical. As nuvens do regime turbulento mantém o *aspect ratio* próximo ao da nuvem original.

Muito estudo ainda deve ser feito nesse regime, até mesmo para confirmar sua natureza turbulenta. Nesse caso, o primeiro passo é torná-lo controlado e reprodutível e em seguida tentar extrair informações como a distribuição espacial e energética dos vórtices, sua conformação espacial e mesmo seu grau de isotropia/anisotropia. Mas esses são apenas alguns dos próximos passos desse trabalho.

5. Conclusões

Neste texto fizemos uma revisão geral dos experimentos em progresso nos laboratórios do IFSC/USP onde amostras de fluidos quânticos são produzidas. O texto permite observar, com algum detalhe, como são produzidas estas amostras, principais detalhes de sua caracterização e aspectos gerais. Sendo este, o único exemplar de fluido atômico quântico da América Latina, procuramos dar detalhes de construção, que possam ser úteis para outros. Na segunda parte do texto, apresentamos os estudos mais recentes que estão em progresso. Estes resultados ainda estão em fase de analise par publicação, de modo que algumas das interpretações aqui fornecidas podem ser modificadas. De qualquer forma, o texto e os resultados apresentados mostram evidencias únicas da ocorrência de efeitos associados com a superfluidez das amostras de gases no regime quântico por nós investigados. Finalmente, cumpre lembrar , que o conteúdo deste texto esta baseado em vários trabalhos do grupo, mas essencialmente, contido na tese de doutorado de Emanuel Henn um dos autores deste texto (E.H).

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Eliminating the chiral anomaly via symplectic embedding approach

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The quantization of the chiral Schwinger model (χQED2) with one-parameter class Faddeevian regularization is hampered by the chiral anomaly, i.e., the Gauss law commutator exhibits Faddeev's anomaly. To overcome this kind of problem, we propose to eliminate this anomaly by embedding the theory through a new gauge-invariant formalism based on the enlargement of the phase space with the introduction of Wess-Zumino(WZ) fields and the symplectic approach [1, 2]. This process opens up a possibility to formulate different, but dynamically equivalent, gauge invariant versions for the model and also gives a geometrical interpretation to the arbitrariness presents on the BFFT and iterative conversion methods. Further, we observe that the elimination of the chiral anomaly imposes a condition on the chiral parameters present on the original model and on the WZ sector.

1. Introduction

It has been shown over the last decade that anomalous gauge theories in two dimensions can be consistently and unitarily quantized for both Abelian[3–5] and non-Abelian[6, 7] cases. In this scenario, the two dimensional model that has been extensively studied is the chiral Schwinger model (CSM)[3]. In these early works were considered to study an effective action with a simple one-parameter class of regularization. The consequences of these constraint structures are that the a>1 class presents, besides the massless excitation also a massive scalar excitation$(m^2 = \frac{e^2a^2}{a-1})$that is not found on the a= 1 class. In order to elucidate the physical spectrum, this model was analyzed by a variety of methods[4, 8–11]. In Refs.[9–11] a gauge invariant formulation of the model with Wess-Zumino (WZ) fields was studied, as suggested in Ref.[12].

Despitethis spate ofinterest,Mitra[13] proposed a new and surprisingbosonized actionfor the Schwinger model with a new regularization prescription, that is different from those involved inthe class of models studied earlier. Inthispaperheproposed a new (Faddeevian) regularization class,incarnatedby a conveniently mass-like term whichleads to a canonicaldescription with three second class constraints,in theDirac's context. In thispaper,ithasbeen shown that theGausslaw commutator exhibits Faddeev's anomaly. This model has an advantage because the Faddeevian mechanism [14], which is related to the anomalous Gauss law algebra in the anomalous gauge theories, workswell. Recall thatin [3] and [4], the Hamiltonian framework was structured in terms of two classes with two and four second class constraints respectively. Mitra's work brings a clear interpretation about the reason leading the bosonization ambiguity to fit into 3 distinct classes, classified according to the number of constraints present in the model. Recently, an extension of theRef. [13] to an one-parameter class of solutions was proposed by one of us in [15] in order to study the restrictions posed by the soldering formalism [16] over this new regularization class. In this context, the gauge field becomes massive once again and its dependence on the ambiguity parameter was shown to be identical to that in Ref. [3], while the massless sector however is more constrained than its counterpart in[3], which corroborates the Mitra's finding outs [13].

In the present paper, the chiral Schwinger model (χQED_2)with one-parameter class Faddeevian regularization will be reformulated as a gauge invariant theory in order to eliminate the chiral anomaly that obstructed the quantization procedure. It is done just enlarging the phase space with the introduction of WZ fields. To this end, we chose to use a new gauge-invariant formalism, thatis anextension of the symplectic gauge-invariant formalism [1,2]. This formalism is developed in a way to handle constrained systems, called the symplectic framework [17,18]. The basic object behind this formalism is the symplectic matrix: if this matrix is singular, the model presents a symmetry. This is achieved introducing an arbitrary function (G), written in terms of the original and WZ variables, into the zeroth-iterative first-order Lagrangian.

In section II, we introduce the symplectic embedding formalism in order to settle the notation and familiarize the reader with the fundamentals of the formalism. In section III, the chiral Schwinger model with one-parameter class Faddeevian regularization will be introduced following so closely the original presentation [15]. Furthermore, this model will be investigated through the symplectic method [17, 18], displaying its chiral noninvariant nature, and the Dirac's brackets among the phase-space coordinates will be also computed.

In section IV, the chiral Schwinger model with one-parameter Faddeevian regularization will be reformulated as a gauge invariant theory with the introduction of WZ field *via* the new symplectic gauge-invariant formalism, presented in the Section II. An immediate consequence produced by this process is the generation of the infinitesimal gauge transformation that keep the Hamiltonian invariant and the elimination of the chiral anomaly. In this way, we have several versions for this model, which are described by different gauge invariant Hamiltonians, but all dynamically equivalent.

The last section is reserved to discuss the physical meaning of our findings together with our final comments and conclusions.

2. General formalism

In this section, we describe the alternative embedding technique that changes the second class natureof aconstrained systemtothe first one. This technique follows the Faddeev and Shatashivilli idea[12] and is based on a contemporary framework that handles constrained models, namely, the symplectic formalism [17, 18].

In order to systematize the symplectic embedding formalism, we consider a general noninvariant mechanical model whose dynamics is governed by a Lagrangian $\mathcal{L}(a_i, \dot{a}_i, t)$ (with i=1,2,...,N), where a_i and $\dot{a}_i$ are the space and velocities variables, respectively. Notice that this model does not result in the loss of generality or physical content. Following the symplectic method the zeroth-iterative first-order Lagrangian is written as

$$\mathcal{L}^{(0)} = A_\alpha^{(0)} \dot{\xi}^{(0)\alpha} - V^{(0)}, \tag{1}$$

where the symplectic variables are

$$\xi^{(0)\alpha} = \begin{cases} a_i, & \text{with } \alpha = 1, 2, \dots, N \\ p_i, & \text{with } \alpha = N+1, N+2, \dots, 2N. \end{cases} \tag{2}$$

with $A_\alpha^{(0)}$ are the one-form canonical momenta and $V^{(0)}$ is the symplecticpotential. The symplectic tensor is given by

$$f_{\alpha\beta}^{(0)} = \frac{\partial A_\beta^{(0)}}{\partial \xi^{(0)\alpha}} - \frac{\partial A_\alpha^{(0)}}{\partial \xi^{(0)\beta}}. \tag{3}$$

If this symplectic matrixis singular,ithas a zero-mode ($\nu^{(0)}$)which can generate a new constraint when contracted with the gradient of symplectic potential,

$$\Omega^{(0)} = \nu^{(0)\alpha} \frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}}. \tag{4}$$

This constraint is introduced into the zeroth-iterative Lagrangian, Eq. (1), through a Lagrange multiplier η, generating the next one

$$\begin{aligned} \mathcal{L}^{(1)} &= A_\alpha^{(0)}\dot{\xi}^{(0)\alpha} + \dot{\eta}\Omega^{(0)} - V^{(0)}, \\ &= A_\gamma^{(1)}\dot{\xi}^{(1)\gamma} - V^{(1)}, \end{aligned} \tag{5}$$

with γ=1,2,..., (2N+1) and

$$\begin{aligned} V^{(1)} &= V^{(0)}|_{\Omega^{(0)}=0}, \\ \xi^{(1)\gamma} &= (\xi^{(0)\alpha}, \eta), \\ A_\gamma^{(1)} &= (A_\alpha^{(0)}, \Omega^{(0)}). \end{aligned} \tag{6}$$

As a consequence, the first-iterative symplectic tensor is computed as

$$f_{\gamma\beta}^{(1)} = \frac{\partial A_\beta^{(1)}}{\partial \xi^{(1)\gamma}} - \frac{\partial A_\gamma^{(1)}}{\partial \xi^{(1)\beta}}. \tag{7}$$

If this tensor is nonsingular, the iterative process stops and the Dirac's brackets among the phase pace variables are obtained from the inverse matrix $(f_{\gamma\beta}^{(1)})^{-1}$ and, consequently, the Hamilton equation of motion can be computed and solved as well, as well discussed in [20]. It is well known that a physical system can be described in terms of a symplectic manifold M, classically at least. From a physical point of view, M is the phase space of the system while a nondegenerate closed 2-form f can be identified as being the Poisson bracket. The dynamics of the system is determined just specifying a real-valued function (Hamiltonian) H on phase space, *i.e.* , one these real-valued function solves the Hamilton equation, namely,

$$\iota(X)f = dH, \tag{8}$$

and the classical dynamical trajectories of the system in phase space are obtained. It is important to mention that if f is nondegenerate, Eq. (8) has an unique solution. The nondegeneracy of f means that the linear map $\flat : TM \to T^*M$ defined by $\flat(X) := \flat$ $(X)f$ is an isomorphism, due to this, the Eq. (8) is solved uniquely for any Hamiltonian $(X = \flat^{-1}(dH))$. On the contrary, the tensor has a zero-mode and a new constraint arises, indicating that the iterative process goes on until the symplectic matrix becomes nonsingular or singular. If this matrix is nonsingular, the Dirac's brackets will be determined. In Ref.[20], the authors consider in detail the case when f is degenerate, which usually arises when constraints are presented on the system. In which case, (*M,f*) is called presymplectic manifold. As a consequence, the Hamilton equation, Eq. (8), may or may not possess solutions, orpossess nonunique solutions. Oppositely,if this matrix is singular and the respective zero-mode does not generate a new constraint, the system has a symmetry.

The systematization of the symplectic embedding formalism begins by assuming that the gauge invariant version of the general Lagrangian $(\tilde{\mathcal{L}}(a_i, \dot{a}_i, t))$is given by

$$\tilde{\mathcal{L}}(a_i, \dot{a}_i, \varphi_p, t) = \mathcal{L}(a_i, \dot{a}_i, t) + \mathcal{L}_{WZ}(a_i, \dot{a}_i, \varphi_p), \quad (p = 1, 2), \tag{9}$$

where $\varphi_p = (\theta, \dot{\theta})$ and the extraterm (L_{WZ})depends on the original $(a_i, \dot{a}_i)$and WZ (φ_p) configuration variables. Indeed, this WZ Lagrangian can be expressed as an expansion in orders of the WZ variable (φ_p)such as

$$\mathcal{L}_{WZ}(a_i, \dot{a}_i, \varphi_p) = \sum_{n=1}^{\infty} \upsilon^{(n)}(a_i, \dot{a}_i, \varphi_p), \quad with \;\; \upsilon^{(n)}(\varphi_p) \sim \varphi_p^n, \tag{10}$$

which satisfies the following boundary condition,

$$\mathcal{L}_{WZ}(\varphi_p = 0) = 0. \tag{11}$$

The reduction of the Lagrangian, Eq. (9), into its first order form precedes the beginning of conversion process, thus

$$\tilde{\mathcal{L}}^{(0)} = A_\alpha^{(0)} \dot{\xi}^{(0)\alpha} + \pi_\theta \dot{\theta} - \tilde{V}^{(0)}, \tag{12}$$

where π_θ is the canonical momentum conjugated to the WZ variable, that is,

$$\pi_\theta = \frac{\partial \mathcal{L}_{WZ}}{\partial \dot{\theta}} = \sum_{n=1}^{\infty} \frac{\partial \upsilon^{(n)}(a_i, \dot{a}_i, \varphi_p)}{\partial \dot{\theta}}. \tag{13}$$

The expanded symplectic variables are $\tilde{\xi}^{(0)\tilde{\alpha}} \equiv (a_i, p_i, \varphi_p)$ and the new symplectic potential becomes

$$\tilde{V}^{(0)} = V^{(0)} + G(a_i, p_i, \lambda_p), \quad (p = 1, 2), \tag{14}$$

where $\lambda_p = (\theta, \pi_\theta)$. The arbitrary function $G(a_i, p_i, \lambda_p)$ is expressed as an expansion in terms of the WZ fields, namely

$$G(a_i, p_i, \lambda_p) = \sum_{n=0}^{\infty} \mathcal{G}^{(n)}(a_i, p_i, \lambda_p), \tag{15}$$

with

$$\mathcal{G}^{(n)}(a_i, p_i, \lambda_p) \sim \lambda_p^n. \tag{16}$$

In this context, the zeroth one-form canonical momenta are given by

$$\tilde{A}^{(0)}_{\tilde{\alpha}} = \begin{cases} A^{(0)}_{\alpha}, & \text{with } \tilde{\alpha} = 1,2,\ldots,\text{N}, \\ \pi_\theta, & \text{with } \tilde{\alpha} = \text{N} + 1, \\ 0, & \text{with } \tilde{\alpha} = \text{N} + 2. \end{cases} \tag{17}$$

The corresponding symplectic tensor, obtained from the following general relation

$$\tilde{f}^{(0)}_{\tilde{\alpha}\tilde{\beta}} = \frac{\partial \tilde{A}^{(0)}_{\tilde{\beta}}}{\partial \tilde{\xi}^{(0)\tilde{\alpha}}} - \frac{\partial \tilde{A}^{(0)}_{\tilde{\alpha}}}{\partial \tilde{\xi}^{(0)\tilde{\beta}}}, \tag{18}$$

is

$$\tilde{f}^{(0)}_{\tilde{\alpha}\tilde{\beta}} = \begin{pmatrix} f^{(0)}_{\alpha\beta} & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{19}$$

which should be a singular matrix.

