

**Modern Topics in Mathematical,
Quantum and Statistical Physics:
Proceedings of the 2025 CIF-UnB conferences**



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(EDITORS)

**Modern Topics in Mathematical,
Quantum and Statistical Physics:
Proceedings of the 2025 CIF-UnB conferences**



2026

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1ª Edição

Direção editorial: Victor Pereira Marinho e José Roberto Marinho

Capa: Alexandre Dodonov

Projeto gráfico e diagramação: Alexandre Dodonov

Edição revisada segundo o Novo Acordo Ortográfico da Língua Portuguesa

Dados Internacionais de Catalogação na publicação (CIP)
(Câmara Brasileira do Livro, SP, Brasil)

Modern topics in mathematical, quantum and statistical physics: proceedings of the 2025 CIF-UnB conferences / Alexandre Dodonov; Aleksandr Pinzul, Ismael Segundo da Silva Carrasco, Carolina Matt'E Gregory, Arsen Melikyan (editors). – São Paulo: LF Editorial, 2026.

Vários colaboradores.
Bibliografia
ISBN 978-65-5563-767-0

1. Congressos 2. Estatísticas 3. Eventos 4. Física matemática 5. Física quântica I. Dodonov, Alexandre. II. Pinzul, Aleksandr. III. Carrasco, Ismael Segundo da Silva. IV. Gregory, Carolina Matt'E. V. Melikyan, Arsen.

26-356770.0

CDD-530.7

Índices para catálogo sistemático:
1. Física: Estudo e ensino 530.7

Maria Alice Ferreira - Bibliotecária - CRB-8/7964

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Aos infratores aplicam-se as sanções previstas nos artigos 102, 104, 106 e 107 da Lei Nº 9.610, de 19 de fevereiro de 1998



EDITORIAL

LF Editorial

www.livrariadafisica.com.br

www.lfeditorial.com.br

(11) 2648-6666 | Loja do Instituto de Física da USP

(11) 3936-3413 | Editora

Preface

This volume contains the proceedings of two major events organized in 2025 by the International Center of Physics (ICP)—also known by its Portuguese acronym CIF, from *Centro Internacional de Física*: the Fourteenth School of Physics Roberto A. Salmeron (EFRAS-14, from *Escola de Física Roberto A. Salmeron*) and the Second ICP Workshop on Quantum and Statistical Physics (QSP-2).

The opening chapter, written by the editors, provides a brief overview and historical context of these events. The remainder of the volume is divided into two parts. Part I comprises chapters contributed by lecturers of EFRAS-14: João Carlos Alves Barata, James Sparks, and Pietro Benetti Genolini.

Part II includes chapters authored by invited and contributed speakers of QSP-2 and their coauthors: Alexandre Dodonov, Pedro Ventura Paraguassú, Nathann Teixeira Rodrigues, Ismael S. S. Carrasco, Márcio Sampaio Gomes Filho, and Frank E. S. Steinhoff. Chapters based on contributed talks were peer-reviewed by the editors.

Alexandre Dodonov, Aleksandr Pinzul, Ismael S. S. Carrasco,
Carolina Matté Gregory and Arsen Melikyan

Brasília – DF – Brazil
April 28, 2026

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Chapter 1

History of EFRAS and QSP at University of Brasília

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1.1 Introduction

The International Center of Physics (ICP)—better known as CIF, an acronym for the Portuguese *Centro Internacional de Física*—is a scientific institution within the Institute of Physics at the University of Brasília. Its main goals are to promote the dissemination of physics through the organization of seminars, colloquia, and other scientific events, as well as to foster academic mobility among students, researchers, and faculty members across all areas of Physics.

The Center has existed since the late 1980s under the name *International Center for Condensed Matter Physics* (ICOMP). In 2023, it underwent major structural changes and was reestablished as CIF.

One of CIF's most traditional events is the School of Physics Roberto A. Salmeron (EFRAS, from the Portuguese *Escola de Física Roberto A. Salmeron*), established in 2012. A brief history of EFRAS, with emphasis on its most recent edition in 2025, is presented in Section 1.2.

Among its scientific meetings, CIF launched in 2024 the series *ICP Workshops on Quantum and Statistical Physics* (QSP). The history of the two workshops held so far is described in Section 1.3.

Further information about CIF and its activities can be found at <http://cif.unb.br> and on the CIF YouTube channel <https://www.youtube.com/@cifunb>.

1.2 School of Physics Roberto A. Salmeron

“Escola de Física Roberto A. Salmeron” (EFRAS) is a national one-week school aimed at graduate and advanced undergraduate students in Physics and related areas. Its program includes mini-courses on current topics in Physics, hands-on workshops and demonstrations, plenary lectures, panel discussions, contributed talks, and poster presentations.

The name EFRAS pays tribute to one of the founding professors of the Institute of Physics at the University of Brasília (IF-UnB), Prof. Roberto A. Salmeron (São Paulo, June 16, 1922 – Paris, June 17, 2020). He was a prominent physicist in the field of elementary particles and worked both at IF-UnB and at CNRS (France), where he later retired.