The implementation of the symplectic embedding scheme consists in computing the arbitrary function ($G\ (a_i, p_i, \lambda_p)$). To this end, the correction terms in order of λ_p, within by $G^{(n)}(a_i, p_i, \lambda_p)$, mustbe computed as well. If the symplectic matrix, Eq. (19), is singular, it has a zero-mode $\tilde{\varrho}$ and, consequently, we have

$$\tilde{\varrho}^{(0)\tilde{\alpha}} \tilde{f}^{(0)}_{\tilde{\alpha}\tilde{\beta}} = 0, \tag{20}$$

where we assume that this zero-mode is

$$\tilde{\varrho}^{(0)} = \left(\gamma^{\alpha} \quad 0 \quad 0 \right), \tag{21}$$

where γ^{α}, is a generic line matrix. Using the relation given in Eq. (20) together with Eq. (19) and Eq. (21), we get

$$\gamma^{\alpha} f^{(0)}_{\alpha\beta} = 0. \tag{22}$$

In this way, a zero-mode is obtained and, in agreement with the symplectic formalism, this zero-mode must be contracted with the gradient of the symplectic potential, namely, ∂V

$$\tilde{\varrho}^{(0)\tilde{\alpha}} \frac{\partial \tilde{V}^{(0)}}{\partial \tilde{\xi}^{(0)\tilde{\alpha}}} = 0. \tag{23}$$

As a consequence, a constraint arise as being

$$\Omega = \gamma^{\alpha} \left[\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right]. \tag{24}$$

Due to this, the first-order Lagrangian is rewritten as

$$\tilde{\mathcal{L}}^{(1)} = A^{(0)}_{\alpha} \dot{\xi}^{(0)\alpha} + \pi_{\theta}\dot{\theta} + \Omega\dot{\eta} - \tilde{V}^{(1)}, \tag{25}$$

where $\tilde{V}^{(1)} = V^{(0)}$. Note that the symplectic variables are now $\tilde{\xi}^{(1)\tilde{\alpha}} \equiv (a_i, p_i, \eta, \lambda_p)$ (with $\tilde{\alpha} = 1,2,\ldots,N+3$) and the corresponding symplectic matrix becomes

$$\tilde{f}^{(1)}_{\tilde{\alpha}\tilde{\beta}} = \begin{pmatrix} f^{(0)}_{\alpha\beta} & f_{\alpha\eta} & 0 & 0 \\ f_{\eta\beta} & 0 & f_{\eta\theta} & f_{\eta\pi_{\theta}} \\ 0 & f_{\theta\eta} & 0 & -1 \\ 0 & f_{\pi_{\theta}\eta} & 1 & 0 \end{pmatrix}, \tag{26}$$

where

$$\begin{aligned} f_{\eta\theta} &= -\frac{\partial}{\partial\theta} \left[\gamma^{\alpha} \left(\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right) \right], \\ f_{\eta\pi_{\theta}} &= -\frac{\partial}{\partial\pi_{\theta}} \left[\gamma^{\alpha} \left(\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right) \right], \\ f_{\alpha\eta} &= \frac{\partial \Omega}{\partial \xi^{(0)\alpha}} = \frac{\partial}{\partial \xi^{(0)\alpha}} \left[\gamma^{\alpha} \left(\frac{\partial V^{(0)}}{\partial \xi^{(0)\alpha}} + \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right) \right]. \end{aligned} \tag{27}$$

Since our goal is to unveil a WZ symmetry, this symplectic tensor must be singular, consequently, it has a zero-mode, namely,

$$\tilde{\nu}^{(1)}_{(\nu)(a)} = \left(\mu^{\alpha}_{(\nu)} \quad 1 \quad a \quad b \right), \tag{28}$$

which satisfies the relation

$$\tilde{\nu}^{(1)\tilde{\alpha}}_{(\nu)(a)} \tilde{f}^{(1)}_{\tilde{\alpha}\tilde{\beta}} = 0. \tag{29}$$

Note that the parameters (a,b) can be 0 or 1 and ν indicates the number of choices for $\tilde{\nu}^{(1)^{-}\alpha}$ [26]. As a consequence, there are two independent set of zero-modes, given by

$$\begin{aligned} \tilde{\nu}^{(1)}_{(\nu)(0)} &= \left(\mu^{\alpha}_{(\nu)} \quad 1 \quad 0 \quad 1 \right), \\ \tilde{\nu}^{(1)}_{(\nu)(1)} &= \left(\mu^{\alpha}_{(\nu)} \quad 1 \quad 1 \quad 0 \right). \end{aligned} \tag{30}$$

Note that the matrix elements $\mu^{\alpha}_{(\nu)}$ present some arbitrariness which can be fixed in order to disclose a desired WZ gauge symmetry. In addition, in our formalism the zero-mode $\tilde{\nu}^{(1)\tilde{\alpha}}_{(\nu)(a)}$ is the gauge symmetry generator, which allows to display the symmetry from the geometrical point of view. At this point, we call attention upon the fact that this is an important characteristic since it opens up the possibility to disclose the desired hidden gauge symmetry from the noninvariant model. Different choices of the zero-mode generates different gauge invariant versions of the second class system, however, these gauge invariant descriptions are dynamically equivalent, *i.e.*, there is the possibility to relate this set of independent zero-modes, Eq. (30), through canonical transformation $(\tilde{\bar{\nu}}^{(\prime,1)}_{(\nu)(a)} = T.\tilde{\bar{\nu}}^{(1)}_{(\nu)(a)})$ where bar means transpose matrix. For example,

$$\begin{pmatrix} \mu^{\alpha}_{(\nu)} \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} . \begin{pmatrix} \mu^{\alpha}_{(\nu)} \\ 1 \\ 1 \\ 0 \end{pmatrix}. \tag{31}$$

While, in the context of the BFFT formalism, different choices for the degenerated matrix X [24] leads to different gauge invariant version of the second class model. It is important to mentioned here that, in Ref.[24], a suitable interpretation and explanation about this result was not given and, also, the author pointed out that not all solutions of the first step of the BFFT method can leadto a solutioninthe second one, which jeopardize the BFFT embedding process. From the symplectic embedding formalism, this kind of problem can be clarified and understood as well: (*i*) first, some choices for the degenerated matrix X lead to different gauge invariant version of the second class model, however, they are dynamically equivalent, as shown by the symplectic formalism; (*ii*) second, some choices for the degenerated matrix X can generate solutions in the first step of the BFFT method that can not lead to a pleasant solution in the second one, which hazards this WZ embedding process. This is interpreted by the symplectic point of view as been the impossibility to introduce some gauge symmetries into the model and, as consequence, an infinite numbers of WZ counter-terms in Hamiltonian [24] are required. This also happens in the iterative constraint conversion [11], since there is an arbitrariness to change the second class nature of the constraints in first one. Now, it becomes clear that the arbitrariness presents on the BFFT and iterative constraint conversions methods has its origin on the choice of the zero-mode, which generates the desired WZ gauge symmetry.

From relation, Eq. (29), together with Eq. (26) and Eq. (28), some differential equations involving $G\,(a_i, p_i, \lambda_p)$are obtained, namely,

$$
\begin{aligned}
0 &= \mu^{\alpha}_{(\nu)} f^{(0)}_{\alpha\beta} + f_{\eta\beta}, \\
0 &= \mu^{\alpha}_{(\nu)} f^{(0)}_{\alpha\eta} + a f_{\theta\eta} + b f_{\pi_\theta\eta}, \\
0 &= f^{(0)}_{\eta\theta} + b, \\
0 &= f^{(0)}_{\eta\pi_\theta} - a \quad .
\end{aligned} \tag{32}
$$

Solving the relations above, some correction terms, within $\sum_{m=0}^{\infty}\mathcal{G}^{(m)}(a_i, p_i, \lambda_p)$, can be determined, also including the boundary conditions $(\mathcal{G}^{(0)}(a_i, p_i, \lambda_p = 0))$.

In order to compute the remaining corrections terms of $G\,(a_i, p_i, \lambda_p)$, we impose that no more constraints arise from the contraction of the zero-mode $(\tilde{\nu}^{(1)\tilde{\alpha}}_{(\nu)(a)})$ with the gradient of potential $\tilde{V}^{(1)}\,(a_i, p_i, \lambda_p)$. This condition generates a general differential equation, which reads as

$$
\begin{aligned}
0 &= \tilde{\nu}^{(1)\tilde{\alpha}}_{(\nu)(a)} \frac{\partial \tilde{V}^{(1)}(a_i, p_i, \lambda_p)}{\partial \tilde{\xi}^{(1)\tilde{\alpha}}} \\
&= \mu^{\alpha}_{(\nu)} \left[\frac{\partial V^{(1)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} + \frac{\partial G(a_i, p_i, \theta, \pi_\theta)}{\partial \xi^{(1)\alpha}} \right] + a \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial G(a_i, p_i, \lambda_p)}{\partial \pi_\theta} \\
&= \mu^{\alpha}_{(\nu)} \left[\frac{\partial V^{(1)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} + \sum_{m=0}^{\infty} \frac{\partial \mathcal{G}^{(m)}(a_i, p_i, \lambda_p)}{\partial \xi^{(1)\alpha}} \right] + a \sum_{n=0}^{\infty} \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \theta} \\
&+ b \sum_{m=0}^{\infty} \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}.
\end{aligned} \tag{33}
$$

The last relation allows us to compute all correction terms in order of λ_p, within $G^{(n)}$ $(a_i p_i, \lambda_p)$. Note that this polynomial expansion in terms of λ_p is equal to zero, subsequently, all the coefficients for each order in this WZ variables must be identically null. In view of this, each correction term in orders of λ_p can be determined as well. For a linear correction term, we have

$$
0 = \mu^{\alpha}_{(\nu)} \left[\frac{\partial V^{(0)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} + \frac{\partial \mathcal{G}^{(0)}(a_i, p_i)}{\partial \xi^{(1)\alpha}} \right] + a \frac{\partial \mathcal{G}^{(1)}(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial \mathcal{G}^{(1)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}, \tag{34}
$$

where the relation $V^{(1)} = V^{(0)}$ was used. For a quadratic correction term, we get

$$
0 = \mu^{\alpha}_{(\nu)} \left[\frac{\partial \mathcal{G}^{(1)}(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right] + a \frac{\partial \mathcal{G}^{(2)}(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial \mathcal{G}^{(2)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}. \tag{35}
$$

From these equations, a recursive equation for $n \geq 2$ is proposed as

$$
0 = \mu^{\alpha}_{(\nu)} \left[\frac{\partial \mathcal{G}^{(n-1)}(a_i, p_i, \lambda_p)}{\partial \xi^{(0)\alpha}} \right] + a \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \theta} + b \frac{\partial \mathcal{G}^{(n)}(a_i, p_i, \lambda_p)}{\partial \pi_\theta}, \tag{36}
$$

which allows us to compute the remaining correction terms in order of θ and π_θ. This iterative process is successively repeated up to Eq. (33) becomes identically null. Then, the new symplectic potential is written as

$$
\tilde{V}^{(1)}(a_i, p_i, \lambda_p) = V^{(0)}(a_i, p_i) + G(a_i, p_i, \lambda_p). \tag{37}
$$

Due to this, the gauge invariant Hamiltonian is obtained explicitly and the zero-mode $\tilde{\nu}^{(1)\tilde{\alpha}}_{(\nu)(a)}$ is identified as being the generator of the infinitesimal gauge transformation, given by

$$\delta\tilde{\xi}^{\tilde{\alpha}}_{(\nu)(a)} = \varepsilon\tilde{\nu}^{(1)\tilde{\alpha}}_{(\nu)(a)}, \tag{38}$$

where ε is an infinitesimal parameter.

3. Realization of the faddeevian regularization in the CSM

In Ref.[3] the authors showed that the χQED_2 can be quantized in a consistent an unitary way just including the bosonization ambiguity parameter satisfying the condition $a \geq 1$ to avoid tachyonic excitations. Afterwards, Rajaraman [4] studied the canonical structure of the model and showed that there are two cases $a>1$ and $a=1$ belonged to distinct classes: the $a=1$ case presenting four second class constraints belongs to an unambiguous class containing only one representative, while the $a>1$ case, presenting only two second class constraints, represents a continuous one-parameter class. Due to the distinct constraint structures, the $a>1$ class presents both massless excitation and massive scalar excitation $(m^2 = \frac{e^2a^2}{a-1})$, while in the other case there is only massless excitation. It occurs because the chiral Schwinger model with the familiar regularization $a>1$ has more physical degrees of freedom than it would have were it gauge invariant. However, it is not match with the Faddeev's case[14] whose the commutator between the Gauss law constraint is non-zero. Here, the second class nature of the set of constraints is due to the Poisson bracket of π_0 (canonical momentum conjugated to the scalar potential A_0)and G (the Gauss law) becomes non-zero.

In[13] the author showed that the Poisson bracket involving the Gauss law constraint is nonzero, indeed exhibits the Faddeev's anomaly. In views of this, the author concluded that the Faddeevian regularization not belong to the class of the usual regularizations. In this new scenario, the gauge field is once again a massive excitation, but the massless fermion that is present has, unlike the usual case, a definite chirality opposite to that entering the interaction with the electromagnetic field. In this work, Mitra showed that with an appropriated choice of the regularization mass term it is possible to close the second class algebra with only three second class constraints. Although this modelis not manifestly Lorentz invariant, the Poincaré generators have been constructed [13] and shown to close the relativistic algebra on-shell. The mainfeature of this new regularization is the presence of a Schwinger term in the Poisson bracket algebra of the Gauss law, which limits the set to only three second class constraints. To see this we start with the CSM Lagrangian with Faddeevian regularization proposed in [15], reads as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + q \left(g^{\mu\nu} + b \, \epsilon^{\mu\nu} \right) \partial_\mu \phi \, A_\nu + \frac{1}{2} q^2 A_\mu M^{\mu\nu} A_\nu \,, \quad (39)$$

where the Mitra's regulator was properly generalized. Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $g^{\mu\nu}$ = diag (+1,−1) and $\epsilon^{01} = -\epsilon^{10} = \epsilon_{10} = 1$. b is a chirality parameter, which can assume the values $b = \pm 1$. The mass-term matrix $M_{\mu\nu}$ is defined as

$$M^{\mu\nu} = \begin{pmatrix} 1 & \alpha \\ \alpha & \beta \end{pmatrix}. \quad (40)$$

To resemble the Rajaraman's a= 1 class it was chosen unity coefficient for A_0^2 term. The Rajaraman's class is a singular case in the *space of parameters* since its canonical description has the maximum number of constraints with no massive excitation. This case is reproduced in Eq. (39) if α= 0 and $\beta = -1$ in Eq. (40). However, a new class appears if we assume a nonvanishing value for α. For example, with Mitra's choice, $\alpha = -1$ and $\beta = -3$, the photon once again becomes massive ($m^2 = 4q^2$), but the remaining massless fermion has a definite chirality, opposite to that entering the interaction with the electromagnetic field. Although this particular choice is too restrictive, another choices are also possible, that leads, eventually, to a new and interesting consequences. In this work the coefficients α and β are arbitrary *ab initio*, but the mass spectrum will impose a constraint between them. This is verified using the symplectic method[17, 18].

Afterhere, the symplectic method will be used to quantize the original second class model, obtaining the Dirac's brackets and the respective reduced Hamiltonian as well. In order to implement the symplectic method, the original second-order Lagrangian in the velocity, given in Eq. (39), is reduced into its first-order, namely,

$$\mathcal{L}^{(0)} = \pi_\phi \dot{\phi} + \pi^1 \dot{A}_1 - U^{(0)}, \quad (41)$$

where the symplectic potential $U^{(0)}$ is

$$\begin{aligned} U^{(0)} &= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_0(\pi_1' + q^2(\alpha - b)A_1 + q\pi_\phi - qb\phi') \\ &- A_1 \left(qb\pi_\phi + \frac{1}{2} q^2 (\beta - b^2) A_1 - q\phi' \right), \end{aligned} \quad (42)$$

where prime represents spatial derivative.

The zeroth-iterative symplectic tensor is given by

$$f^{(0)}(x,y) = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \delta^2(x-y). \tag{43}$$

This matrix is obviously singular, thus, it has the following zero-mode,

$$\nu^{(0)} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \tag{44}$$

that when contracted with the gradient of the potential $U^{(0)}$ generates a constraint, given by,

$$\begin{aligned} \Omega_1 &= \int \nu_\alpha^{(0)}(x) \frac{\partial U^{(0)}(y)}{\partial \xi_\alpha^{(0)}(x)} \, \mathrm{d}y \\ &= \pi_1' + q^2(\alpha - b)A_1 + q\pi_\phi - qb\phi', \end{aligned} \tag{45}$$

that is identified as being the Gauss law, which satisfies the following Poisson algebra,

$$\{\Omega_1(x), \Omega_1(y)\} = -2q^2\alpha\partial_x\delta^2(x-y), \tag{46}$$

where ∂x represents $\frac{\partial}{\partial x}$. The corresponding bracketin the familiar regularization scheme [3] is zero. That is why the Mitra's model is the one which is in accordance with the Faddeev's scenario[14] in which the Gauss law commutator has a chiral anomaly.

Bringing back the constraint Ω_1 into the canonical sector of the first-order Lagrangian by means of a Lagrange multiplier η, we get the first-iterative Lagrangian $L^{(1)}$, reads as

$$\mathcal{L}^{(1)} = \pi_\phi\dot{\phi} + \pi^1\dot{A}_1 + \Omega_1\dot{\eta} - U^{(1)}, \tag{47}$$

with the first-order symplectic potential,

$$U^{(1)} = \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_1\left(qb\pi_\phi + \frac{1}{2}q^2(\beta - b^2)A_1 - q\phi'\right). \tag{48}$$

The corresponding matrix $f^{(1)}(x,y)$ is then[27]

$$f^{(1)}(x,y) = \begin{pmatrix} 0 & -1 & 0 & 0 & -qb\partial_y \\ 1 & 0 & 0 & 0 & q \\ 0 & 0 & 0 & -1 & q^2\sigma \\ 0 & 0 & 1 & 0 & \partial_y \\ qb\partial_x & -q & -q^2\sigma & -\partial_x & 0 \end{pmatrix} \delta^2(x-y), \tag{49}$$

that is singular and has a zero-mode, given by,

$$\nu^{(1)} = (\, -q \quad -qb\partial_x \quad -\partial_x \quad q^2\sigma \quad 1\,), \tag{50}$$

that generates the following constraint,

$$\begin{aligned} \Omega_2 &= \int \nu_\alpha^{(1)}(x) \frac{\partial U^{(1)}(y)}{\partial \xi_\alpha^{(1)}(x)} \mathrm{d}y \\ &= q^2\sigma\pi_1 + q^2(\beta+1)A_1'. \end{aligned} \tag{51}$$

The twice-iterated Lagrangian, obtained after including the constraint, given in Eq. (51), into Lagrangian, Eq. (47), by means of a Lagrange multiplier ζ, reads

$$\mathcal{L}^{(2)} = \pi_\phi\dot{\phi} + \pi^1\dot{A}_1 + \Omega_1\dot{\eta} + \Omega_2\dot{\zeta} - U^{(2)}, \tag{52}$$

where $U^{(2)} = U^{(1)}\,|\Omega_2=0$.

The matrix $f^{(2)}(x,y)$ is [28]

$$f^{(2)}(x,y) = \begin{pmatrix} 0 & -1 & 0 & 0 & -qb\partial_y & 0 \\ 1 & 0 & 0 & 0 & q & 0 \\ 0 & 0 & 0 & -1 & q^2\sigma & q^2\varrho\partial_y \\ 0 & 0 & 1 & 0 & \partial_y & q^2\sigma \\ qb\partial_x & -q & -q^2\sigma & -\partial_x & 0 & 0 \\ 0 & 0 & -q^2\varrho\partial_x & -q^2\sigma & 0 & 0 \end{pmatrix} \delta(x-y), \tag{53}$$

that is a nonsingular matrix. Then we can identify it as the symplectic tensor of the constrained theory. The inverse of $f^{(2)}\,(x,y)$ gives, after a straightforward calculation, the Dirac brackets among the physical fields,

$$
\begin{aligned}
\{\,\phi(x)\,,\,\phi(y)\,\}^{*} &= -\frac{1}{2\,\alpha}\,\Theta(x-y)\,,\\
\{\,\phi(x)\,,\,\pi_{\phi}(y)\,\}^{*} &= \frac{(2\alpha-b)}{2\alpha}\delta(x-y)\,,\\
\{\,\phi(x)\,,\,A_1(y)\,\}^{*} &= -\frac{1}{2q\alpha}\,\delta(x-y)\,,\\
\{\,\phi(x)\,,\,\pi_1(y)\,\}^{*} &= \frac{q\sigma}{2\,\alpha}\,\Theta(x-y)\,,\\
\{\pi_{\phi}(x),\pi_{\phi}(y)\}^{*} &= \frac{1}{2\,\alpha}\,\partial_x\delta(x-y)\,,\\
\{\pi_{\phi}(x),A_1(y)\}^{*} &= \frac{b}{2q\,\alpha}\partial_x\delta(x-y)\,,\\
\{\pi_{\phi}(x),\pi_1(y)\}^{*} &= -\frac{qb\sigma}{2\alpha}\delta(x-y)\,,\\
\{A_1(x),A_1(y)\}^{*} &= \frac{1}{2\,q^2\,\alpha}\,\partial_x\delta(x-y)\,,\\
\{A_1(x),\pi_1(y)\}^{*} &= \left(\frac{\alpha+b}{2\,\alpha}\right)\delta(x-y)\,,\\
\{\pi_1(x),\pi_1(y)\}^{*} &= -\frac{q^2\sigma^2}{2\,\alpha}\Theta(x-y)\ ,
\end{aligned}
\tag{54}
$$

where $\Theta\,(x-y)$ represents the signfunction. This means that the Mitra model is not a gauge invariant theory. The second-iterative symplectic potential $U^{(2)}$ is identified as the reduced Hamiltonian, given by,

$$
\begin{aligned}
H_r = \int\ dx\ \Big\{&\frac{1}{2}\pi_1^2+\alpha\,\pi_1A_1'+q\,(1-b\,\alpha)\,A_1\phi'+\phi'^2+\\
+\ &\frac{b}{q}\,\phi'\pi_1'+\frac{1}{2\,q^2}\pi_1'^2+\frac{1}{2}\,q^2\left(\alpha^2-\beta\right)A_1^2\Big\}\ ,
\end{aligned}
\tag{55}
$$

where the constraints $\Omega 1$ and $\Omega 2$ were assumed equal to zero in a strong way.