EFRAS has been held annually since 2012. One of its main features is the selection of one or two central topics for each edition, thereby complementing the education of undergraduate and graduate students. The past and planned editions are:

1. Quantum Mechanics: Foundations and Applications (2012)
2. Fundamental Interactions (2013)
3. Critical Phenomena and Phase Transitions (2014)
4. New Frontiers in Materials Science (2015)
5. Advances in Light–Matter Interaction: Temperature, Dissipation, and Entanglement (2016)
6. Physics Education (2017)
7. New Trends in Quantum Field Theory (2018)
8. Machine Learning in Physics and Exact Sciences (2019)
9. Quantum Mechanics: Foundations and New Trends (2020)
10. Gravity: New Trends and Perspectives (2021)
11. Physics and Its Teaching (2022)
12. Scientific and Academic Journey in Physics, Celebrating 50 Years of the Undergraduate Physics Program at UnB (2023)
13. Hands-on Activities in Quantum Technologies and Artificial Intelligence (2024)
14. Modern Topics in Mathematical Physics (2025)
15. Statistical Mechanics, Quantum Physics, and Physics Education (2026, planned)

1.2.1 EFRAS-14

The XIV School of Physics Roberto A. Salmeron (EFRAS-14) took place from September 8 to 12, 2025. It was organized by professors of the Institute of Physics at UnB: Alexandre Dodonov, Aleksandr Pinzul, Arsen Melikyan, and Carolina Matté Gregory. The official poster and a photograph of the organizers are shown in Fig. 1.1.



Figure 1.1: Official poster (left) and photograph of the organizers (right) of EFRAS-14.

The school featured the following mini-courses, each consisting of three or four lectures:

1. Operator Algebras and the Formulation of Quantum Field Theory, by Prof. João Carlos Alves Barata (USP)
2. Introduction to Black Hole Thermodynamics, by Prof. Pietro Benetti Genolini (Université de Genève)
3. Knots and Quantum Mechanics, by Prof. Dmitry Melnikov (IIP-UFRN)
4. Introduction to Integrable Models, by Prof. José Francisco Gomes (IFT–UNESP)

The program also included a poster session, contributed talks by graduate students, and two plenary lectures:

- Localization on Supergravity, by Prof. James Sparks (University of Oxford)

- Graphs and Quantum: A Valuable Interplay, by Prof. Marcelo Terra Cunha (IMECC–UNICAMP)

EFRAS-14 was attended by approximately 60 participants. The official photograph of the event is shown in Fig. 1.2. All talks are available on the CIF YouTube channel: <https://www.youtube.com/@cifunb/playlists>. Additional information and presentation slides can be found at <http://www.cif.unb.br/en/efras/190-historico-de-coloquios/94-14efras>.

We acknowledge that the organization of the school was made possible thanks to the financial support of FINATEC–UnB, the Sociedade Brasileira de Física (SBF), CIF, and the Institute of Physics at UnB.



Figure 1.2: Official photograph of the participants of EFRAS-14.

1.3 ICP Workshops on Quantum and Statistical Physics

The first ICP Workshop on Quantum and Statistical Physics (QSP-1) took place from April 24 to 26, 2024, with the aim of disseminating and discussing recent advances in all areas of Quantum and Statistical Physics. These themes were chosen in view of the large number of researchers working in these and related areas at the University of Brasília.

QSP-1, whose poster is shown in Fig. 1.3 (left), was organized by Alexandre Dodonov and Ismael Segundo da Silva Carrasco as co-chairs, with Alexandre Cavalheiro Dias and Mariana Malard Sales Andrade serving also composing the organizing committee. The program included nine poster presentations and the following invited talks:

- Alexandre Cavalheiro Dias (UnB)
- André Avelino Pasa (UFSC)
- Demétrio Antônio da Silva Filho (UnB)
- Eduardo Miranda (UNICAMP)

- Fábio David Alves Araújo Reis (UFF)
- Fernando Albuquerque de Oliveira (UnB)
- Filippo Giovanni Ghiglieno (UFSCar)
- Halyne Silva Borges (Instituto Federal do Triângulo Mineiro)
- Helena de Souza Bragança Rocha (UnB)
- Ismael Segundo da Silva Carrasco (UnB)
- Jorlândio Francisco Felix (UnB)
- Lauro Tomio (IFT–UNESP)
- Márcio Santos (UFSC)
- Maria Carolina de Oliveira Aguiar (UFMG)
- Mariana Malard Sales Andrade (UnB)
- Nail Khusnutdinov (UFABC)
- Nathann Teixeira Rodrigues (UFF)
- Tarcísio Marciano da Rocha Filho (UnB)
- Viktor Dodonov (UnB)

The second edition (QSP-2) took place from July 2 to 4, 2025, and its poster is shown in Fig. 1.3 (right). Alexandre Dodonov and Ismael Segundo da Silva Carrasco again served as co-chairs, with Caio Cesar Holanda Ribeiro and Nathann Teixeira Rodrigues (UnB) completing the organizing committee. QSP-2 had broader participation, with 13 poster presentations and 23 invited and contributed talks delivered by:

- Alexandre Dodonov (UnB)
- Alexandre Martins de Souza (CBPF)
- Caio Cesar Holanda Ribeiro (UnB)
- Carlos Eduardo Fiore dos Santos (USP)
- Eduardo Inácio Duzzioni (UFSC)
- Edwin Edgar Mozo Luis (UFF)
- Fábio David Alves Araújo Reis (UFF)
- Fernando Albuquerque de Oliveira (UnB)
- Frank Eduardo da Silva Steinhoff (UFMT)
- Giuliano Pavan Ribeiro (UFSCar)

1° ICP Workshop on Quantum and Statistical Physics
24-26 April, 2024
 International Center of Physics
 Institute of Physics - University of Brasilia

INVITED SPEAKERS
 Alexandre Cavalheiro Dias (UnB)
 Alexandre Dodonov (UnB)
 André Avelino Pasa (UFSC)
 Demétrio Antônio da Silva Filho (UnB)
 Eduardo Miranda (UNICAMP)
 Fábio David Alves Aarão Reis (UFF)
 Fernando Albuquerque de Oliveira (UnB,UFF)
 Filippo Giovanni Ghiglieno (UFSCar, UnB)
 Halyne Silva Borges (IFTM)
 Helena de Souza Bragança Rocha (UnB)
 Ismael Segundo da Silva Carrasco (UnB)
 Jorlandio Francisco Felix (UnB)
 Lauro Tomio (IFT-UNESP, UnB)
 Márcio Santos (UFSC)
 Maria Carolina de Oliveira Aguiar (UFMG)
 Mariana Malard Sales Andrade (UnB)
 Nail Khusnutdinov (UFABC)
 Nathann Rodrigues (UFF)
 Tarcísio Marciano da Rocha Filho (UnB)
 Viktor Dodonov (UnB)

ORGANIZING COMMITTEE
 Ismael Segundo da Silva Carrasco (UnB)
 Alexandre Dodonov (UnB)
 Mariana Malard Sales Andrade (UnB)
 Alexandre Cavalheiro Dias (UnB)

Invited talks
Contributed talks
Poster presentations

Registration deadline
14 April

Support: fapdf SBF CIF UnB

2° ICP Workshop on Quantum and Statistical Physics
July 2 - 4, 2025
 International Center of Physics
 Institute of Physics - University of Brasilia

INVITED SPEAKERS
 Alexandre Dodonov (UnB)
 Alexandre Martins de Souza (CBPF)
 Caio Cesar Holanda Ribeiro (UnB)
 Carlos Eduardo Fiore dos Santos (USP)
 Eduardo Inácio Duzzioni (UFSC)
 Edwin Edgar Mozo Luis (UFF)
 Fábio David Alves Aarão Reis (UFF)
 Fernando Albuquerque de Oliveira (UnB,UFF)
 Frank Eduardo Da Silva Steinhoff (UFMT)
 Giuliano Pavan Ribeiro (UFSCar)
 Ismael Segundo da Silva Carrasco (UnB)
 José Soares de Andrade Júnior (UFC)
 Nathann Teixeira Rodrigues (UnB)
 Rafael Besse (UnB)
 Tarcísio Marciano da Rocha Filho (UnB)
 Tiago de Sousa Araújo Cassiano (UnB)
 Tiago José Oliveira (UFV)

ORGANIZING COMMITTEE
 Ismael Segundo da Silva Carrasco (UnB)
 Alexandre Dodonov (UnB)
 Nathann Rodrigues (UnB)
 Caio Cesar Holanda Ribeiro (UnB)

Deadline for submission of Contributed talks and Posters: June 1, 2025

Information and registration: www.cif.unb.br/qsp2

Support: IF FINATEC SBF CIF fapdf

Figure 1.3: Posters of the QSP workshops.

- Ismael Segundo da Silva Carrasco (UnB)
- José Soares de Andrade Júnior (UFC)
- Nathann Teixeira Rodrigues (UnB)
- Rafael Besse (UnB)
- Tarcísio Marciano da Rocha Filho (UnB)
- Tiago de Sousa Araújo Cassiano (UnB)
- Tiago José Oliveira (UFV)
- Diego Paiva Pires (UFMA)
- Márcio Sampaio Gomes Filho (UFABC)
- Gustavo Forão (USP)
- Pedro Ventura Paraguassú (PUC-Rio)
- Carlos Maciel de Oliveira Bastos (UnB)
- Jonas Pedro Pereira (UnB)

Figure 1.4 shows the official photographs of participants of the two editions. The third edition is planned as a satellite event of the EFRAS-15 in August 2026. Additional information can be found at <http://www.cif.unb.br/en/destaques/92-2wqsp>, and all presentations are available on the CIF YouTube channel.



Figure 1.4: Official photographs of the participants of the first and second ICP Workshops on Quantum and Statistical Physics (2024 and 2025).

Part I

Contributions of the invited lecturers of EFRAS-14