At this stage, we are interested to compute the energy spectrum. To this end, we use the reduced Hamiltonian, Eq. (55), and the Dirac's brackets, Eq. (54), to obtain the following equations of motion for the fields,

$$
\begin{aligned}
\dot\phi &= b\,\phi'-\frac{1}{q}\pi_1'+\frac{q}{2\,\alpha}\left(1-2\alpha^2+\beta\right)A_1,\\
\dot\pi_1 &= -b\pi_1'+\frac{q^2}{2\,\alpha}\Big[(b-\alpha)(1-\alpha^2)-(b+\alpha)(\alpha^2-\beta)\Big]\,A_1,\\
\dot A_1 &= \left(\frac{\alpha+b}{2\alpha}\right)\pi_1-\left(\frac{1+\beta}{2\alpha}\right)A_1'\ .
\end{aligned}
\tag{56}
$$

Now, we are ready to determine the spectrum of the model. Isolating π^1 from the constraint $\Omega 2$, given in Eq. (51), and substituting in Eq. (56), we get

$$\left(\frac{2\,\alpha}{\alpha+b}\right)\ddot{A}_1 \;+\; b\left(\frac{1+\beta}{\alpha+b}\right)A_1'' = -\left(\frac{2\,b\,\alpha}{\alpha+b}+\frac{1+\beta}{\alpha+b}\right)\dot{A}_1' \;+ \\ +\;\frac{q^2}{2\alpha}\left[(b-\alpha)\left(1-\alpha^2\right)-(b+\alpha)\left(\alpha^2-\beta\right)\right]A_1\,. \tag{57}$$

To get a massive Klein-Gordon equation for the photon field we must set

$$(1+\beta)+b\,(2\alpha)=0, \tag{58}$$

which relates α and β and shows that the regularization ambiguity adopted in [13] canbe extended to a continuous one-parameter class (for a chosen chirality). Wehave, using Eq. (57) and Eq. (58), the following mass formula for the massive excitation of the spectrum,

$$m^2 = q^2\,\frac{1+b\,\alpha}{b\,\alpha}\,. \tag{59}$$

Note that to avoid tachyonic excitations, α is further restricted to satisfy $b\alpha = |\alpha|$, so $\alpha \rightarrow -\alpha$ interchanges from one chirality to another. Observe that in the limit $\alpha \rightarrow 0$ the massive excitation becomes infinitely heavy and decouples from the spectrum. This leads us back to the four-constraints class. It is interesting to see that the redefinition of the parameter as $a=1+|\alpha|$ leads to,

$$m^2 = \frac{q^2 a^2}{a-1}, \tag{60}$$

which is the celebrate mass formula of the chiral Schwinger model, showing that the parameter dependence of the mass spectrum is identical to both the Jackiw-Rajaraman and the Faddeevian regularizations.

Let us next discuss the massless sector of the spectrum. To disclose the presence of the chiral excitation we need to diagonalize the reduced Hamiltonian, Eq. (55). This procedure may, at least in principle, impose further restrictions over α. This all boils down to find the correct linear combination of the fields leading to the free chiral equation of motion. To this end we substitute π^1 from its definition and $A1$ from the Klein-Gordon equation into Eq. (56) to obtain

$$
\begin{aligned}
0 \;=\; & \frac{\partial}{\partial t}\left\{\phi + \frac{q}{2\alpha}\left(\frac{2+2\,b\,\alpha-\alpha^2}{m^2}\right)\dot{A}_1 + \frac{1}{q}\left(\frac{\alpha}{\alpha+b}\right)A_1'\right\} - \\
- \; & \frac{\partial}{\partial x}\left\{b\,\phi - \frac{1}{q}\left(\frac{\alpha}{\alpha+b}\right)\dot{A}_1 + \left[\frac{q}{2\alpha}\left(\frac{2+2\,b\,\alpha-\alpha^2}{m^2}\right) - \frac{1}{q}\left(\frac{2\,b\,\alpha}{\alpha+b}\right)\right]\right\}\;. \quad (61)
\end{aligned}
$$

This expression becomes the equation of motion for a self-dual boson χ, given by,

$$\dot{\chi} - b\,\chi' = 0, \tag{62}$$

if we identify the coefficients for $\dot{A}_1$ and A_1' in the two independent terms of Eq. (61) with,

$$\chi = \phi + \frac{1}{q}\left(\frac{\alpha}{\alpha+b}\right)\left(A_1' - b\dot{A}_1\right). \tag{63}$$

This field redefinition, differently from the case of the massive field whose construction imposed condition, Eq. (58), does not restrain the parameter α any further. Using the constraints $\Omega 1$ and $\Omega 2$, given in Eq. (45) and Eq. (51) respectively, and Eq. (62), all the fields can be expressed as functions of the free massive scalar $A1$ and the free chiral boson χ, interpreted as the bosonized Weyl fermion. The main result of this section is now complete, i.e., the construction of the one-parameter class regularization generalizing Mitra's proposal. In the next section, this general model will be reformulate as a gauge theory in order to eliminate the chiral anomaly.

4. Embedding the chiral Schwinger model with the Faddeevian regurarization

In order to begin with the WZ embedding formulation, some WZ counter-terms, embraced by LWZ, are introduced into the original Lagrangian, leading to the gauge invariant Lagrangian, namely,

$$\tilde{\mathcal{L}} = \mathcal{L} + \mathcal{L}_{WZ}\;. \tag{64}$$

In agreement with the symplectic embedding formalism, the invariant Lagrangian above, Eq. (64), must be reduced into its first-order form, given by

$$\tilde{\mathcal{L}}^{(0)} = \pi_\phi\dot{\phi} + \pi^1\dot{A}_1 + \pi_\theta\dot{\theta} - \tilde{U}^{(0)}, \tag{65}$$

where

$$\begin{aligned}\tilde{U}^{(0)} &= \frac{1}{2}(\pi_1^2+\pi_\phi^2+\phi'^2) - A_0(\pi_1' + q^2(\alpha-b)A_1 + q\pi_\phi - qb\phi') \\ &- A_1\left(qb\pi_\phi + \frac{1}{2}q^2(\beta-b^2)A_1 - q\phi'\right) + G(\phi,\pi_\phi,A_0,A_1,\pi_1,\lambda_p), \end{aligned} \tag{66}$$

where the arbitrary function G is

$$G(\phi,\pi_\phi,A_0,A_1,\pi_1,\lambda_p) = \sum_{n=0}^{\infty} \mathcal{G}^{(n)}(\phi,\pi_\phi,A_0,A_1,\pi_1,\lambda_p), \tag{67}$$

where the function expanded in terms the WZ variables ($\lambda_p = (\theta, \pi_\theta)$)is given by

$$\mathcal{G}^{(n)} \sim (\lambda_p)^n. \tag{68}$$

The symplectic variables are given by $\xi_{\tilde\alpha} = (\phi, \pi_\phi, A_0, A_1, \pi_1, \lambda p)$. The corresponding symplectic matrix $f^{(0)}(x,y)$ reads

$$\tilde{f}^{(0)}(x,y) = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \delta^{(2)}(x-y). \tag{69}$$

As the matrix is singular, it has a zero-mode,

$$\tilde{\nu}^{(0)} = (\,0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,) \tag{70}$$

which generates the following constraint

$$\Omega = -\pi_1' - q^2(\alpha-b)A_1 - q\pi_\phi + qb\phi' + \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \tag{71}$$

after the contraction with the gradient of the symplectic potential.

Following the symplectic embedding formalism, this constraint is introduced into the kinetical sector ofthe zeroth-iterative Lagrangian, Eq. (65), through a Lagrange multiplier (η), which leads to the first-iterative Lagrangian, given by

$$\widetilde{\mathcal{L}}^{(1)} = \pi_\phi \dot{\phi} + \pi_1 \dot{A}^1 + \pi_\theta \dot{\theta} + \Omega \dot{\eta} - \widetilde{U}^{(1)}, \tag{72}$$

where $U^{(1)} = U^{(0)} |\Omega{=}0$. Now, the symplectic variables are $\xi_{\alpha}^{-} = (\phi, \pi_\phi, A_0, A_1, \pi_1, \eta, \lambda_p)$ with the respective symplectic matrix,

$$\tilde{f}^{(1)} = \begin{pmatrix} 0 & -\delta^{(2)}(x-y) & 0 & 0 & 0 & f^{(1)}_{\phi\eta} & 0 & 0 \\ \delta^{(2)}(x-y) & 0 & 0 & 0 & 0 & f^{(1)}_{\pi_\phi\eta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f^{(1)}_{A_0\eta} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta^{(2)}(x-y) & f^{(1)}_{A_1\eta} & 0 & 0 \\ 0 & 0 & 0 & \delta^{(2)}(x-y) & 0 & f^{(1)}_{\pi_1\eta} & 0 & 0 \\ f^{(1)}_{\eta\phi} & f^{(1)}_{\eta\pi_\phi} & f^{(1)}_{\eta A_0} & f^{(1)}_{\eta A_1} & f^{(1)}_{\eta\pi_1} & 0 & f^{(1)}_{\eta\theta} & f^{(1)}_{\eta\pi_\theta} \\ 0 & 0 & 0 & 0 & 0 & f^{(1)}_{\theta\eta} & 0 & -\delta^{(2)}(x-y) \\ 0 & 0 & 0 & 0 & 0 & f^{(1)}_{\pi_\theta\eta} & \delta^{(2)}(x-y) & 0 \end{pmatrix}, \tag{73}$$

with

$$\begin{aligned}
f^{(1)}_{\phi\eta} &= qb\partial_y \delta^{(2)}(x-y) + \frac{\delta}{\delta\phi(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\
f^{(1)}_{\pi_\phi\eta} &= -q\delta^{(2)}(x-y) + \frac{\delta}{\delta\pi_\phi(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\
f^{(1)}_{A_0\eta} &= \frac{\delta}{\delta A_0(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\
f^{(1)}_{A_1\eta} &= -q^2(\alpha-b)\delta^{(2)}(x-y) + \frac{\delta}{\delta A_1(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\
f^{(1)}_{\pi_1\eta} &= -\partial_y \delta^{(2)}(x-y) + \frac{\delta}{\delta\pi_1(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)},
\end{aligned} \tag{74}$$

$$\begin{aligned}
f^{(1)}_{\theta\eta} &= \frac{\delta}{\delta\theta(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)}, \\
f^{(1)}_{\pi_\theta\eta} &= \frac{\delta}{\delta\pi_\theta(x)} \int \mathrm{d}\omega \sum_{n=0}^{\infty} \frac{\delta \mathcal{G}^{(n)}(\omega)}{\delta A_0(y)},
\end{aligned}$$

Now, in order to unveil the gauge symmetry and to put our result in perspective with others, we make a "educated guess" for the zero-mode, namely,

$$\widetilde{\nu}^{(1)\widetilde{\alpha}} = \left(q \quad qb\partial_x \quad 0 \quad \partial_x \quad 0 \quad 1 \quad \Delta \quad -q^2 c\partial_x/\Delta \right), \tag{75}$$

where$\Delta^2 = q^2\,(\beta+1)$ and c is a chiral parameter in the WZ sector. The contraction of this zero-mode with the symplectic tensor leads to eight differential equations, which are written as

$$\begin{aligned}
0 &= \int \mathrm{d}x \frac{\delta}{\delta\phi(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\
0 &= \int \mathrm{d}x \frac{\delta}{\delta\pi_\phi(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\
0 &= \int \mathrm{d}x \frac{\delta}{\delta A_0(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\
0 &= \int \mathrm{d}x \left[q^2(\alpha-b)\delta^{(2)}(x-y) - \frac{\delta}{\delta A_1(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right], \\
0 &= \int \mathrm{d}x \frac{\delta}{\delta\pi_1(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)}, \\
0 &= \int \mathrm{d}x \left[-q^2(\alpha+b)\partial_x\delta^{(2)}(x-y) + \partial_x \frac{\delta}{\delta A_1(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} + \right. \\
&+ \left. \Delta \frac{\delta}{\delta\theta(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} - \frac{q^2}{\Delta} c\partial_x \frac{\delta}{\delta\pi_\theta(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right], \\
0 &= \int \mathrm{d}x \left[-\frac{q^2}{\Delta} c\partial_x\delta^{(2)}(x-y) - \frac{\delta}{\delta\theta(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right], \\
0 &= \int \mathrm{d}x \left[-\Delta\delta^{(2)}(x-y) - \frac{\delta}{\delta\pi_\theta(y)} \int_\omega \sum_{n=0}^{\infty} \frac{\delta\mathcal{G}^{(n)}(\omega)}{\delta A_0(x)} \right].
\end{aligned} \tag{76}$$

From the fourth relation above, we get the boundary condition, which is written as

$$\mathcal{G}^{(0)} = q^2(\alpha-b)A_0A_1. \tag{77}$$

So, the zeroth correction term is

$$\mathcal{G}^{(0)} = q^2(\alpha - b)A_0A_1 + \mathcal{G}^{(0)}(A_1), \tag{78}$$

since from the third relation above, we see that $G^{(0)}$ has no quadratic dependence in terms of A_0. While from the seventh and eighth relation above, the first correction term, at least, is obtained partially as being

$$\mathcal{G}^{(1)} = -\frac{q^2}{\Delta}c\partial_1\theta A_0 - \Delta\pi_\theta A_0. \tag{79}$$

It is important to notice that the relation above can not envelop all of the first-correction terms, because some of them can not depend on the temporal component of the potential field. In view of this, we rewrite this term as

$$\mathcal{G}^{(1)} = -\frac{q^2}{\Delta}c\partial_1\theta A_0 - \Delta\pi_\theta A_0 + \mathcal{G}^{(1)}(A_1, \lambda_p). \tag{80}$$

Thus, the constraint, given in Eq. (71), becomes

$$\Omega = -\pi_1' - q\pi_\phi + qb\phi' - \frac{q^2}{\Delta}c\theta' - \Delta\pi_\theta. \tag{81}$$

This constraint satisfies the following Poisson algebra

$$\{\Omega(x),\ \Omega(y)\} = 2q^2(b - c)\partial_y\delta(x - y). \tag{82}$$

Note that the elimination of the chiral anomaly above imposes a condition on the chiral parameters present on the original model and on the WZ sector, b and c, respectively, *i.e*, $b= c$. Hence, there is a specific set of WZ gauge symmetries that can deal with the chiral anomaly.

The symplectic potential can be expressed as

$$\begin{aligned}
\tilde{U}^{(1)} &= \tilde{U}^{(0)}\,|_{\Omega=0} \\
&= \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) - A_1\left(qb\pi_\phi + \frac{1}{2}q^2(\beta - b^2)A_1 - q\phi'\right) \\
&+ \sum_{n=1}^{\infty}\mathcal{G}^{(n)}(A_1, \lambda_p) + \mathcal{G}^{(0)}(A_1).
\end{aligned} \tag{83}$$

Now, it becomes necessary to guarantee that no more constraint arises. To this end, weimpose that the contraction of the zero-mode, Eq. (75), with the gradient of the symplectic potential, Eq. (83), does not produce a new constraint, namely,

$$0 = \int \tilde{\nu}^{(1)\tilde{\alpha}} \frac{\partial U^{(1)}(y)}{\partial \tilde{\xi}^{(1)\tilde{\alpha}}(x)} \, \mathrm{d}y,$$

$$\begin{aligned} 0 = & \int \mathrm{d}y \left[-q^2(\beta+1)A_1(y)\partial_y \delta^{(2)}(x-y) + \sum_{n=1}^{\infty} \partial_x \frac{\delta \mathcal{G}^{(n)}(A_1,\lambda_p)(y)}{\delta A_1(x)} + \right. \\ + & \left. \partial_x \frac{\delta \mathcal{G}^{(0)}(A_1)(y)}{\delta A_1(x)} + \Delta \sum_{n=1}^{\infty} \frac{\delta \mathcal{G}^{(n)}(A_1,\lambda_p)(y)}{\delta \theta(x)} - \frac{q^2}{\Delta} b \partial_x \sum_{n=1}^{\infty} \frac{\delta \mathcal{G}^{(n)}(A_1,\lambda_p)(y)}{\delta \pi_\theta(x)} \right]. \end{aligned} \tag{84}$$

Which is a polynomial expression in order of the WZ fields (λ_p). For the zeroth relation written in terms of WZ fields, we get

$$\begin{aligned} 0 = & \int \mathrm{d}y \left[-q^2(\beta+1)A_1(y)\partial_y \delta^{(2)}(x-y) + \partial_x \frac{\delta \mathcal{G}^{(0)}(A_1)(y)}{\delta A_1(x)} + \right. \\ + & \left. \Delta \frac{\delta \mathcal{G}^{(1)}(A_1,\lambda_p)(y)}{\delta \theta(x)} - \frac{q^2}{\Delta} b \partial_x \frac{\delta \mathcal{G}^{(1)}(A_1,\lambda_p)(y)}{\delta \pi_\theta(x)} \right]. \end{aligned} \tag{85}$$

At this point, it is important to notice that some degeneracy appears, since we can not solve the relation above leading to an unique result, *i.e*, this relation has solution but it is not unique. At first, this sound quite bad, however, this shows how powerful the symplectic embedding formalism canbe: different choicesfor $G^{(0)}(A_1)$and $G^{(1)}(A_1,\lambda_p)$ leads to distinct gauge invariant Hamiltonian descriptions for the noninvariant model, but with the same WZ gauge symmetry. It is a new feature in the WZ embedding concept that could be revealed in the symplectic embedding formalism. On the other hand, this relation can lead to a hard computation of the gauge invariant symplectic potential just assuming a bad solution for $G^{(0)}(A_1)$ and $G^{(1)}(A_1,\lambda_p)$.

As we are interested in comparing our result with others, we tackle a fine solution which becomes the computation of correction terms in WZ fields an easy task. The chosen solutions for Eq. (85) are

$$\begin{aligned} \mathcal{G}^{(0)}(A_1) &= -\frac{q^4}{2\Delta^2} A_1^2, \\ \mathcal{G}^{(1)}(A_1,\lambda_p) &= \Delta A_1 \partial_1 \theta - \frac{q^2}{\Delta} b A_1 \pi_\theta. \end{aligned} \tag{86}$$

So, the symplectic potential becomes

$$\begin{aligned}\tilde{U}^{(1)} &= \frac{1}{2}(\pi_1^2+\pi_\phi^2+\phi'^2) - A_1\left(qb\pi_\phi + \frac{1}{2}q^2(\beta-b^2)A_1 - q\phi' +\right.\\ &+ \left.\frac{q^4}{2\Delta^2}bA_1 - \Delta\partial_1\theta + \frac{q^2}{\Delta}b\pi_\theta\right) + \sum_{n=2}^{\infty}\mathcal{G}^{(n)}(A_1,\lambda_p).\end{aligned} \tag{87}$$

Again, we use the Eq. (84), which allows us to compute the quadratic correction term in WZ fields,

$$\begin{aligned}0 &= \int \mathrm{d}y\left[\Delta\partial_1\theta(y)\partial_y\delta^{(2)}(x-y) - \frac{q^2}{\Delta}\pi_\theta(y)\partial_x\delta^2(x-y)+\right.\\ &+ \left.\Delta\frac{\delta\mathcal{G}^{(2)}(A_1,\lambda_p)(y)}{\delta\theta(x)} - \frac{q^2}{\Delta}\partial_x\frac{\delta\mathcal{G}^{(2)}(A_1,\lambda_p)(y)}{\delta\pi_\theta(x)}\right],\end{aligned} \tag{88}$$

which leads to the following solution,

$$\mathcal{G}^{(2)} = \frac{q^2}{2}(\partial_1\theta)^2 - \frac{1}{2}(\pi_\theta)^2. \tag{89}$$

As this last correction term $G^{(2)}$ has dependence only on the WZ fields, the correction terms $G^{(n)} = 0$ for $n \geq 3$. Due to this, the symplectic potential, identified as being the gauge invariant Hamiltonian, is

$$\begin{aligned}\tilde{\mathcal{H}} = \tilde{U}^{(1)} &= \frac{1}{2}\left(\pi_1^2+\pi_\phi^2+(\partial_1\phi)^2+(\partial_1\theta)^2-(\pi_\theta)^2\right)\\ &+ A_0\left(-\partial_1\pi_1 - q\pi_\phi + qb\partial_1\phi - \frac{q^2}{\Delta}b\partial_1\theta - \Delta\pi_\theta\right)\\ &+ A_1\left(-qb\pi_\phi + \frac{q^2}{2}\frac{\beta^2}{(\beta+1)}A_1 + q\partial_1\phi - \frac{q^2}{\Delta}b\pi_\theta + \Delta\partial_1\theta\right),\end{aligned} \tag{90}$$

which is the same result obtained in [11], when $\beta = -a$.

To complete the gauge invariant reformulation of the model, the infinitesimal gauge transformation will be also computed. In agreement with the symplectic method, the zero-mode, $\tilde{\nu}^{(1)}$, Eq. (75), is the generator of the infinitesimal gauge transformation ($\delta O = \varepsilon\tilde{\nu}$), given by

$$
\begin{aligned}
\delta\phi &= q\varepsilon, \\
\delta\pi_\phi &= -qb\partial\varepsilon, \\
\delta A_0 &= 0, \\
\delta A_1 &= -\partial\varepsilon, \\
\delta\pi_1 &= 0, \\
\delta\eta &= \varepsilon, \\
\delta\theta &= \Delta\varepsilon, \\
\delta\pi_\theta &= \frac{q^2}{\Delta}b\partial\varepsilon,
\end{aligned}
\tag{91}
$$

where ε is an infinitesimal time dependent parameter.

5. Final discussions

In this work the bosonized form of the CSM fermionic determinant parameterized by a single real number, which extends early regularizations [3, 13], was studied from the symplectic point of view. Afterwards, we reproduce the spectrum that has been shown in Ref. [15] to consist of a chiral boson and a massive photon field. The mass formula for the scalar excitation was shown to reproduce the Jackiw-Rajaraman result, while the massless excitation was shown to reproduce the Mitra result. In views of this, unitarity condition restraints the range of the regularization parameter in a similar way. This noninvariant model was reformulated as a gauge invariant model *via* the symplectic embedding formalism, where we were able to produce the gauge invariant version of the model, in order to eliminate the chiral anomaly in the Gauss law commutator. It is important to mention here that the gauge invariant version of CSM, given in Ref. [11], can be also obtained when $\beta = -a$. Notice that the invariant theory was obtained with the introduction of a finite numbers of WZ terms. It is also important to emphasize that we can obtain different Hamiltonian formulations for the model. Different choices of the zero-mode generates different gauge invariant versions of the second class system, however, these gauge invariant descriptions are dynamically equivalent, *i.e.*, there is the possibility to relate this set of independent zero-modes through canonical transformation [25]. Another interesting point discussed in this workis the geometrical interpretation given to the degeneracy of the matrix *X*presents on the BFFT formalism and the arbitrariness on the iterative method. Different choices for this matrix X, or way to turn second class constraints to first one in the iterative method, leads to distinct gauge invariant version of the second class model. But, from the symplectic embedding point of view, these invariant descriptions are equivalent. Besides, it was also possible to reveal an interesting feature in the symplectic embedding formulation: the possibility to introduce boundary conditions that has no dependence on the WZ variables, which opens new *modus* to runthe embedding process. Thisis a new concept which lifts the WZ embedding idea to the next level.

Acknowledgments

This work is supported in part by FAPEMIG and CNPq, Brazilian Research Agencies. One of us, W. Oliveira, would like to thank Prof. Dr. José Maria Filardo Bassalo, his friend and teacher, for the great stimulation gave in the beginning of his career.

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[26] It is important to notice thatvis not a fixed parameter

[27] $\sigma = \alpha - b$

[28] g= β+1

Letters & Testimonies

Geneva, September 2008
My Dear Bassalo,

As I said earlier commitments had the dates of the Symposium in his honor as a member and advisor of the International Conference diffraction 2008. I wish however that you forgive me for my lack of consideration with you as a dear, colleague and friend.

We met in Brasilia, when it was marked our lives professionally and humanely. We had the good fortune to meet men who influenced our scientific career but above all gave us courage to continue to build this country so unequal and unfair. Although we have gone in different directions in physics always had our professional contact. You have been an example to the young students. But you also was one of the most important of our group of Brasilia with your lidership. As Brecht would say, you are one of those men needed my dear.

I would like to wish you many years of work with that good humor as always and I hope to see you soon,
The friend and colleague,
Santoro

Geneve, setembro de 2008
Meu Caro Bassalo,

Como lhe disse havia compromissos anteriores às datas do Simpósio em sua homenagem como membro fundador e conselheiro internacional da Conferência Diffraction 2008. Gostaria no entanto que você me perdoe por esta minha falta de consideração com você um tão caro, colega e amigo.

Nos conhecemos em Brasília, ocasião em que nossas vidas foram marcadas profissional e humanamente. Tivemos a sorte de conhecer homens ilustres que influenciaram nossa carreira científica mas sobre tudo nos deram coragem para continuarmos a construção desse País tão desigual e injusto. Apesar de termos ido por diferentes direções na Física sempre mantivemos nosso contato profissional. Você tem sido um exemplo para os mais jovens. Mas você também foi uma das figuras mais importantes de nosso grupo de Brasília. Como diria Brecht, você é um daqueles homens imprescindíveis meu caro.

Eu gostaria de te desejar muitos anos de vida profissional com aquele bom humor de sempre e espero te ver em breve,

O amigo e colega,

Santoro

United States, September 2008

J. M. F. Bassalo: a great champion, a Samurai friend and an exemplary teacher.

I do not know of anybody else that has had a greater influence on the scientists and engineers of our state than my great brother-friend Bassalo. Countless generations of colleagues of our UFPA, directly or indirectly, have been inspired by this great champ. And I am no exception. Back in the times of being his teaching assistant for one of his physics courses, to the countless chats at his home office, my path towards science was traced out. It has also been a tremendous privilege to have collaborated scientifically with him.

He possesses impeccable honesty, coherent character and a sense of true friendship, that is, the friendship of a Samurai. He is one of the very few friends in my life who always spoke the truth. As another great friend Paulo de Tarso can testify, I have always mentioned that I could sign at the bottom of a blank piece of paper and hand it to Bassalo without worries.

His strong self-motivation, will power and never-ending dedication to teaching of science and technology are impressive. Indeed, he has been a great force for the good of science and the epicenter of a great scientific shockwave in our homeland.

Long live, Bassalo.

Antonio Nassar

Estados Unidos, setembro de 2008.

J. M. F. Bassalo: Um grande campeão, um amigo Samurai, e um exemplo de professor.

Não conheço ninguém que tenha mais influenciado os cientistas e engenheiros do nosso Pará do que o meu grande amigo-irmão de sempre Bassalo. Inúmeras gerações de colegas da nossa UFPA, direta ou indiretamente, foram inspiradas por este grande campeão. E eu não fui exceção. Desde os meus tempos de monitor de um dos seus cursos de Física, aos grandes e inúmeros bate-papos na sua antiga casa da São Jerônimo, meu caminho acadêmico foi iluminado. E ter colaborado cientificamente com ele foi um grande privilégio.

Um ser humano de honestidade impecável, caráter invariante ao tempo, e acima de tudo, um amigo Samurai. Um dos poucos amigos de vida que sempre falou a verdade. Como pode atestar meu grande amigo Paulo de Tarso, sempre disse que poderia assinar uma página em branco e confiar a este nosso amigo sem nenhuma reserva.

Sua motivação, força de vontade e dedicação incansáveis ao ensino de ciência *e tecnologia são impressionantes. Realmente, ele é a grande força para o bem e o epicentro da maior onda de choque científica que aconteceu na nossa terrinha.*

Vida longa, Bassalo.

Antonio Nassar

Macaé – RJ, September 9, 2008

To: The Organizing Committee
Amazonian Symposium on Physics
"Celebration in honor of Prof. José M. F. Bassalo"

Dear Sirs

I join myself to those who celebrate Prof. José M. F. Bassalo in this event, as part of the Amazonian Symposium on Physics, happening at Belém-PA, 9-12 Sept. 2008 .

Bassalo is the main founder of the Physics teaching and research in the State of Pará, and co-founder (together with me) of the Geosciences in the Federal University of Pará. No one, like Bassalo, spotted so many talents among the youth of Pará, leading them toward a research career either in Physics and Mathematics or Geosciences. He helped them on searching for a competent advisorship anywhere in the country or outside, or even in doing the follow-up of the students along their career.

Bassalo has been restless on doing this mission, facing the innumerous obstacles that underdevelopment imposes on Brazil and Amazonia, sometimes having to demand from the depth of his soul everything that the environment or the people were unable to provide.

Bassalo got his MSc and PhD degrees in Physics late, in the USP (São Paulo University), at a very hight personal sacrifice, then married having two sons, having to let his family behind for several years, and also to renunciate his previous profession of a successful civil engineer.

Bassalo nurse always multiple dreams, struggling for them for years, without loosing his faith and hope, even when the first results do not come all right. He tries again and again, even with more vigor if facing a second time.

Myself and Bassalo, we planned to create an Amazonian Geophysics Institute at the Federal University of Pará. We started a very long and well structured plan of formation of skilled personnel, who together with us would make this dream a reality, 10 years later. Both of us were persecuted, and excluded from certain positions and opportunities. We were almost arrested by the military dictatorship regime which took over the Government in Brasil for more than 20 years. Without fear and convinced of our purpose, we went on the battle field. We wone, since from such a dream the Geosciences Center of Federal University of Pará was raised as a respectable institution of teaching and research, and Geophysics became an arena of knowledge that is well set at various State of Brazil, as well as in the Amazonia.

The Physical Science itself in the Federal University of Pará, however moving at a lower velocity, comes improving considerably toward a better plateau of excellence and prestige, pushed on by the idealism, stimulus and inspiration of its founder, Prof. José Maria Filardo Bassalo.
Long life, my friend Bassalo !

Carlos Alberto Dias, Ph. D.
Doctor Honoris Causa by Federal University of Pará
and by Federal University of Bahia

Macaé – RJ, 09 de Setembro de 2008

Ao : Professor José Maria Filardo Bassalo

Junto-me aos que homenageiam o Prof. Bassalo neste evento, durante a realização do Simpósio Internacional de Física, em Belém – PA.

Bassalo é o fundador da pesquisa em Física no Estado do Pará, e o co-fundador (junto comigo) das geociências na UFPA. Ninguém, como ele, descobriu tantos talentos na juventude do Pará, orientando-os para uma carreira de pesquisa em Física, ou em Matemática ou em Geociências, os conduzindo em busca de um orientador competente onde quer que fosse, e os pastoreando durante o processo de formação deles, na busca de uma carreira de pesquisador profissional.

Bassalo tem sido incansável neste sagrado mister, enfrentado todo o desânimo que o atraso do Brasil e do Pará impõem , buscando no fundo de sua alma tudo aquilo que não pôde obter do meio ou dos circunstantes.

Bassalo formou-se físico na USP, a um elevado custo de sacrifício pessoal, já casado e com filhos, interrompendo seu convívio familiar, tendo de abandonar a carreira de engenheiro civil que brilhantemente conquistara em Belém do Pará.

Bassalo acalenta múltiplos sonhos, os persegue por anos a fio, sem deixar sucumbir nunca a esperança, mesmo quando o resultado de algum empreendimento não é de todo satisfatório. Parte novamente e com maior vigor, para o próximo embate.

Junto com Bassalo, sonhamos fazer surgir na UFPA um Instituto de Geofísica da Amazônia. Partimos para uma longa e bem elaborada formação do pessoal, que junto conosco iria fazer desse sonho uma realidade reprodutiva, 10 anos mais tarde. Fomos perseguidos, excluídos de certas entidades ou mesmo *oportunidades. Quase fomos presos pelo regime militar de exceção, que assolou o Brasil por mais de 20 anos. Destemidos e resolutos, partimos para a luta. Vencemos : deste sonho surgiu o atual Centro de Geociências da UFPA, e a Geofísica é hoje uma bandeira vitoriosa que tremula em vários Estados do território brasileiro, sobretudo na Amazônia.*

A Física da UFPA, propriamente dita, embora em marcha mais lenta, vem também conquistando novos patamares de excelência e reconhecimento, impulsionada pelo idealismo, incentivo e inspiração do seu fundador, o Prof. José Maria Filardo Bassalo.

Vamos à luta, Bassalo !

Prof. Carlos Alberto Dias, Ph. D
*Doutor **Honoris Causa** pela UFPA*
*Doutor **Honoris Causa** pela UFBA*

Rio de Janeiro, September 2008.
For a Friend: Professor José Maria Filardo Bassalo

Let me start this letter by telling all of you that I met Professor Bassalo :an Italian /Amazônida/Paraense Physicist and Civil Engineer; in the old days of September / 1975. I were fifteen years old at that time and Professor Bassalo had just arrived from São Paulo with a Doctor of Science (Ph.D) degree in Plasma Theoretical Physics. I were regarded as a gifted high school student and were somewhat excited by being called by him to have a scientific interview.

This interview, took place in the small office at his home (the so called "Puleirinho" da Avª Governador José Malcher by his close friends!). There, I told him that I had read by myself the Books of Butkov, Kreyszig, Leite Lopes and Adhemar Fonseca. I showed either to him my first "paper" (An assembling of Advanced Calculus Techniques) where I show how to solve formally The Schrödinger Equation as a composed power series as an independent variable the full potential –considered as a harmonic function in the Space. Apparently, this idea of mine were original and worked for the above remarked class of potentials!

After the interview, I were shaken by his "judgment" ; I should convice my parents to allow me to go to Rio de Janeiro in order to start my Post-Graduate Courses directly from my first year in High-School.

As a consequence (solely at age of 15!) , I saw myself reading under his (and Paulo de Tarso) advising (" just leisurely read the books and do not worry if you do not grasp them fully in the first time") ; the following "Post-Graduate" Books :

1) Bjorken and Drell –Relativistic Quantum Mechanics

2) Morse and Feedsbach –Methods of Mathematical Physics vol I and II.

Bassalo!, as a result of those memorable days of my adolescence (September 1975 /June 1978) , I am sure that from there all my works come from. By the way, I were approved (with astonishiment of the instructors) in two disciplines of the IM/UFRJ summer course (1976) for preparation for the full Master Program in Mathematics –I were not enroled in the Program because people asked me my Bachelor Degree in Math! (What Bizarre!)

Basalo.I call you an Italian firstly by the simple reason that your higher cherishment for studies and culture certainly come from yours Italian ancestors. You are an Amazônida too : you had the chance to went to places where you could have had a more research focused career.But you decided to remain here,in Pará.. Finally, I regard you as the foremost historical figure on Mathematical/ Physics on the Province of Pará in the second half of twenty century, as a direct result of yours books, research articles, scientific collaborations and most importantly, by yours tenacious fighting against academic darkness and academic "Gulags" which unfortunately still prevails in so many Departaments and Institutes

in Brazilian Universities.
Thank you and Congratulations

Luiz C. L. Botelho
Professor of Mathematics and Theoretical Physics

Rio de Janeiro, setembro de 2008.
Ao Professor e Amigo de longa data (33anos) José Maria Filardo Bassalo

Conheci o ilustre Professor Ítalo- Amazônida-Paraense J.Bassalo, nos idos de setembro de 1975 quando somente tinha 15 anos de idade e o Professor Bassalo recém-chegado do seu Doutoramento realizado na USP. A razão deste encontro foi relacionada ao fato de eu ter sido considerado à época um daqueles alunos especiais, o qual tinha apresentado um desenvolvimento intelectual/cognitivo de destaque em relação as turmas do 1º ano do curso técnico da saudosa ETFPa. Já naquela época, eu já estava "lendo" os livros do Butkov , Kreyszg (vol I & II), Adhemar Fonseca (vol I, III& IV) e Leite Lopes (Introdução à Física Atômica) –É importante ressaltar que eu já tinha estudado os 05 (cinco) volumes da LISA Biblioteca da Matemática Moderna e o vol I&III da coleção do A.Bravo Rey Física/ Química Modernas.

Para um adolescente, ter sido recebido pelo Professor Bassalo para uma entrevista no saudoso "Puleirinho" da Avª Gov José Malcher (escritório de estudos do Bassalo na residência do seu sogro) foi um fato muito marcante . Observo que os professores Eloy e "Carioca" –Professores de Matemática da Escola Técnica foram aqueles que me recomendaram para uma entrevista com o Professor Bassalo .Depois de conversar com o Bassalo e o estimado Paulo de Tarso, eles decidiram que eu realmente já deveria passar diretamente do 1º ano do Curso Técnico para o Curso de Mestrado no Rio de Janeiro; a exemplo de um tal de Mário Abud :paulista que dizia-se ter estudado toda a coleção dos livros do Landau ,antes de ter entrado na Graduação da USP!.

Eis que somente com 15 anos, eu me vi envolvido na preparação que o Bassalo e o Paulo planejaram para um curso de nivelamento à Pós-Graduação no IM/ UFRJ (verão de 1976) e sob o patrocínio de saudoso Professor de La Penha. A preparação consistia na "leitura" dos seguintes livros (pasmem!).
1-Bjorken and Drell –Relativistic Quantum Mechanics
2-Morse and Feesbach –Methods of Mathematical Physics/vol I E vol II.
3-Landau & Lifschitz –Theorie du Champ (em francês!)
4-Leech –Mecânica Analítica.

Resultado desta "Amazônida Orientação", iniciada em 1975: os meus Trabalhos

Científicos, as minhas Teses e as minhas 03 (três) Monografias recentemente publicadas .

Caro Bassalo! , chamo-lhe de Italiano porque certamente a cultura refinada da sociedade dos seus ancestrais contribuíram para a sua visão humanística da prática científica e estudos.Também chamo-lhe de Amazônida , porque o " tamanho" dos seus sonhos acadêmicos para as Instituições Culturais da Amazônia corretamente merecem esta classificação .E finalmente o considero um destacado Paraense –pela sua contribuição fundamental a modernização acadêmica de toda a Universidade Federal do Pará , através dos seus Livros, Artigos Científicos e "Lutas Hercúleas" travadas contra o obscurantismo acadêmico.

Parabéns e Obrigado
Luiz Carlos Lobato Botelho

Belém, September 10th 2008.

It is a great pleasure to take part in this meeting to honor my friend José Maria Bassalo, since he was undoubtedly one of the persons who had a decisive influence in my studies to become a physicist.

I have known Bassalo since I was a child, because I was his sister's Madalena student at elementary school. Later on, I was a student of his, when in the early sixties, he taught Physics at the Colégio Estadual Paes de Carvalho, a traditional high school in Belém.

Bassalo was a great teacher not only regarding his profound knowledge of the subject, but also in the sense of guiding the youth towards the scientific career. In my case, I recall that I had concluded the first year of studies in engineering, when he recommended myself and Carlos Alberto Lima for two scholarships with the intent of studying Physics at the Faculty of Philosophy of the Universidade do Brasil, in Rio de Janeiro. These two scholarships had been obtained by Carlos Dias that had given a course in the so called Nucleus of Physics and Mathematics of the Universidade do Pará and convinced the rector of the necessity of having a group of physicists in the Amazon region.

Carlos Alberto and I went to Rio and in the following years we returned many times to Belém. In those opportunities, we talked with Bassalo that would recommend students that had been outstanding in the courses he taught. Due to these initiatives, a whole group of students that would later constitute the Geophysics group in the Universidade do Pará, went to study in Rio.

In 1965, along with my class I was motivated to finish my course in the newly created Universidade de Brasília. Bassalo had decided to professionally face the career of a Physicist and also moved to Brasília. With him, came other students from Pará, as Antonio Gomes de Oliveira, José Augusto Dias and Fernando Pena. It was a fantastic time. The Professors were of highest quality, and we worked very hard, waking up really early and sleeping quite late. During the day, we would attend courses and teach as tutors. Unfortunately, at those days Brazil was not prepared for the advances that the Universidade de Brasília represented. There was a military intervention that resulted in the resignation of the Professors.

Bassalo returned to Belém until 1968 when he moved to São Paulo. We shared a flat at CRUSP, the residential dormitory of the Universidade de São Paulo until the time that the University was also invaded by the military.

When Prof. Jayme Tiomno, who was the leader of the theory group, was compulsorily retired I decided to move abroad to complete my education in Physics. Bassalo started to work with Mauro Cattani in Atomic Physics concluding his Master's and Doctorate degrees in Physics . He then returned to Belém where he has fought in every way possible to improve the scientific standards, through good scientific publications, articles about the history of science, teaching and advising students, and writing books.

I think that the Universidade do Pará has profited greatly from Bassalo's dedication and should be proud of having him as a member. I feel extremely honored to have his friendship and I wish him only the very best in all the years to come.
Marcelo Gomes

Belém, 10 de setembro de 2008.
É para mim um motivo de grande satisfação participar desta reunião em homenagem ao meu amigo José Maria Bassalo pois ele foi, sem dúvida, uma das pessoas que tiveram influência decisiva em minha formação de físico.
Conheço o Bassalo desde criança pois fui aluno da sua irmã Madalena na escola primária. Mais tarde, fui aluno do Bassalo quando ,no início da década de sessenta, ele ensinava Física no Colégio Estadual Paes de Carvalho, um colégio tradicional de Belém. O Bassalo foi um grande professor não somente no que concerne o seu conhecimento profundo da matéria como no sentido da orientação de jovens para a carreira cientifica. No meu caso, lembro que havia terminado o primeiro ano de engenharia civil quando ele indicou eu e o Carlos Alberto Lima para duas bolsas de estudos com a finalidade de estudar física na Faculdade de Filosofia da antiga Universidade do Brasil, no Rio de Janeiro. Essas bolsas tinham sido conseguidas por intermédio do Carlos Dias que havia dado um curso no então Núcleo de Física e Matemática da Universidade do Pará e convenceu o reitor da necessidade de ter um grupo de físicos na região Amazônica.
Eu e o Carlos Alberto fomos para o Rio e nos anos seguintes voltamos muitas vezes a Belém. Nessas oportunidades, conversávamos com o Bassalo que então nós indicava estudantes que haviam se destacado nos curso, que ele lecionava. Devido a essas iniciativas, foi para o Rio de Janeiro todo um grupo de estudantes que mais tarde nucleou a Geofísica na Universidade do Pará.
Em 1965, juntamente com minha classe , fui motivado para terminar o curso na recém criada Universidade de Brasília. O Bassalo havia decidido se dedicar profissionalmente à carreira de Físico e foi também para Brasília. Com ele também vieram outros paraenses, como o Antonio Gomes de Oliveira, José Augusto Dias e o Fernando Pena.
Era fantástico. Os Professores eram da mais alta qualidade e nós trabalhávamos bastante, acordando bem cedo e dormindo tarde. Durante o dia assistíamos os cursos e lecionávamos como monitores. Infelizmente o Brasil daquela época não estava preparado para os avanços que a Universidade de Brasília representava. Houve intervenção militar na Universidade o que ocasionou o pedido de demissão dos professores.
Bassalo voltou para Belém até que em 1968 foi para São Paulo. Moramos juntos no CRUSP, o conjunto residencial da Universidade de São Paulo até quando a Universidade foi também invadida pelos militares.

Quando aposentaram compulsoriamente o Prof. Jayme Tiomno que era o líder do grupo teórico, decidi ir para o exterior para completar minha formação em Física. O Bassalo começou a trabalhar com o Mauro Cattani em Física Atômica concluindo o mestrado e o doutorado em Física. Ele então retornou para Belém onde tem lutado de todas as formas possíveis para melhorar o nível cientifico, quer seja através de boas publicações cientificas, artigos sobre a história da ciência, dando aulas e orientando estudantes e escrevendo livros didáticos.

Eu penso que a Universidade do Pará lucrou grandemente com a dedicação do Bassalo e deveria se orgulhar em tê-lo em seu corpo docente. Eu me sinto honrado pela sua amizade e lhe desejo muitas felicidades nos anos vindouros.
Marcelo O. C. Gomes

Paris, September 9, 2008

Dear Bassalo,

I would like to be present with you and the other friends in the homage which is being paid to you for the commemoration of your seventieth birthday. I deeply regret not being able to travel from Paris to Belém to be with you. I would like however to say something about our friendship, which grows up with time.

We met at the University of Brasília, when you went from Belém to Brasília to work with me. There was immediately comprehension in the ideals that we shared: we wanted to work for the future. We dreamed of a better Brazil and a future with more justice. That dream was interrupted by the violence of a dictatorialship government, but was not extinguished. You have not changed, you continue to be the same Bassalo.

You have been persecuted during the dictatorship. You received a fellowship from the French government to make a Ph. D. in France, but the president of the University of Pará of that period did not resist the pressure of the political police and refused to give you a permission of absence.

After that you had other difficulties in the period you have spent in the University of São Paulo.

In spite of those difficulties nothing, absolutely nothing, affected your determination to work for education, for science, for the intellectual formation of the new generations. You have continued the task you have assigned for yourself in matter of education, giving examples with your lectures, your activities as a professor, your articles, your books, your life.

Such courageous attitudes do affect the life of a person in several aspects. You had the great chance of counting with the support of Célia, the companion for ever, present in all occasions.

Not being able to travel to Belém to participate in the homage which will be paid to you, I send to you and Célia my best thoughts and fortes abraços,
Salmeron

Paris, setembro de 2008.

Caro Bassalo,

Eu gostaria de estar presente, juntamente com outros amigos e admiradores, na homenagem que lhe é prestada. Lamento profundamente não poder estar com vocês. Mas quero deixar meu testemunho da grande amizade que nos liga há anos.

Conhecemo-nos na UnB, quando você foi de Belém para Brasília para trabalhar comigo. Muita compreensão mútua em ideais comuns e muita esperança nos *uniam; queríamos trabalhar para o futuro. Sonhávamos com um Brasil melhor, com uma sociedade mais justa. O sonho foi interrompido com a violência de um regime ditatorial, mas não acabou. Você não mudou, continuou sendo o mesmo.*

Você foi perseguido. Tendo obtido uma bolsa para fazer doutorado na França, o reitor da Universidade do Pará da época não resistiu a pressões e não lhe deu licença para se ausentar da universidade. Depois você teve outras dificuldades no período que passou na USP.

Nada, absolutamente nada, abalou o seu caráter e sua determinação em trabalhar para a educação, para a ciência, para a formação de jovens das novas gerações. Você continuou, dando exemplo com seus cursos, suas atividades como professor, seus artigos, seus livros, sua vida.

Atitudes como as suas afetam a vida. Para enfrentar as dificuldades, você teve a sorte de poder contar com o apoio da Célia, a companheira de sempre, presente em todas as ocasiões.

Não podendo fazer a viagem para Belém para participar da festa em sua homenagem, daqui de longe envio para você e para Célia um forte abraço,
Salmeron

Bassalo: Teacher, Scientist, Chronicler and Motivator

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There wasn't a semester in my academic life, as a Physics teacher, that I hadn't done the following comment to my students: "Professor JOSÉ MARIA FILARDO BASSALO was, no doubt, the best teacher I have had in my academic life as a student". And the reason for this assertion, I believe, will be easier to be understood in the lines below. I consider myself an idealist. Since I was a boy, I was fascinated by Mathematics and Science. I wished I would be a teacher, ascientist. In 1976, as a student of Curso Cientfico (HighSchool) at "Colégio Estadual Paes de Carvalho", as he himself says in his article "25 ANOS DE MEU DOUTORADO EM FÍSICA" (CBPF-CS-003/00),"dearest and famous CEPC", I decided to make the "vestibular" to Curso de Bacharelado em Matemtica (Bachelor of Science) from Universidade Federal do Pará (UFPa). I started attending that course in the following year. I wished I would be a theoretical physicist, so I decided to study Mathematics. My contact with Bassalo started through a classmate from UFPa called Luis Carlos Lobato Botelho, a brilliant Physicist and Mathematical, today's professor at Instituto de Matemtica from Universidade Federal Fluminense (UFF). Botelho had an academic guiding by Bassalo since high school (at that time called Científico and nowadays called Ensino Médio). It was in 1977 that our friendship began and still lasts until now. I was bornin Belm, but I live in Juiz de Fora (UFJF). Unfortunately, I was never a regular student in his classes, but I attended many of them. I remember, in 1977, I was going to my Chemistry class and passed in front of Bassalos class. It was a Physics class. I stopped to see what Master Bassalo was saying and, then, I was completely fascinated by his way of teaching. I remember that in the left border of the blackboard, he put the names of all scientists that contributed to the point he was going to show in his class, he detailed with their birth and death dates and when they won the Nobel Prize. His classes had, at the same time, historical, philosophical andanalytical contents. In that semester, because of my absences, I failed in Chemistry because I decided to attend every class given by Bassalo at that time. Master Bassalo invited me to talk about Physics. So, I started going to his house. He lived in Governador José Malcher Avenue, Belms central region, filled with beautiful mango trees. It was an old but very beautiful house. But what really caught my attention was the little room where Master Basssalo worked. Not exactly the room, but the books all around there. I remember he said: "The books

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that lie in the horizontal are those that I didn't read". And I noticed they were just a few. I also remember tha the had a hammock in his little room and I found him many times lying in it. In some of my visits I sat on that hammock. Those visits are carefully tattooed in my memories. I miss our talks. I also remember one time that I told him about my fascination to the interrelation between Physics and Mathematics. And also, if the Universe began as an initial stage, described by a mathematical equation, I thought Nature was a mathematical representation. He said it was a mathematician point of view. In my first visit, he lent me the book "Diálogos sobre Física Atômica" by German physicist Werner Heisenberg (Portuguese edition in Brazil, the same text by Heisenberg is called "A Parte e o Todo", Editora Contraponto). This book is an autobiography based in dialogs remembrances with Bohr, Planck, Einstein, Dirac, Fermi, Pauli, Sommerfeld, Rutherford and many of his colleagues. This book had a strong influence in my life and increased my desire to be a theoretical physicist. I remember I was anxious to find my Master, I was very shy at that time, and was worried about what we would talk about. However, he made me feel at home, so that in a few minutes that heavy sensation had disappeared. I'd like to visit him every week, but I knew Master Bassalo used to work alot andmy visits would disturb. I myself had many things to study and read. Talk about Bassalo is talk about his "Crônicas de Física". I began to read them in 1978, in articles published in Belems newspapers. I always asked my mother to buy the Sunday's edition, when his articles were published. And his articles, fortunately, were put together and published, later, in many books called "Crônicas da Física" e "Nascimentos da Física". They are always on my desk when I prepare my classes. And they are either in the bibliography that I recommend to my students, in any subject. I want to register here my thanks for this work. Reference books that cant and musntt be lacking in any Physics teacher library.

At the end of the 70s, he gave me an article to read called "A Cadeia de Cognio da Física". In this article he wanted to show that Physics knowledge is obtained by a sequence of cognition with two fundamental links: reality-physics (observation-phenomena) and reality-mathematics (concept-law-formula-pattern), without having a preferential sense on the way of these links. I remember I commented something about them in our talks. And, as a good surprise, I received the article already published, with a dedication to me, (Ciência e Cultura, 33 (6), junho de 1981) and I could notice that my name was in his thanks comments. This event has made me honored and I saw how humble he is. In 1978 he got a me scholarship at Laboratório de Matemática Aplicada e Computação no Núcleo de Ciências Geofísicas e Geológicas (NCGG) at UFPa. I stayed in NCGG about three months until I traveled to Rio de Janeiro. I was so motivated that I seemed to be a researcher. During the months I stayed in NCGG I created a numerical integration method that I never applied. Bassalo was and is very worried about a *good scientifically formation of the students. In 1978, realizing my huge interest in study Mathematics and Physics, asked me if I would like to study outside Belém. I said yes. He talked about the possibility of being transferred to another institution. And then it happened. In 1979, January ,I was astudent of Mathematics at Universidade Federal do Rio de Janeiro (UFRJ). I remember in that summer I attended asubject (for the master level) about ordinary diferenttial equations with an excellent professor Maria Isabel Camacho. I was excellent in that subject, although I was only a undergraduation student. I think I was very lucky and this helped me to justify my transfer. As I know, my transfer was achieved with a help from professor Guilherme Maurício Souza Marcos de La Penha, a brilliant mathematician from Par, that was the director of Instituto de Matemática da UFRJ. Professor La Penha was either a student and Bassalo steacher (See the article written by Bassalo called "La Penha: Gerador e Gerenciador da Ciência" , CBPF-CS-014/97). I went through some diffculties far from my family and friends, especially from my friend Bassalo. But my idealism in becoming a scientist was higher. I undergraduated in Curso de Bacharel em Matemática in 1982, Mestre em Física at Pontifcia Universidade Católica do Rio de Janeiro (PUC-RIO) in 1991 and PH.D. in Science (Physics) in 1997 in the Physics Institute of UFRJ. From 1983 to 1990 I was a professor at the Mathematics Department of PUC-RIO. In 1991 I was a permanent teacher at the Physics Department of UFJF, where I am a teacher until nowadays. I am a scholar in research in Conselho Nacional de Pesquisa (CNPq) since 2003, I've oriented many Bachelor's Conclusion Monographies, Master's essays and a thesis of Doctorate. In 1997, Bassalo was at Juiz de Fora as a guest for the IX Physics Week from the Physics Department. He gave some courses about the History of Physics. They were two days of joy for me. In the night he stayed here in Juiz de Fora, we walked together downtown and talked about things, especially about Physics and our dearest Belm. A dream come true. In the next year, at his invitation, I was in Belm and gave two seminars at UFPa. Coming back to my first university, now as a Ph.D. in Physics, talk with students and teachers about my job, mycarrier, was reassuring. It was also a dream come true. I want to leave here my eternal gratitude to Master Bassalo, not only by his incentive given to me in my career, but for his support during all these years. His figure, his work, and his fight as a generator and manager of science are a good support for everyone who loves science. You, my friend, gave more than received. You have my recognition for ever and ever. I hope that the State of Pará will give you all the value you deserve. You, friend Bassalo, is an alive treasure from Pará. My thank you very much.*

Bassalo: Professor, Cientista, Cronista e Incentivador
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Não houve um semestre nesta minha vida acadêmica, como professor de Física, que não fizesse o seguinte tipo de comentário para com os meus alunos: "O professor JOSÉ MARIA FILARDO BASSALO foi, sem sombra de dúvida o melhor professor que tive em minha vida de estudante ". E a razão desta minha afirmação, acredito, será fácil de ser percebida nas linhas que se seguem.

Considero-me um idealista. Desde meu tempo de menino, alimento um fascínio pelas Ciências e a Matemática. Desejava ser um professor, um cientista. Em 1976, quando estudante do Curso Científico no Colégio Estadual "Paes de Carvalho", como ele mesmo diz em seu artigo "25 ANOS DE MEU DOUTORADO EM FíSICA" (CBPF-CS-003/00), "o querido e famoso CEPC", decidi prestar o vestibular para o Curso de Bacharelado em Matemática da Universidade Federal do Pará (UFPa). Entrei, então, para o Curso de Bacharelado em Matemática no ano seguinte. Eu desejava ser um físico teórico, daí minha opção por fazer Matemática. Meu contato com o Bassalo se deu por intermédio de um colega de curso, Luíz Carlos Lobato Botelho, um físico e matemático paraense brilhante, hoje professor titular no Instituto de Matemática da Universidade Federal Fluminense (UFF). O Botelho teve uma espécie de orientação acadêmica do Bassalo desde à época do científico (hoje, ensino médio). E foi a partir daí, em 1977, que nasceu nossa amizade, amizade esta que perdura até os dias de hoje.

Nasci em Belém, mas no momento resido em Juiz de Fora, MG. Sou professor associado no Departamento de Física da Universidade Federal de Juiz de Fora (UFJF). Nunca fui aluno regularmente inscrito, infelizmente, em qualquer de seus cursos, mas cheguei a assistir muitas das suas aulas. Lembro-me que na época, 1977, estava indo para minha aula de Química Geral e passei em frente à sala que Bassalo estava ministrando uma aula da disciplina Física Geral. Dei uma parada para assistir ao que o Mestre estava falando e fiquei, então, completamente fascinado pela maneira como ele conduzia sua aula. Lembro-me que, na margem esquerda do quadro, ele colocava o nome de todos os cientista que tomaram participação no desenvolvimento do tópico que seria abordado naquela aula, juntamente com os anos de nascimento, falecimento e de quando ganhou o Prêmio Nobel. Sua aula tinha, ao mesmo tempo, conteúdos histórico, filosófico e analítico. Naquele semestre fiquei reprovado por falta na disciplina Química Geral, pois tinha resolvido assistir a todas as aulas do Bassalo que ainda restavam no período. No término das aulas, ficávamos em volta do Mestre

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conversando sobre Física. Foi então que recebi um convite, por parte do Mestre, para estender esse bate papo sobre Física. E passei, então, a frequentar sua residência.

Ele residia na avenida Governador José Malcher, região central de Belém, arborizada por lindas mangueiras. Era uma casa antiga e muito bonita, com assoalho em tábua corrida e de um enorme corredor. Mas, o que me chamou mesmo à atenção, foi a salinha onde o Mestre trabalhava. Na realidade, não muito a salinha, mas sim a enorme quantidade de livros que contornavam suas paredes. Lembro-me que ele me disse o seguinte: "Os livros que estão na horizontal, ainda não os li". E pelo que pude perceber, eram poucos. Lembro-me, também, que ele tinha uma rede na salinha e por várias vezes o encontrei lendo deitado sobre ela. Em algumas de minhas visitas cheguei a sentar naquela rede. Estas visitas ao Mestre foram carinhosamente tatuadas em minhas memórias. Sinto muita falta de nossas conversas. Também lembro-me que, em uma de nossas conversas, falei-lhe de meu fascínio pela inter-relação existente entre Física e Matemática. E que se o Universo partiu de um estágio inicial, sendo este descrito por uma equação matemática, achava ser, então, a Natureza uma representação da Matemática. E ele comentou que esta era a visão de um matemático.

Em minha primeira visita ao Mestre, ele emprestou-me o livro "Diálogos sobre Física Atômica" do físico alemão Werner Heisenberg (edição portuguesa; no Brasil, o mesmo texto do Heisenberg é intitulado "A Parte e o Todo", da editora Contraponto). Este livro é uma autobiografia baseada em recordações de diálogos que Heisenberg travou com Bohr, Planck, Einstein, Dirac, Fermi, Pauli, Sommerfeld, Rutherford e muitos outros colegas seus. Este livro teve uma influência muita grande sobre mim e aumentou sobremaneira meu desejo de tornar-me um físico teórico. Lembro-me que estava ansioso por esse encontro com o Mestre, era muito tímido na época, e estava preocupado sobre o que conversaríamos. Ele me deixou tão à vontade, que, em minutos, aquela sensação pesada desapareceu. Minha vontade era visitá-lo toda semana, mas sabia que o Mestre trabalhava muito e minhas visitas poderiam atrapalhá-lo e, por outro lado, eu tinha, também, muita coisa para estudar e ler.

Falar do Bassalo é falar de suas "Crônicas da Física ". Comecei a lê-las em 1978, em artigos publicados em jornais de Belém. Sempre pedia para minha mãe comprar o jornal de domingo, pois era quando seus artigos eram publicados. E tais artigos, felizmente, foram reunidos, mais tarde, em vários livros com os títulos Crônicas da Física e Nascimentos da Física. Eles sempre estão presentes em minha mesa quando preparo minhas notas de aula. E estão, também, sempre presentes na bibliografia recomendada aos alunos, em qualquer que seja a disciplina. Eu quero registrar aqui o meu agradecimento por tamanha obra; obra que não pode e não deve faltar à biblioteca de um professor de física.

No final dos anos 70, ele deu-me um artigo seu para leitura intitulado "A Cadeia de Cognição da Física". Neste artigo ele procura mostrar que o conhecimento físico é obtido por meio de uma cadeia de cognição com dois elos fundamentais: realidade-física (observação/fenômeno) e realidade-matemática (conceito/lei/fórmula/modelo), sem, no entanto, haver um sentido preferencial no percurso desses elos. Lembro-me que cheguei a fazer alguns comentários sobre o mesmo em uma de nossas conversas. E para minha grata surpresa recebi, depois, o artigo, com dedicatória, já publicado (Ciência e Cultura, 33 (6), junho de 1981) e percebi que meu nome constava nos agradecimentos. Isto deixou-me muito honrado e também pude perceber a grandeza de sua humildade.

Em 1978 ele conseguiu arrumar-me uma bolsa de estudos no Laboratório de Matemática Aplicada e Computação no Núcleo de Ciências Geofísicas e Geológicas (NCGG) da UFPa. Fiquei no NCGG por uns três meses antes de viajar para o Rio de Janeiro. O incentivo científico que recebia do Mestre era tanto, que já me portava como um pesquisador. Nos meses que passei no NCGG cheguei a desenvolver um método de integração numérica, o qual nunca apliquei.

O Bassalo tinha e tem uma enorme preocupação no que tange à boa formação científica dos alunos. Em 1978, percebendo meu enorme interesse em estudar Matemática e Física, perguntou-me se eu gostaria de estudar fora de Belém. Disse-lhe que sim. E ele falou-me da possibilidade de me transferir para outra instituição. E assim aconteceu. Em janeiro de 1979 eu já era aluno do Instituto de Matemática da Universidade Federal do Rio de Janeiro (UFRJ). Lembro-me que naquele verão cursei uma disciplina (de nivelamento para mestrado) sobre equações diferenciais ordinárias, com a excelente professora Maria Isabel Camacho. Tive um rendimento excelente na disciplina, embora ainda estudante de graduação. Acho que dei muita sorte e isto ajudou a justificar minha transferência. Pelo que soube, minha transferência foi conseguida com a ajuda do professor Guilherme Maurício Souza Marcos de La Penha, brilhante matemático paraense, que chegou a ser diretor do Instituto de Matemática da UFRJ. O professor La Penha foi aluno e também professor do Bassalo (Veja o artigo escrito pelo Bassalo "La Penha: Gerador e Gerenciador da Ciência" , CBPF-CS-014/97). Passei algumas dificuldades longe da família e dos amigos, principalmente do amigo Bassalo. Mas meu idealismo em tornar-me um cientista falava mais alto. Formei-me no curso de Bacharel em Matemática em 1982, Mestre em Física na Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio) em 1991 e Doutor em Ciências (Física) em 1997 no Instituto de Física da UFRJ. De 1983 a 1990 fui professor horista no Departamento de Matemática da PUC-Rio. Em 1991 fui efetivado no Departamento de Física da UFJF, onde sou professor até hoje. Tenho uma bolsa de Produtividade em Pesquisa do Conselho Nacional de Pesquisas (CNPq) desde 2003, já orientei várias Monografias de Conclusão de Curso de Bacharelado, Dissertações de Mestrado e uma Tese de

Doutorado.

Em 1997 ele esteve em Juiz de Fora como convidado para a IX Semana de Física do Departamento de Física da UFJF. Ele ministrou alguns cursos sobre História da Física. Foram dois dias de muita alegria para mim. Na noite em que ele ficou aqui, em Juiz de Fora, caminhamos pelo centro da cidade e conversamos sobre muitas coisas, principalmente sobre Física e nossa querida Belém. Foi um sonho realizado. No ano seguinte, a convite seu, estive em Belém e ministrei dois seminários na UFPa. Voltar à minha universidade de origem, já com o título de doutor em física, e conversar com os alunos e professores sobre meu trabalho e minha trajetória, foi muito gratificante. Também foi um sonho realizado.

Quero deixar aqui registrada minha eterna gratidão para com o Mestre Bassalo, não só pelo grande incentivo dado a mim no início de minha carreira, mas por todo o incentivo dado a todos ao longo de todos esses anos. Sua pessoa humana, seu trabalho de divulgação científica, seu trabalho científico e sua luta incessante como gerador e gerenciador da ciência constituem um grande incentivo para todos aqueles que amam a ciência. Você, meu amigo, deu muito mais do que recebeu. Você tem meu reconhecimento por todo o sempre. Espero que o Pará saiba lhe dar todo o valor que você merece. Você, amigo Bassalo, é um tesouro paraense vivo. O meu muito obrigado.